

# Quantum Foundations

## Lecture 24

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HSC112

# Announcements



- ◉ Assignments: Final Version due May 2.
- ◉ Homework 5 due May 25.
- ◉ Final Exam to be issued later this week.

# Counterintuitive Features of dBB Trajectories

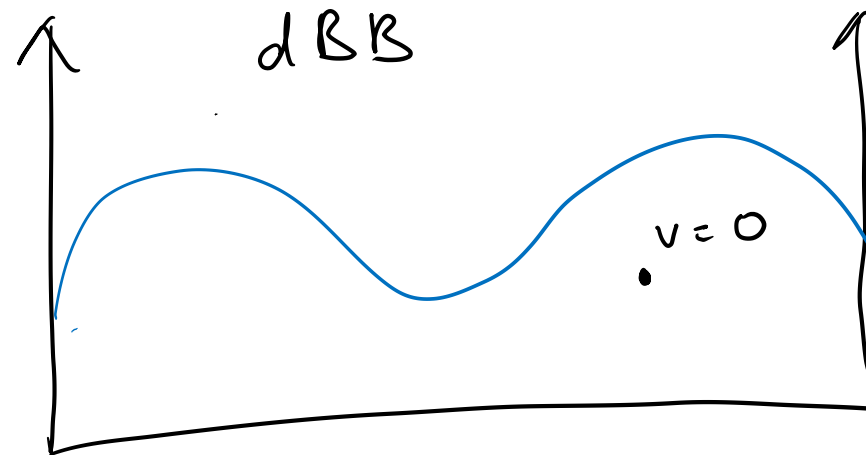
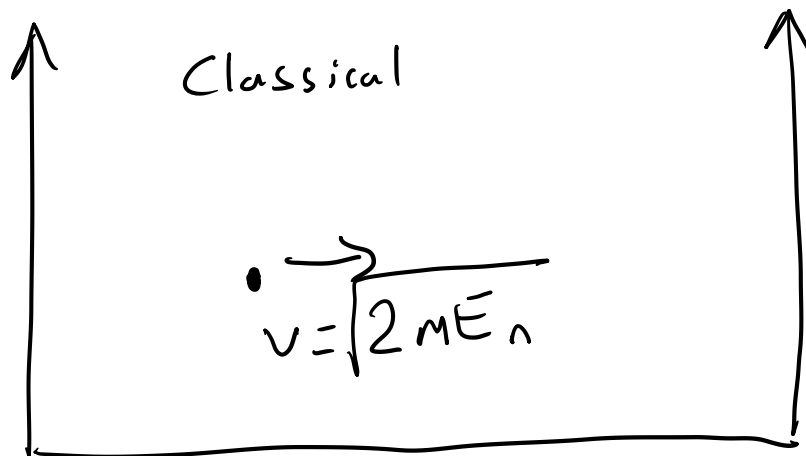
- ◉ dBB trajectories display several features that violate classical intuitions about particle trajectories.
- ◉ It is important to note that, if decoherence occurs in an environmental basis that is localized in position, dBB trajectories of the system will approximately follow classical trajectories.
- ◉ dBB doesn't owe us anything more than that. So long as:
  - ◉ It reproduces the predictions of quantum theory in measurements.
  - ◉ Macroscopic systems typically have approximately classical trajectories.then the theory saves the phenomena.
- ◉ Since quantum and classical predictions are different, dBB trajectories *must* differ from classical ones in some situations.
- ◉ The question is only if they are weirder than absolutely necessary to reproduce quantum theory, and whether that is a bad thing.

# Real Stationary States

- Consider a stationary state:  $\psi(\mathbf{q}, t) = \psi_n(\mathbf{q})e^{-iE_n t/\hbar}$
- The current is:  $\vec{J}_k(\mathbf{q}) = \frac{\hbar}{m_k} \text{Im}(\psi_n^* \vec{\nabla}_k \psi_n)(\mathbf{q})$ , i.e. is independent of  $t$ .
- However, if  $\psi_n(\mathbf{q})$  is also a real valued function then:

$$\vec{J}_k(\mathbf{q}) = \frac{\hbar}{2im_k} (\psi_n^* \vec{\nabla}_k \psi_n - \psi_n \vec{\nabla}_k \psi_n^*)(\mathbf{q}) = 0$$

- The particles are also stationary, e.g. particle in an infinite well, hydrogen atom eigenstates.



# The No-Crossing Rule

- ◉ In classical mechanics, phase space trajectories do not cross (except at singularities) because equations are 2<sup>nd</sup> order and so  $(\mathbf{q}, \mathbf{p})$  contains enough data to specify a unique trajectory.
- ◉ In dBB the guidance equations is 1<sup>st</sup> order and there is no back action on the quantum state from the configuration space point:
- ◉  $[\psi(\mathbf{q}, t_0), \mathbf{Q}(t_0)]$  and  $[\psi(\mathbf{q}, t_0), \mathbf{Q}'(t_0)]$  specify unique trajectories.
- ◉ Trajectories associated with the same wavefunction evolution cannot cross in *configuration space*.
- ◉ This is responsible for almost all the weird features of dBB trajectories.
- ◉ Note: with decoherence into localized environment states:

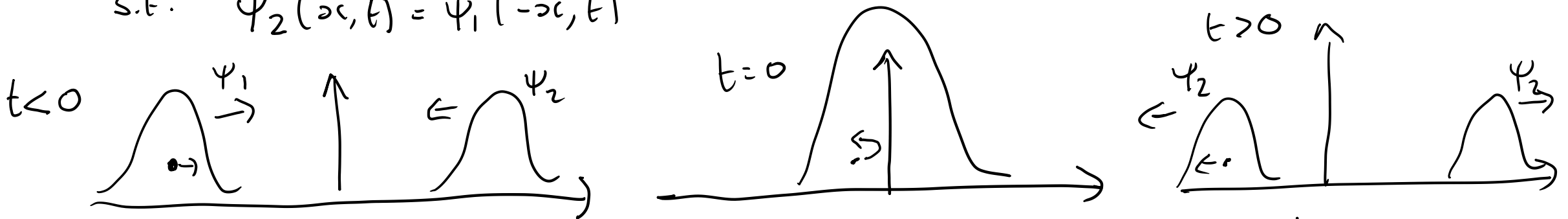
$$\alpha\psi_0(\mathbf{q}_S)\Phi_0(\mathbf{q}_E) + \beta\psi_1(\mathbf{q}_S)\Phi_1(\mathbf{q}_E)$$

trajectories can cross in the *system* configuration space because  $\mathbf{Q}_E$  is necessarily different in the two branches. This is needed to recover classical trajectories.

