Quantum Foundations Lecture 23

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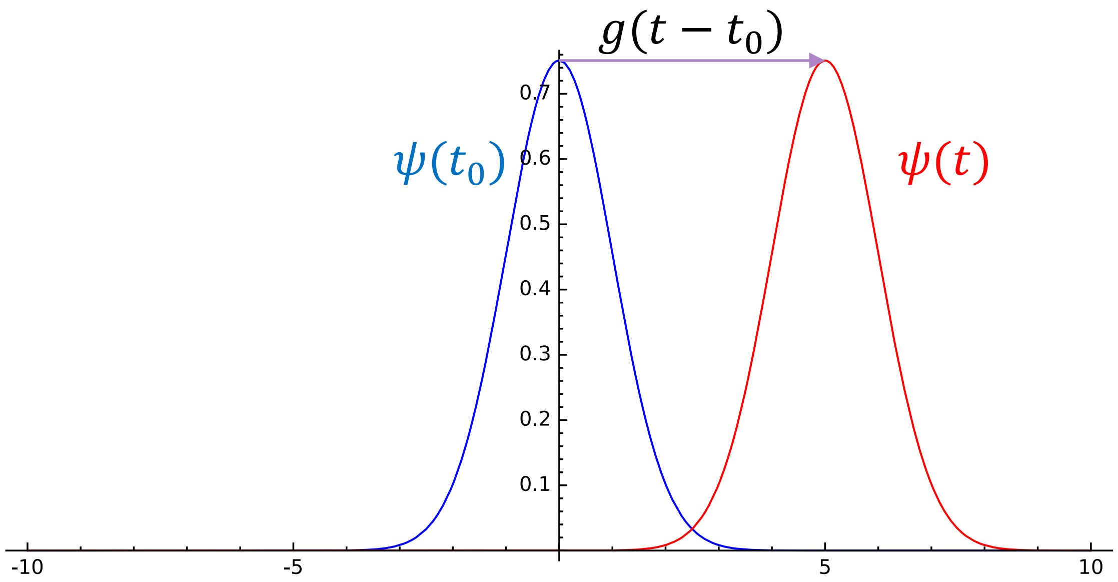
HSC112

Announcements

- Assignments: Final Version due May 2.
- Homework 4 due April 30.
- Homework 5 due May 25.
- Final Exam to be issued later this week.

Translation Hamiltonian

- Suppose now that the Hamiltonian of our system is proportional to the momentum
$$\hat{H} = g\hat{p}$$
- The propagator $\hat{U}(t, t_0) = e^{-ig(t-t_0)\hat{p}}$ is a translation operator, so the wavefunction will move to the right at a rate g.



Von Neumann Measurement Model

- Now we want to show how a measurement of any Hermitian observable $\hat{A}$ can be accomplished by coupling the system to the position of a pointer, and then measuring the position of the pointer.
- Suppose $\hat{A}$ has eigenstates $\hat{A}|\phi_j\rangle = a_j|\phi_j\rangle$ and suppose, for now, that the system is prepared in one of its eigenstates $|\phi_j\rangle_S$.
- We prepare our pointer in a narrow Gaussian wavepacket, centered at $x = 0$, i.e. $|\Psi(t_0)\rangle_M$ with

$$\langle x|\Psi(\mathrm{t}_0)\rangle = \Psi(x, t_0) = \frac{1}{\sqrt{\sqrt{\pi}\sigma}} \exp\left(\frac{-x^2}{2\sigma^2}\right)$$

Von Neumann Measurement Model

- We now couple the system and pointer using the Hamiltonian

$$\hat{H} = \hat{A}_S \otimes \hat{p}_M$$

- We run the dynamics for time $t - t_0 = 1$, which will generate the propagator

$$\hat{U}_{SM} = e^{-i\hat{A}_S \otimes \hat{p}_M} = \sum_{n=0}^{\infty} \frac{(-i)^n \hat{A}_S^n \otimes \hat{p}_M^n}{n!}$$