# Empty Waves Steal the Particle

⊙ Consider a superposition of 2 wavepackets  $\psi(x,t) = \frac{1}{\sqrt{2}}(\psi_1(x,t) + \psi_2(x,t))$

s.t.  $\psi_2(x,t) = \psi_1(-x,t)$



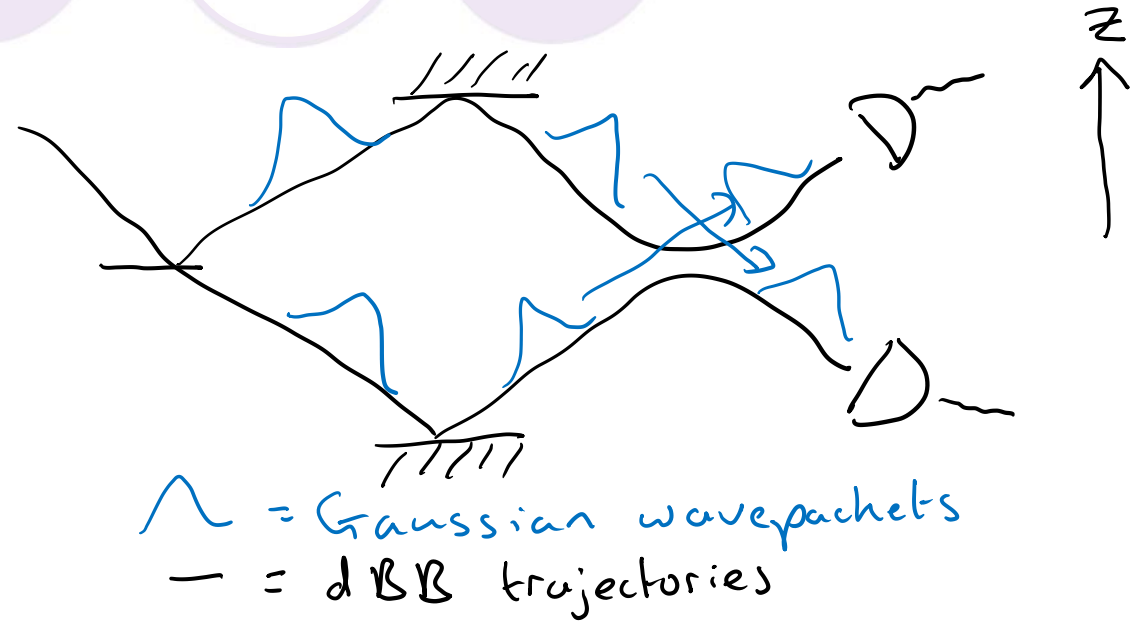
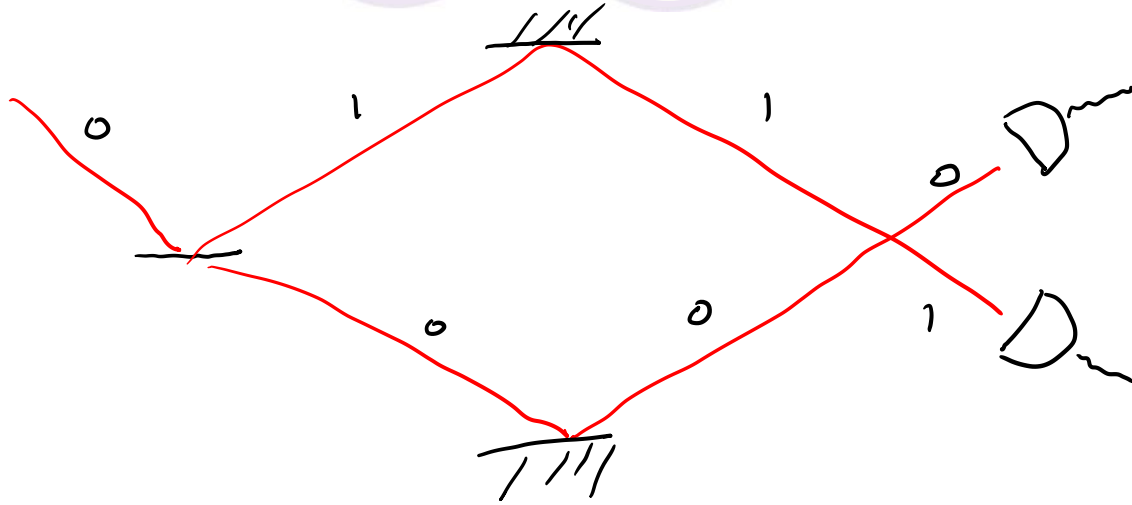
⊙ The dBB particle will switch wavepackets during the interference due to the no-crossing rule: empty wave steals the particle.

⊙ Here we can explicitly see that  $J(0,t) = 0$  for all times b.c.  $J(x,t)$  is an odd function of  $x$

$$J(x,t) = [J_{11}(x,t) + J_{22}(x,t) + J_{12}(x,t) + J_{21}(x,t)] = [J_{11}(x,t) - J_{11}(-x,t) + J_{12}(x,t) - J_{12}(-x,t)]$$

where  $J_{km}(x,t) = \frac{\hbar}{m} \text{Im} \left( \psi_k^* \frac{\partial \psi_m}{\partial x} \right)$

# Consequences for Mach-Zehnder



⊙ If we remove the final beamsplitter from a Mach-Zehnder, many physicists would be inclined to say that detector 0 firing is evidence that the particle took path 0.

⊙ The opposite happens in dBB. No crossing  $\Rightarrow$  the empty wave steals the particle

Detector 0 clicks  $\Rightarrow$  The particle travelled along path 1.