Von Neumann Measurement Model

- When this acts on the state of the system and pointer, we get

$$\hat{U}_{SM}\left|\phi_j\right\rangle_S \otimes |\Psi\rangle_M = \sum_{n=0}^{\infty} \frac{(-i)^n \hat{A}_S^n \left|\phi_j\right\rangle_S \otimes \hat{p}_M^n |\Psi\rangle_M}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{(-i)^n a_j^n \left|\phi_j\right\rangle_S \otimes \hat{p}_M^n |\Psi\rangle_M}{n!}$$

$$= \left[\sum_{n=0}^{\infty} \frac{\left(-i a_j \hat{p}_M\right)^n}{n!}\right] \left|\phi_j\right\rangle_S \otimes |\Psi\rangle_M$$

$$= \left|\phi_j\right\rangle_S \otimes e^{-i a_j \hat{p}_M} |\Psi\rangle_M$$

Von Neumann Measurement Model

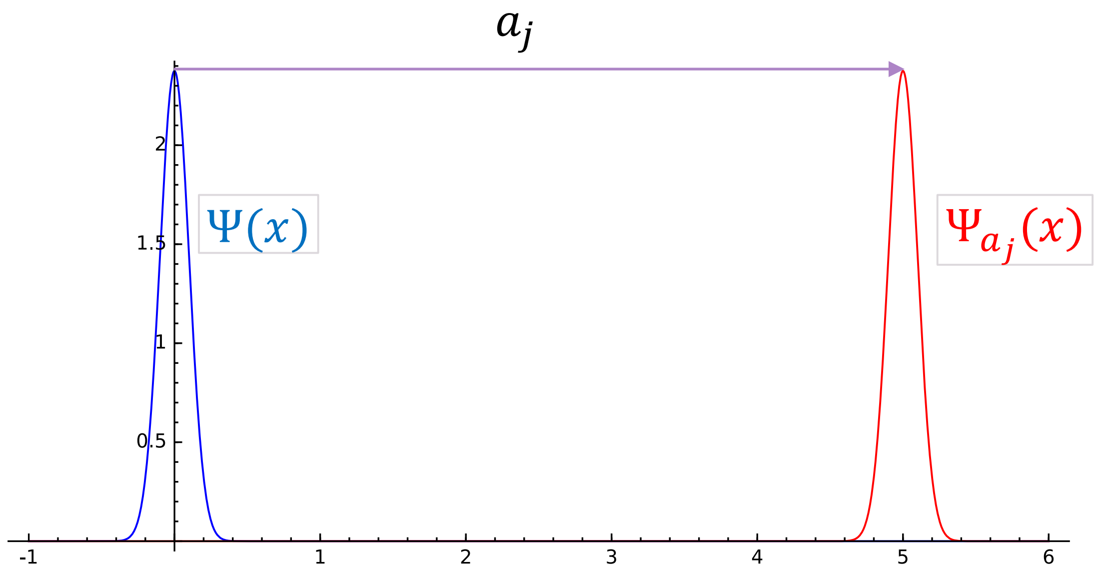
- In other words, the wavefunction $\Psi(x)$ of the pointer will be translated to

$$\Psi_{a_j}(x) = \Psi(x - a_j)$$

- If the width σ of the initial Gaussian is sufficiently small, measuring the position of the pointer will yield the probability density

$$\mathrm{p}(x) = \left|\Psi_{a_j}(x)\right|^2$$

which will be very close to a_j with near certainty.



Von Neumann Measurement Model

- Now consider what happens if we start the system in an arbitrary state $|\psi\rangle_S$, which can be written as a superposition of eigenstates of $\hat{A}$.

$$|\psi\rangle_S = \sum_j \alpha_j |\phi_j\rangle_M$$

- By the superposition principle, the evolution will be

$$\hat{U}_{SM}|\psi\rangle_S \otimes |\Psi\rangle_M = \sum_j \alpha_j |\phi_j\rangle_S \otimes e^{-ia_j\hat{p}}|\Psi\rangle_M$$
$$= \sum_j \alpha_j |\phi_j\rangle_S \otimes \left|\Psi_{a_j}\right\rangle_M$$

where $\left\langle x \middle| \Psi_{a_j} \right\rangle = \Psi_{a_j}(x) = \Psi(x - a_j)$.