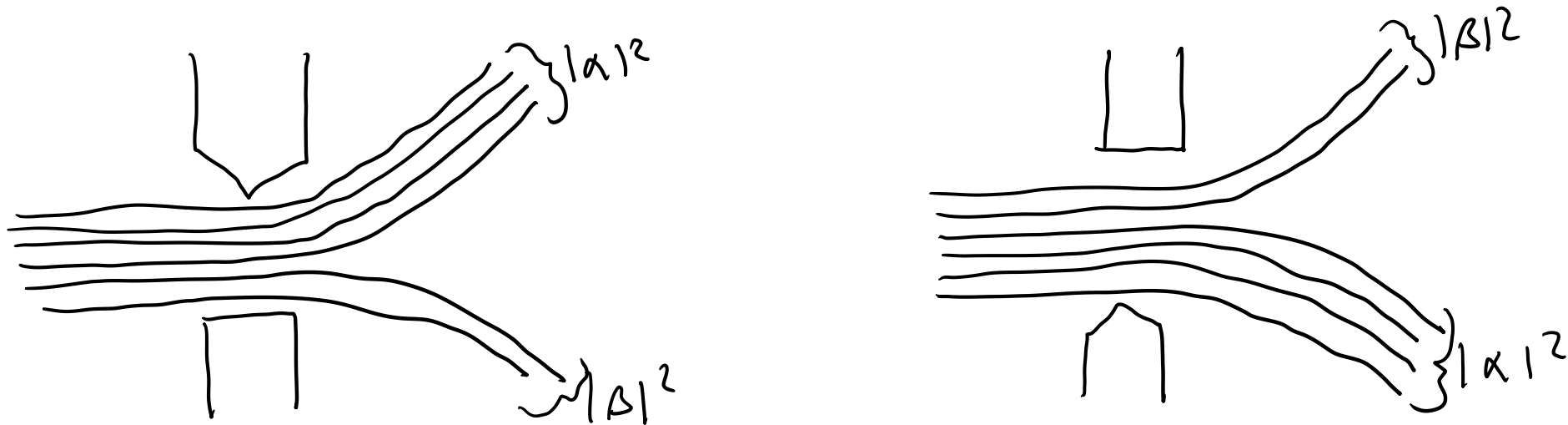
# Surreal Trajectories

- ⊙ To make things more dramatic, we can place a localized spin- $\frac{1}{2}$  system in path 0 initially placed in  $|\uparrow\rangle$  and have the interaction
$$\psi_0(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\uparrow\rangle \rightarrow \psi_0(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\downarrow\rangle$$
$$\psi_1(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\uparrow\rangle \rightarrow \psi_0(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\uparrow\rangle$$
- ⊙ Because  $\vec{Q}_P$  is unaffected by this interaction the current will still be zero in the interference region.
- ⊙ If we detect the particle at detector 0 and subsequently measure the spin, we will find it spin down.
- ⊙ You might want to take this as evidence that the particle travelled along path 0, but the dBB trajectory is path 1.
- ⊙ This can happen because the spin flip does not lead to decoherence that is localized in position.



# KS Contextuality in de Broglie-Bohm

- ◉ KS Contextuality occurs in dBB because the outcome of an experiment depends on  $\mathbf{Q}_S, \psi(\mathbf{q}_S), \mathbf{Q}_E, \Phi_R(\mathbf{q}_E)$ , and the interaction Hamiltonian, and not on  $\mathbf{Q}_S, \psi(\mathbf{q}_S)$  alone.
- ◉ Example: Stern-Gerlach measurement of  $\psi(\mathbf{q}_S) \otimes (\alpha|\uparrow\rangle + \beta|\downarrow\rangle)$



- ◉ No-crossing rule  $\Rightarrow$  some  $\mathbf{q}_S$  switch between giving spin up and spin down outcomes when we rotate the magnets by  $180^\circ$ .
- ◉ This is *more contextual* than implied by KS, which can only be proved in  $d \geq 3$ .

# Underdetermination

- ◉ The only property of the guidance equation needed to reproduce the quantum predictions is equivariance:  $\rho(\mathbf{Q}, t_0) = |\psi(\mathbf{Q}, t_0)|^2 \rightarrow \rho(\mathbf{Q}, t) = |\psi(\mathbf{Q}, t)|^2$  for all other  $t$ .
- ◉ Any other equivariant dynamics would do just as well, e.g. (E. Deotto, G. Ghiradri, Found.Phys. 28:1-30 (1998))

$$\frac{d\vec{Q}_k}{dt} = \frac{\hbar}{m_k} \frac{\text{Im}(\psi^* \vec{\nabla}_k \psi)}{\psi^* \psi}(\mathbf{Q}) + \frac{\vec{J}_0(\vec{Q}_k)}{\psi^* \psi(\mathbf{Q})} \quad \text{with} \quad \vec{\nabla} \cdot \vec{J}_0 = 0$$

- ◉ Further:
  - ◉ We could add more primitive variables, e.g. spin with stochastic dynamics.
  - ◉ We could use a different basis, e.g. momentum.
  - ◉ We could even use a POVM, e.g. coherent states.

# The Equilibrium Hypothesis

- ◉ The quantum state plays two roles in dBB:
  - ◉ Dynamical: it appears in the guidance equation.
  - ◉ Probabilistic: We set  $\rho(\mathbf{q}, t_0) = |\psi(\mathbf{q}, t_0)|^2$  as a postulate – quantum equilibrium hypothesis.
- ◉ These two roles are independent, we could set the probability density to anything else.
- ◉ There is evidence (analytic and numerical) that, under suitable coarse-graining, other densities relax to  $|\psi(\mathbf{q}, t_0)|^2$  over time, akin equilibration in statistical mechanics.
- ◉ Valentini posits that nonequilibrium states may have occurred in the early universe.
  - ◉ This would resolve some of the underdetermination, but leads to the bold hypothesis that superluminal signaling occurs in our universe.