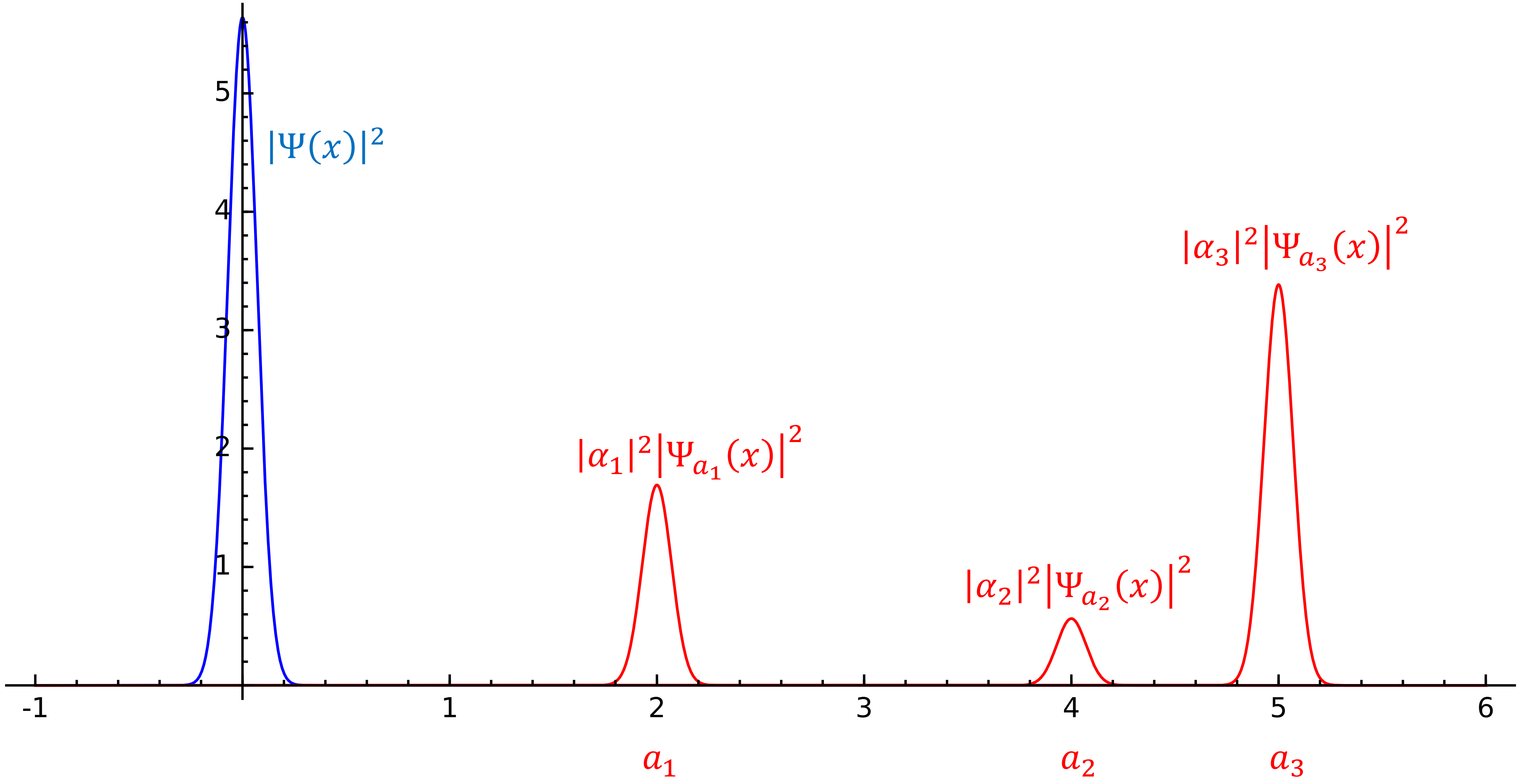
Von Neuman Measurement Model

- If we look at the probability density for the pointer position, we will get

$$\mathrm{p}(x) = \sum_{jk} \left(\alpha_j^* \langle \phi_j \Big|_S \otimes \Big\langle \Psi_{a_j} \Big|_M \right) |x\rangle_M \langle x| \left(\alpha_k |\phi_k\rangle_S \otimes \Big| \Psi_{a_j} \Big\rangle_M \right)$$

$$= \sum_j \left|\alpha_j\right|^2 \left|\Psi_{a_j}(x)\right|^2$$

Von Neumann Measurement Model



A Map Of The Madness

	Realist		Copenhagenish	
	Ontological Model	**Exotic Ontology**	**Objective**	**Perspectival**
ψ-epsitemic		Ironic Many Worlds	Copenhagen	QBism
		Quantum Logical Realism	Healy's Quantum Pragmatism	Rovelli's Relational Quantum Mechanics
			Bub's "Information" Interpretation	
ψ-ontic	de Broglie-Bohm	Everett/Many Worlds		
	Spontaneous Collapse			
	Modal Interpretations			

10.ii) de Broglie-Bohm Theory

- A brief history:
  - The 1st order form of dBB theory was discovered and then abandoned by de Broglie in the 1920’s.
  - dBB was rediscovered, in 2nd order form, by Bohm in 1952.
  - The forgotten 1st order form was promoted by Bell in the 1970’s and 80’s.
  - Proponents still fight over which form is better. I will follow Bell’s approach here.
- See T. Norsen, “Foundations of Quantum Mechanics” (Springer,2017) for an overview of this theory.

Ontology of dBB Theory

- The goal of any interpretation is to:
  - Provide an ontology: a statement of what exists and how it behaves.
  - Save the phenomena: Explain the quantum predictions and our everyday experience in terms of the ontology.
- Bohmians typically divide the ontology into two pieces:
  - Primitive ontology: The things that determine what we experience. Usually assumed to be localized in spacetime – local beables. In dBB this is particle trajectories.
  - The rest: Needed to determine how the primitive ontology behaves. In dBB this is the quantum state.

Single Particle Theory in 1-Dimension

- For particles with no internal degrees of freedom (spin), we use the wavefunction

$$\psi(x,t) = \langle x|\psi(t)\rangle$$

- The quantum state obeys the Schrödinger equation: $i\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle$
- dBB also has an actual particle with position X.

- This obeys the guidance equation:

$$\frac{dX}{dt} = \frac{1}{m}\frac{\mathrm{Im}\left(\psi^*(x,t)\frac{\partial\psi(x,t)}{\partial x}\right)}{\psi^*(x,t)\psi(x,t)}\Bigg|_{x=X}$$

Single Particle Theory in 3-Dimensions

- In 3-dimensions, we introduce the basis $|\vec{q}\rangle = |x\rangle \otimes |y\rangle \otimes |z\rangle$
- For particles with no internal degrees of freedom (spin), we use the wavefunction

$$\psi(\vec{q}, t) = \langle \vec{q} | \psi(t) \rangle = \langle x | \langle y | \langle z | \psi(t) \rangle$$

- The quantum state obeys the Schrödinger equation: $i \frac{\partial |\psi\rangle}{\partial t} = H|\psi\rangle$
- dBB also has an actual particle with position vector $\vec{Q}$
- This obeys the guidance equation:

$$\frac{d\vec{Q}}{dt} = \frac{1}{m} \frac{\mathrm{Im}\left(\psi^*(\vec{q}, t) \vec{\nabla} \psi(\vec{q}, t)\right)}{\psi^*(q, t) \psi(q, t)} \Bigg|_{\vec{q} = \vec{Q}}$$