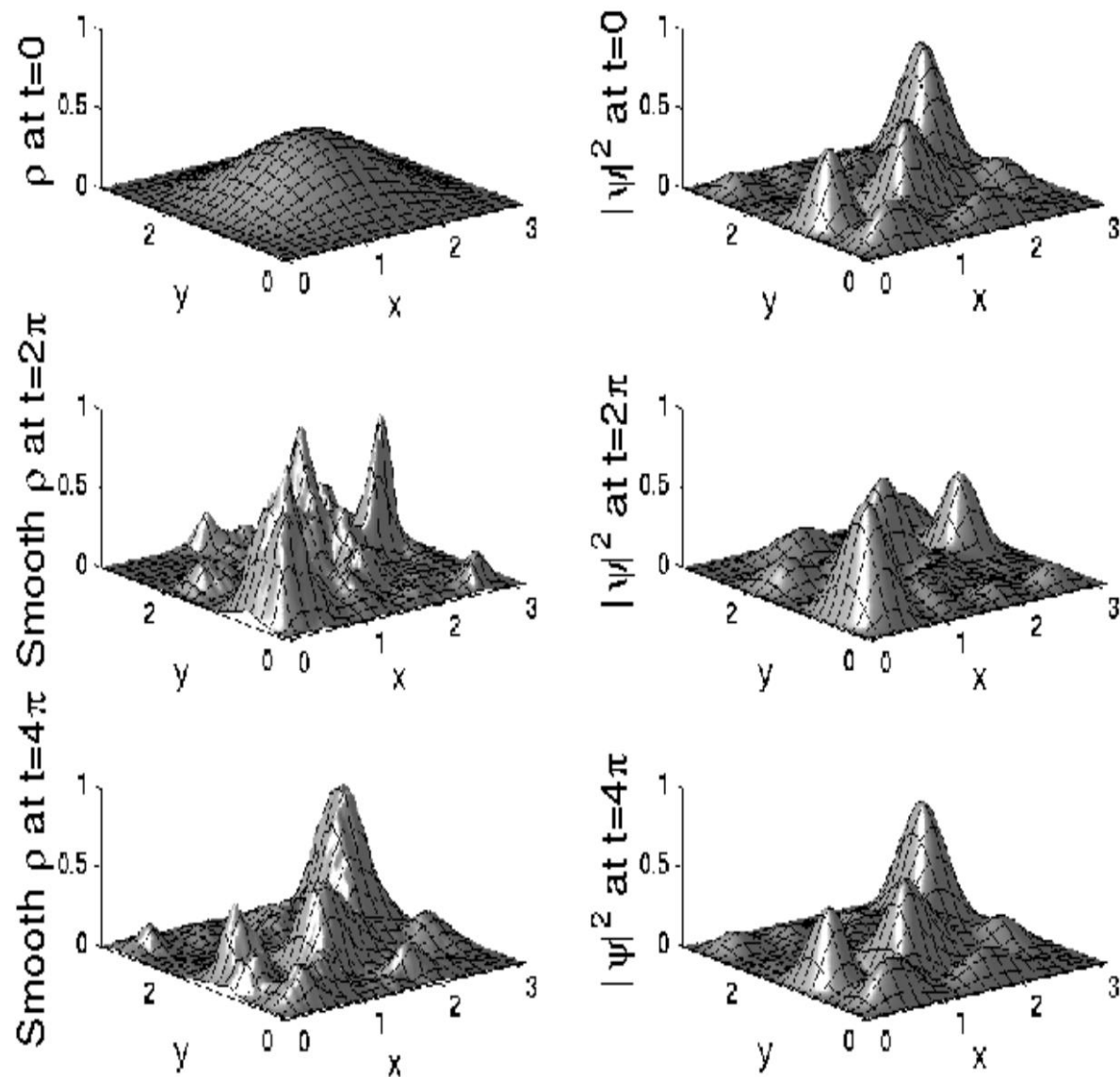
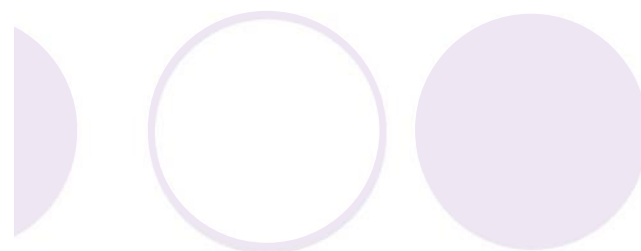


Figure 7: Smoothed  $\tilde{\rho}$ , compared with  $|\psi|^2$ , at times  $t = 0, 2\pi$  and  $4\pi$ . While  $|\psi|^2$  recurs to its initial value, the smoothed  $\tilde{\rho}$  shows a remarkable evolution towards equilibrium.



A. Valentini, H. Westman, Proc. Roy. Soc. Lond. A 461:253-272 (2005)

# Relativistic Generalizations of de Broglie-Bohm

- ◉ Generalizations of dBB to relativistic QFT have been developed. There are various versions:
  - ◉ Particle ontology vs. field ontology.
  - ◉ An ontology with particle occupation numbers requires stochastic dynamics.
  - ◉ A mixture of the two, e.g. particles for fermions and fields for bosons, only fermions and treat bosons like spin or vice versa.
- ◉ These theories cannot be *fundamentally* Lorentz invariant:
  - ◉ Under the equilibrium hypothesis, the operational predictions are Lorentz invariant.
  - ◉ But the theories violate parameter independence – there is superluminal signaling at the ontic level.
  - ◉ These effects would become observable in nonequilibrium states.

# Summary



- ◉ dBB provides a coherent ontology with straightforward equations of motion, and saves the phenomena.
- ◉ Trajectories do not obey common intuitions, but arguably this must be so if they are to reproduce quantum phenomena.
- ◉ dBB arguably *more weird* than an interpretation has to be, i.e.
  - ◉ Contextual in ways that QM does not require.
  - ◉ Nonlocal in experiments that have local explanations.
  - ◉  $\psi$ -ontic even for experiments that have good  $\psi$ -epistemic explanations.
- ◉ Taking the equilibrium hypothesis as a postulate is a fine tuning and leads to underdetermination of the theory.
- ◉ Viewing it as emergent removes the underdetermination, but leads to the bold hypothesis that we should expect to see explicit Lorentz violation, i.e. signaling, somewhere in nature.
- ◉ dBB is a good counterexample to many exaggerated claims about QM.

# 10.iii) Spontaneous Collapse Theories

- ◉ In orthodox quantum theory, the system evolves according to the Schrödinger equation, except if there is a “measurement” when the state randomly collapses.
- ◉ The idea of spontaneous collapse theories is to modify the Schrödinger dynamics so that collapses are included as a natural dynamical process.
  - ◉ **Microscopic** systems obey Schrödinger dynamics to a good approximation.
  - ◉ **Macroscopic** systems quickly collapse to localized states with high probability.
- ◉ This means that the predictions of a collapse theory will differ from those of standard quantum theory. They can in principle be empirically refuted.

# The Girhardi-Rimini-Weber (GRW) Model

- ◉ Consider a single particle in one dimension for simplicity.
- ◉ Most of the time, the system obeys Schrödinger dynamics

$$i \frac{\partial |\psi(t)\rangle}{\partial t} = \hat{H} |\psi(t)\rangle$$

- ◉ There is a constant probability per unit time for a spontaneous localization to occur

$$\frac{dP}{dt} = \lambda$$

- ◉ This will give rise to a Poisson distributed sequence of times  $t_1, t_2, \dots$  at which localizations occur. The average waiting time will be

$$\tau = \overline{t_{n+1} - t_n} = \frac{1}{\lambda}$$

- ◉ GRW recommend  $\lambda \approx 10^{-16} \text{ s}^{-1}$  or  $\tau \approx 10^{16} \text{ s} = 3 \times 10^8 \text{ years}$ . Localizations occur extremely rarely.



# GRW Model

- When a localization occurs, the wavefunction is updated to

$$\psi(x, t) \rightarrow \psi'(x, t) = \frac{1}{p(X)} g_X(x) \psi(x, t)$$

where

$$g_X(x) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{4}}} e^{-(x-X)^2/4\sigma^2}$$

- The value of  $X$  at which the localization occurs is chosen with probability density

$$p(X) = \int_{-\infty}^{+\infty} |g_X(x) \psi(x, t)|^2 dx$$

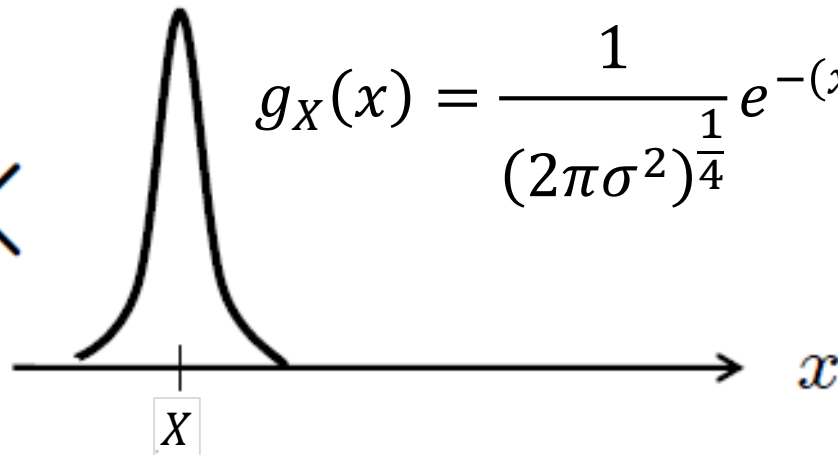
- This introduces a new parameter  $\sigma$ . GRW recommend  $\sigma \approx 10^{-7}$  m.

# GRW Model

$$\psi(x, t) = \frac{\sqrt{3}}{2} \phi_1(x) + \frac{1}{2} \phi_2(x)$$

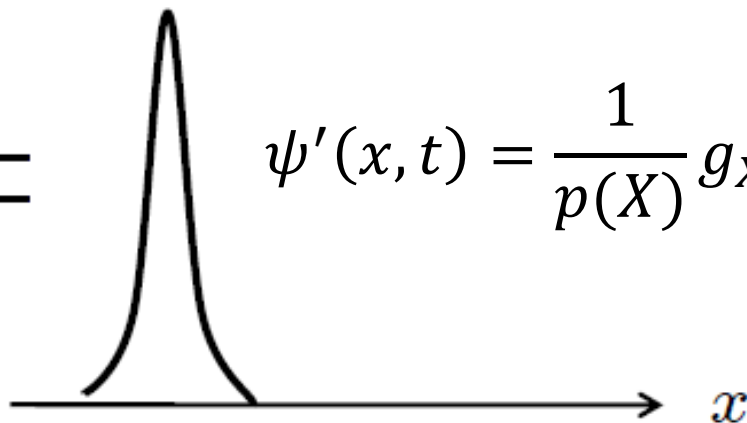


$$g_X(x) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{4}}} e^{-(x-X)^2/4\sigma^2}$$



$=$

$$\psi'(x, t) = \frac{1}{p(X)} g_X(x) \psi(x, t)$$



# GRW in terms of a POVM

- ◉ We can rewrite the spontaneous collapse in terms of a (continuous) POVM

$$E(X) = M^\dagger(X)M(X), \quad M(X) = \int_{-\infty}^{+\infty} dx \, g_X(x) |x\rangle\langle x|, \quad \int_{-\infty}^{+\infty} dX \, E(X) = I$$

- ◉ Then

$$p(X) = \langle \psi(t) | E(X) | \psi(t) \rangle, \quad |\psi'(t)\rangle = \frac{M(X) |\psi(t)\rangle}{\sqrt{p(X)}}$$

- ◉ Or, in terms of density operators

$$p(X) = \text{Tr}(E(X)\rho(t)), \quad \rho(t) \rightarrow \rho'(t) = \frac{M(X)\rho(t)M^\dagger(X)}{p(X)}$$

# GRW in terms of a POVM

- $X$  is unknown to the experimenter, so they will observe the average state update

$$\rho(t) \rightarrow \int_{-\infty}^{+\infty} dX M(X) \rho(t) M^\dagger(X')$$

- Recall that a CPT map has the form

$$\mathcal{E}(\rho) = \sum_j M^{(j)} \rho M^{(j)\dagger}$$

- The GRW map is a continuous analogue of this. The spontaneous collapse process will look like an approximate position decoherence to an experimenter.
- The same dynamics could be achieved by unitary interaction with the environment. Cannot tell GRW from decoherence via experiments.