General Case

- To describe N particles, we need to specify a position vector for each of them

$$\boldsymbol{q} = (\vec{q}_1, \vec{q}_2, \cdots, \vec{q}_3)$$

- Notation: $\vec{q}$ denotes a vector in $\mathbb{R}^3$. $\boldsymbol{q}$ denotes a vector in $\mathbb{R}^{3N}$, called a *configuration vector*.
- $\mathbb{R}^{3N}$ is called *configuration space*.
- We can write a quantum state as a wavefunction on configuration space:

$$\psi(\boldsymbol{q}, t) = \psi(\vec{q}_1, \vec{q}_2, \dots, \vec{q}_N, t) = \langle \boldsymbol{q} | \psi(t) \rangle = \langle \vec{q}_1, \vec{q}_2, \dots, \vec{q}_N | \psi(t) \rangle$$

- The wavefunction obeys the Schrödinger equation: $i \frac{\partial |\psi\rangle}{\partial t} = H|\psi\rangle$
- dBB also has an actual point in configuration space:

$$\boldsymbol{Q} = (\vec{Q}_1, \vec{Q}_2, \dots, \vec{Q}_N)$$

- This obeys the guidance equation:

$$\frac{d\vec{Q}_k}{dt} = \frac{\hbar}{m_k} \frac{\mathrm{Im}\left(\psi^*(\boldsymbol{q}, t) \vec{\nabla}_k \psi(\boldsymbol{q}, t)\right)}{\psi^*(\boldsymbol{q}, t)\psi(\boldsymbol{q}, t)} \Bigg|_{\boldsymbol{q}=\boldsymbol{Q}}$$

Equilibrium Hypothesis and Equivariance

- One more postulate is required to obtain the same predictions as standard quantum theory - Quantum Equilibrium Hypothesis:
  - At time $t = t_0$, the probability density of the system occupying configuration point $\boldsymbol{Q}$ is:

$$\rho(\boldsymbol{Q}) = |\psi(\boldsymbol{Q})|^2$$

- Under the dBB evolution we will show that if this holds at $t = t_0$ then it holds at all times. This is known as equivariance.
- There is controversy about what $\rho(\boldsymbol{Q})$ means as dBB is applied to the *entire universe*, which only has a single configuration space point.
  - Roughly speaking, if we prepare many systems in the state $|\psi\rangle \otimes |\psi\rangle \otimes \cdots \otimes |\psi\rangle$, the probability density of configurations is $\rho(\boldsymbol{Q})$.
- Note that the quantum state is playing two *independent* roles:
  - It governs dynamics via the guidance equation.
  - It is used to set the probability density.

Continuity Equations

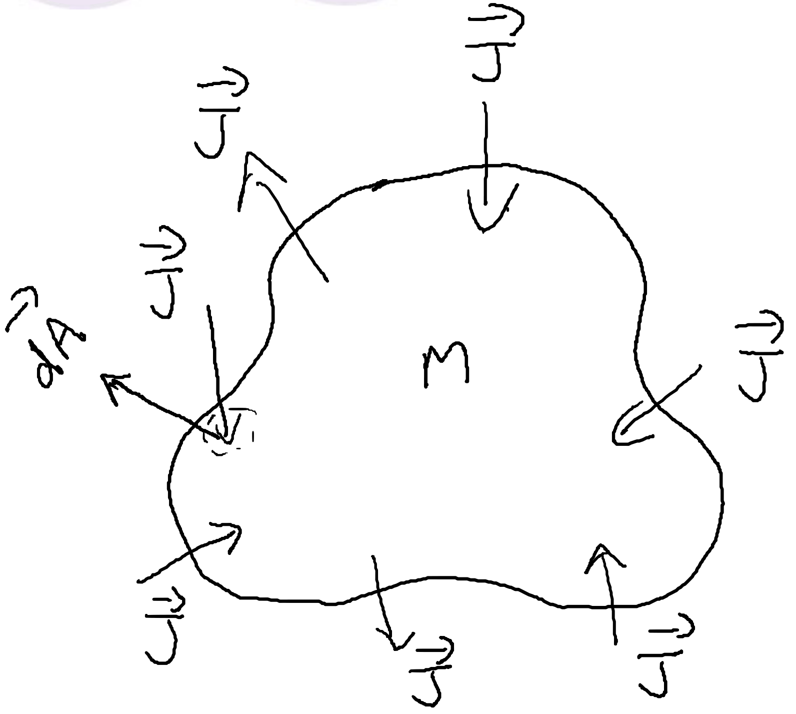
- Consider a volume of space in which there is a total mass $m(t)$.
- Let $\vec{J}(\vec{r},t)$ be the mass current, i.e.