# Multi-particle generalization

- ◉ Each particle experiences localizations at a rate  $\lambda$ .
- ◉ The total rate of localizations for  $N$  particles will be  $N\lambda$ .
- ◉ Average time between localizations is  $\tau/N$ .
- ◉ For a macroscopic system,  $N \approx 10^{23}$ , this gives

$$N\lambda \approx 10^7 \text{ s}^{-1}, \quad \frac{\tau}{N} \approx 10^{-7} \text{ s}$$

- ◉ Collapses occur very frequently. For noninteracting unentangled particles

$$\psi(x_1, x_2, \dots, x_N, t) = \psi_1(x_1, t)\psi_2(x_2, t) \cdots \psi_N(x_N, t)$$

this won't make a difference. Each particle collapses extremely rarely.

# Multi-particle generalization

- ◉ For entangled particles, it makes a big difference.
- ◉ On average, every  $\tau/N$ , one particle is selected at random (suppose it is particle 1). The whole wavefunction gets updated to

$$\psi'(x_1, x_2, \dots, x_N, t) = \frac{1}{p(X)} g_X(x_1) \psi(x_1, x_2, \dots, x_N, t)$$

$$p(X) = \int_{-\infty}^{+\infty} |g_X(x_1) \psi(x_1, x_2, \dots, x_N, t)|^2 dx_1 dx_2 \cdots dx_N$$

# Multi-particle generalization

- Suppose

$$\psi(x_1, x_2, \dots, x_N, t) = \alpha \phi_a(x_1) \phi_a(x_2) \cdots \phi_a(x_N) + \beta \phi_b(x_1) \phi_b(x_2) \cdots \phi_b(x_N)$$

where  $\phi_a(x)$  and  $\phi_b(x)$  are localized around  $x = a$  and  $x = b$  with small width compared to  $\sigma$  and  $|a - b| \gg \sigma$ .

- Then  $P(X \approx a) \approx |\alpha|^2$ ,  $P(X \approx b) \approx |\beta|^2$ . For  $X \approx a$ , the state will collapse to

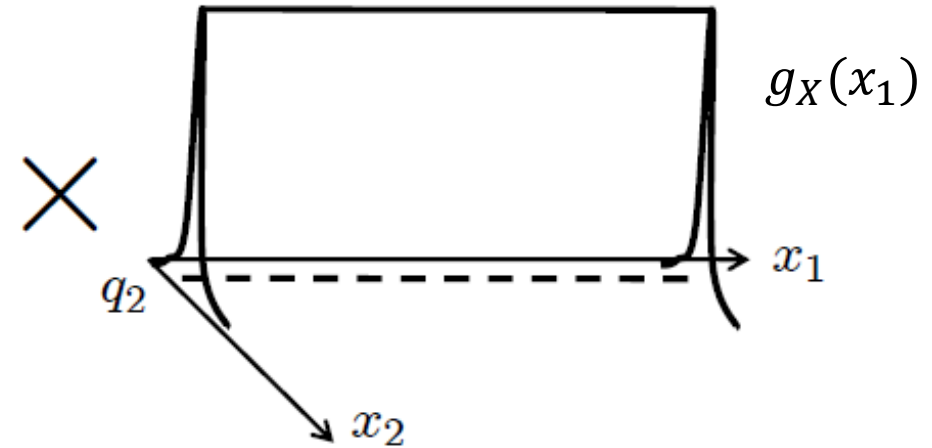
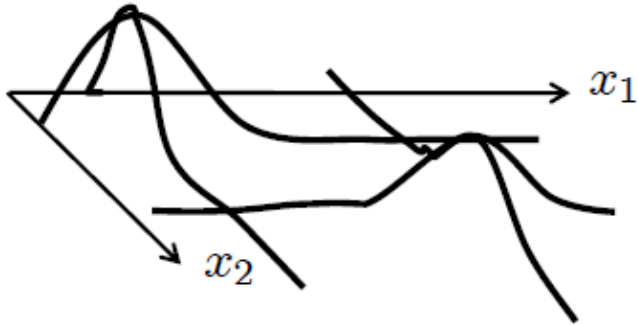
$$\psi'(x_1, x_2, \dots, x_N, t) \approx \phi_a(x_1) \phi_a(x_2) \cdots \phi_a(x_N)$$

and similarly for  $X \approx b$ .

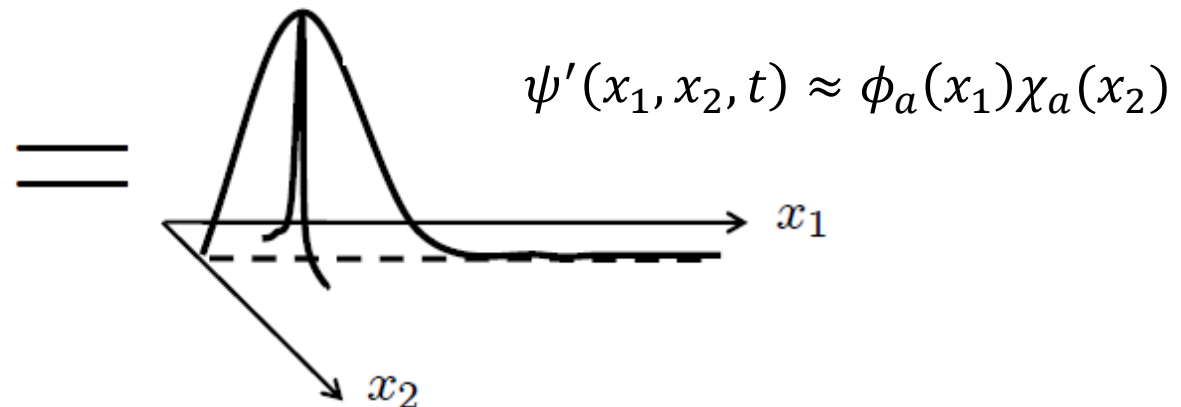
- The spontaneous collapse of a single particle localizes the entire wavefunction.

# Multi-particle generalization

$$\psi(x_1, x_2, t) = \alpha \phi_a(x_1) \chi_a(x_2) + \beta \phi_b(x_1) \phi_b(x_2)$$



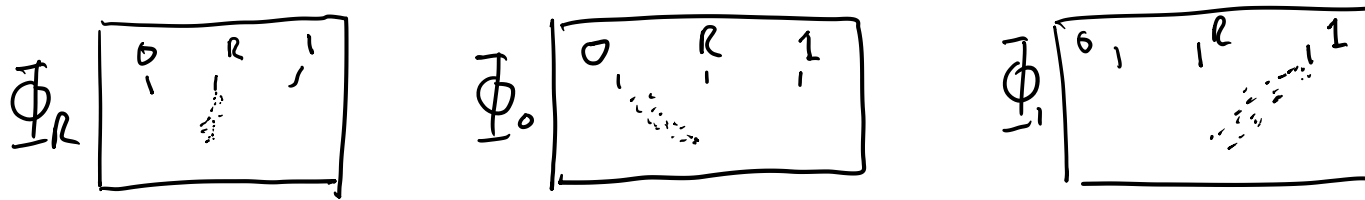
$\times$





# Measurement in GRW

- The pointer of a measuring device is made of a macroscopic number  $N \approx 10^{23}$  of particles.



- In a measurement interaction

$$[\alpha\psi_0(\mathbf{q}_S) + \beta\psi_1(\mathbf{q}_S)]\Phi_R(\mathbf{q}_E) \rightarrow \alpha\psi_0(\mathbf{q}_S)\Phi_0(\mathbf{q}_E) + \beta\psi_1(\mathbf{q}_S)\Phi_1(\mathbf{q}_E)$$

but

$$\Phi_j(\mathbf{q}_E) = \phi_j(\vec{q}_1)\phi_j(\vec{q}_2)\cdots\phi_j(\vec{q}_N)$$

so the pointer and system will collapse extremely rapidly to either

$$\psi_0(\mathbf{q}_S)\Phi_0(\mathbf{q}_E) \quad \text{or} \quad \psi_1(\mathbf{q}_S)\Phi_1(\mathbf{q}_E)$$

# Ontology and the Tails Problem

- ◉ GRW gives us wavefunctions that are approximately localized in configuration space. But they are still functions on a  $3N$  dimensional space. How is this related to what we see in 3D space?
  - ◉ In other words, does GRW have a primitive ontology of local beables like de Broglie-Bohm theory?
- ◉ The localizations are only approximate.  $g_X(x)$  is a Gaussian function with exponentially small tails that stretch to infinity. So there are still tiny components of the wavefunction that remain in superposition. Why don't we see these?
  - ◉ Note: We have to use a smooth  $g_X(x)$  to avoid dynamics that causes the wavefunction to spread extremely rapidly.

# Three Primitive Ontologies for GRW

- ⦿ Three primitive ontologies have been proposed for GRW
  1. GRWw (wavefunction ontology). The wavefunction itself is the only ontology.
    - ⦿ We have to use ideas similar to Everett/many-worlds to understand what a wavefunction means for everyday experience.
    - ⦿ The tails problem is serious here because we have no reason to believe that components of the wavefunction with small amplitude are less important.
  2. GRWm (mass density ontology)
  3. GRWf (flash ontology)

# GRWm

- ◉ We can define a mass density for particle  $j$  as

$$\rho_j(x) = m_j \int_{-\infty}^{+\infty} |\psi(x_1, x_2, \dots, x_N)|^2 dx_1 dx_2 \cdots dx_{j-1} dx_{j+1} \cdots dx_N$$

- ◉ The total mass density is then

$$\rho(x) = \sum_{j=1}^N \rho_j(x)$$

- ◉ Without spontaneous collapses, this would tend to spread out and cover all space – does not capture everyday experience.
- ◉ With GRW collapses, the mass density tends to get localized in blobs that look like classical reality.
  - ◉ There are still blobs with very small mass spread out everywhere (tails problem). Need to argue that you cannot experience or perceive things with small mass.

# GRWf



- ◉ The localization events themselves happen at specific points  $X, t$  in spacetime.
- ◉ For macroscopic systems they happen extremely frequently.
- ◉ The flash ontology proposes that the world is made of small “matter events” in spacetime, where a piece of matter appears that is localized at  $(X, t)$  for each spontaneous collapse.
- ◉ What we see are these flashes. Because they happen rapidly, it looks like continuous motion of particles.
- ◉ Flashes happen with very small probability where the wavefunction has small amplitude. Because you need several flashes in a row to perceive something, this arguably solves the tails problem.