Net amount off mass flowing through a unit cross-sectional area per unit time.

- We use surface area vectors $\mathrm{d}\vec{A}$ pointing out of the volume.
- Then

$$\frac{\mathrm{d}m}{\mathrm{d}t} + \int \vec{J}(\vec{r},t) \cdot \mathrm{d}\vec{A} = 0$$

- We can write $m(t) = \int \rho(\vec{r},t)dV$, where $\rho(\vec{r},t)$ is the mass density.



Continuity Equations

- Using the divergence theorem, we can also write

$$\int \vec{J}(\vec{r},t)\cdot \mathrm{d}\vec{A} = \int \vec{\nabla}\cdot\vec{J}(\vec{r},t)\mathrm{d}V$$

so we have

$$\int \left[\frac{\mathrm{d}\rho(\vec{r},t)}{\mathrm{d}t} + \vec{\nabla}\cdot\vec{J}\,(\vec{r},t)\right] dV = 0$$

- Since this has to hold for any volume, we have

$$\boxed{\frac{\mathrm{d}\rho(\vec{r},t)}{\mathrm{d}t} + \vec{\nabla}\cdot\vec{J}(\vec{r},t) = 0}$$

- This is called the *continuity equation*.

Hamiltonian For a Nonrelativistic Particle

- The Hamiltonian operator $\widehat{H}$ represents the energy of a particle.
- For a nonrelativistic particle in 1D we have

$$\widehat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$$

where $V(x)$ is the potential energy of the particle.

- Last lecture we saw that, in the position representation

$$\hat{p} = -i\frac{\partial}{\partial x}$$

so $\hat{p}^2 = -\frac{\partial^2}{\partial x^2}$ and

$$\widehat{H} = -\frac{1}{2m}\frac{\partial^2}{\partial x^2} + V(x)$$

Hamiltonian for Nonrelativistic Particles

- In 3-dimensions, this generalizes to

$$\hat{H} = -\frac{1}{2m}\nabla^2 + V(\vec{q})$$

and if we have N particles, this generalizes to

$$\hat{H} = -\sum_{k=1}^{N} \frac{1}{2m_k}\nabla_k^2 + V(\boldsymbol{q})$$

where $\boldsymbol{q} = (\vec{q}_1, \vec{q}_2, \cdots, \vec{q}_N)$, $\vec{q}_k = (x_k, y_k, z_k)$, and

$$\nabla_k^2 = \frac{\partial^2}{\partial x_k^2} + \frac{\partial^2}{\partial y_k^2} + \frac{\partial^2}{\partial z_k^2}$$

Continuity Equation for Probability

- We can derive a continuity for the probability density $\rho(\boldsymbol{q}) = |\psi(\boldsymbol{q})|^2$ in quantum theory.
- Consider a single particle in 1D

$$\frac{\partial \rho}{\partial t} = \frac{\partial\left(\psi^*(x,t)\psi(x,t)\right)}{\partial t} = \psi^*(x,t)\frac{\partial \psi(x,t)}{\partial t} + \frac{\partial \psi^*(x,t)}{\partial t}\psi(x,t)$$