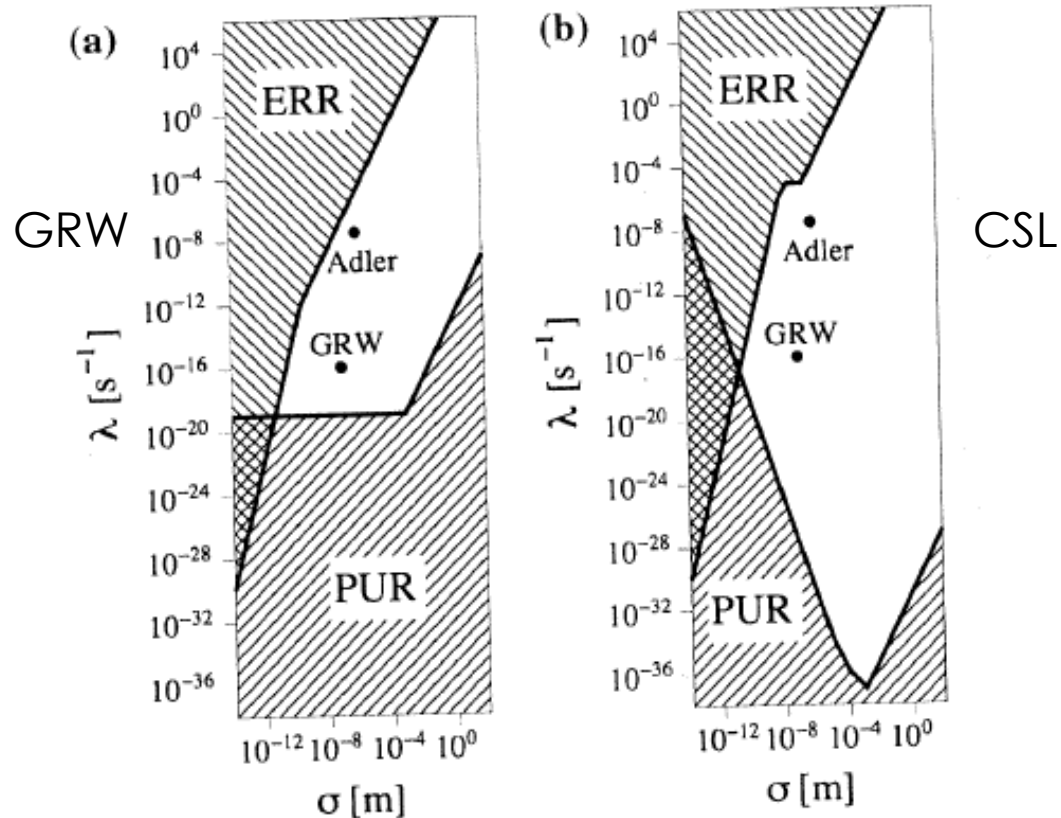
# Generalizations of GRW

- ◉ In GRW, the localizations happen at discrete times, via a dynamics that is not unified with the Schrödinger equation.
- ◉ It is possible to have a continuous time stochastic process causing the collapses, which can be unified with Schrödinger dynamics as a stochastic differential equation. This is called Continuous Spontaneous Localization (CSL).
- ◉ Just as GRW is indistinguishable from decoherence, CSL is indistinguishable from the theory of quantum continuous measurements (talk to Prof. Dressel for details).
- ◉ Some people have proposed explicit mechanisms where classical fluctuating fields cause the collapse.
  - ◉ Gravity (Penrose)
  - ◉ Integrated Information (McQueen, Chalmers)

# Empirical Tests of GRW

- ◉ Because GRW implies that there is necessarily decoherence when the system consists of enough particles, various parameter ranges for  $\lambda$  and  $\sigma$  can be ruled out empirically if we see coherence in large systems. It can be distinguished from standard quantum theory.
- ◉ We can also rule out some parameter ranges as Perceptually Unsatisfactory, e.g. if it implies that a dust particle can be in a superposition of two observably distinct positions for more than a few microseconds then we would not have a solution to the measurement problem.

# Empirical Tests of GRW



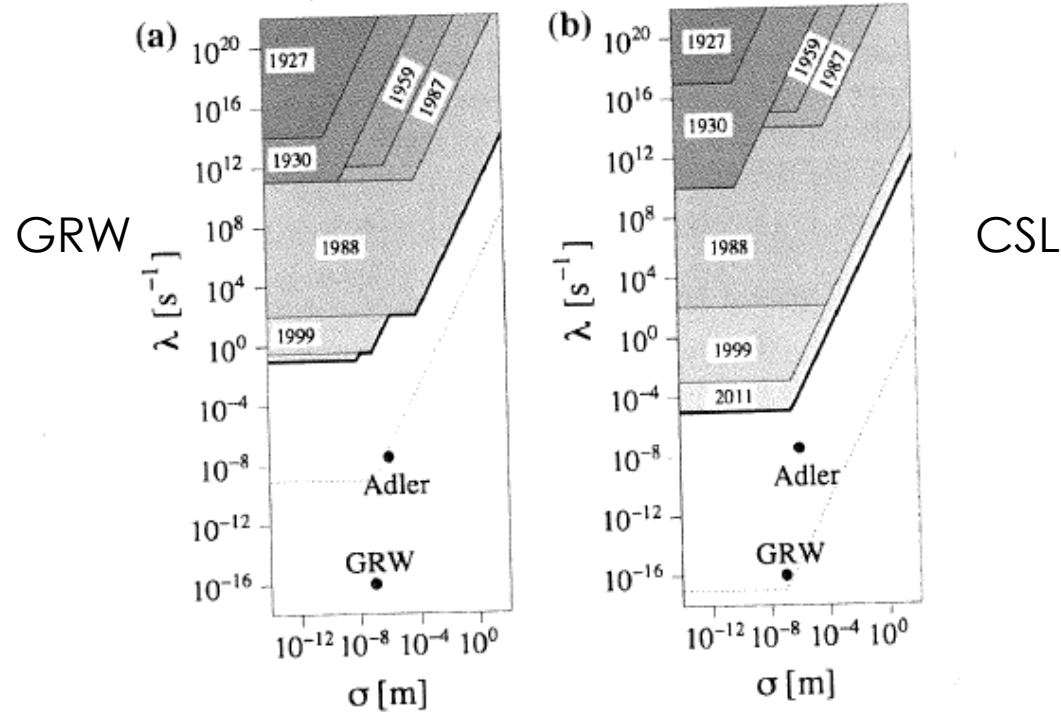
From W. Feldman, R. Tumulka, Parameter diagrams of the GRW and CSL theories of wavefunction collapse, J. Phys. A, 45:065304 (2012)

As reproduced in T. Norsen, Foundations of Quantum Mechanics, (Springer, 2017)

**Fig. 9.10** Map of parameter space, again for both GRW and CSL theories, showing now both the “Empirically Refuted Region” (ERR) and the “Perceptually Unsatisfactory Region” (PUR) as discussed in the text. From Ref. [8]. Figure © IOP Publishing. Reproduced with permission. All rights reserved. <https://doi.org/10.1088/1751-8113/45/6/065304>



# Empirical Test of GRW



From W. Feldman, R. Tumulka, Parameter diagrams of the GRW and CSL theories of wavefunction collapse, J. Phys. A, 45:065304 (2012)

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**Fig. 9.11** The “Empirically Refuted Region” (ERR) of the GRW and CSL parameter spaces has steadily advanced, in recent decades, leaving an ever-narrowing window of parameter values which are compatible both with experimental and perceptual evidence. This suggests that, within perhaps a couple of decades, we will either have direct experimental evidence in support of the spontaneous collapse models, or the models will have been ruled out as either empirically or perceptually unacceptable. From Ref. [8]. Figure © IOP Publishing. Reproduced with permission. All rights reserved. <https://doi.org/10.1088/1751-8113/45/6/065304>

# Summary



- ◉ Spontaneous collapse theories supplement Schrödinger dynamics with a physical collapse mechanism that localizes the state.
- ◉ These theories can be ruled out empirically by generating superpositions involving large numbers of particles in different locations.
- ◉ The ontology of these theories is less clear than de Broglie-Bohm. Three ontologies have been proposed, but it is not clear if they all solve the tails problem.
- ◉ It is not obvious how to generalize these theories to quantum field theory. Can it be done in a Lorentz invariant way?