- From the Schrödinger equation $i\frac{\partial|\psi(t)\rangle}{\partial t} = \hat{H}|\psi(t)\rangle$, we have

$$\frac{\partial \psi(x,t)}{\partial t} = \frac{i}{2m}\frac{\partial^2 \psi(x,t)}{\partial^2 x} - iV(x)\psi(x,t)$$

$$\frac{\partial \psi^*(x,t)}{\partial t} = \frac{-i}{2m}\frac{\partial^2 \psi^*(x,t)}{\partial^2 x} + iV(x)\psi^*(x,t)$$

Continuity Equation For Probability

- Substituting these into

$$\frac{\partial \rho}{\partial t} = \psi^*(x,t)\frac{\partial \psi(x,t)}{\partial t} + \frac{\partial \psi^*(x,t)}{\partial t}\psi(x,t)$$

gives

$$\frac{\partial \rho}{\partial t} = \frac{i}{2m}\left[\psi^*(x,t)\frac{\partial^2 \psi(x,t)}{\partial^2 x} - \frac{\partial^2 \psi^*(x,t)}{\partial^2 x}\psi(x,t)\right]$$

$-iV(x)[\psi^*(x,t)\psi(x,t) - \psi(x,t)\psi^*(x,t)]$ (this term cancels)

Continuity Equation For Probability

$$\psi^*(x,t)\frac{\partial^2\psi(x,t)}{\partial^2 x}-\frac{\partial^2\psi^*(x,t)}{\partial^2 x}\psi(x,t)$$

$$=\psi^*(x,t)\frac{\partial^2\psi(x,t)}{\partial^2 x}+\frac{\partial\psi^*(x,t)}{\partial x}\frac{\partial\psi(x,t)}{\partial x}-\frac{\partial\psi(x,t)}{\partial x}\frac{\partial\psi^*(x,t)}{\partial x}-\frac{\partial^2\psi^*(x,t)}{\partial^2 x}\psi(x,t)$$

$$=\frac{\partial}{\partial x}\left[\psi^*(x,t)\frac{\partial\psi(x,t)}{\partial x}-\frac{\partial\psi^*(x,t)}{\partial x}\psi(x,t)\right]$$

$$=\frac{\partial}{\partial x}\left[2i\mathrm{Im}\left(\psi^*(x,t)\frac{\partial\psi(x,t)}{\partial x}\right)\right]$$

Continuity Equation for Probability

- Therefore, if we define

$$J(x,t) = \frac{1}{m}\mathrm{Im}\left(\psi^*(x,t)\frac{\partial\psi(x,t)}{\partial x}\right)$$

  we get

$$\frac{\partial\rho(x,t)}{\partial t} + \frac{\partial J(x,t)}{\partial x} = 0$$

- This has the form of a continuity equation.
- $J(x,t)$ is the probability current, i.e. the rate of flow of probability out of point x.

Continuity Equation in 3D

- For a single particle in 3D, this generalizes to

$$\frac{\partial \rho(\vec{q},t)}{\partial t} + \vec{\nabla} \cdot \vec{J}(\vec{q},t) = 0$$

with probability current

$$\vec{J}(\vec{q},t) = \frac{1}{m} \operatorname{Im}\left(\psi^*(\vec{q},t)\vec{\nabla}\psi(\vec{q},t)\right)$$

Multiple particles

- For multiple particles in 3D, this generalizes to

$$\frac{\partial \rho(\boldsymbol{q},t)}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{J}(\boldsymbol{q},t) = 0$$

with probability current $\boldsymbol{J} = (\vec{J}_1, \vec{J}_2, \cdots, \vec{J}_n)$

$$\vec{J}_k(\boldsymbol{q},t) = \frac{1}{m_k} \mathrm{Im}\left(\psi^*(\boldsymbol{q},t)\vec{\nabla}_k \psi(\boldsymbol{q},t)\right)$$

Bell's derivation of the guidance equation and equivariance

- Solutions of the Schrödinger equation satisfy the continuity equation:

$$\frac{\partial|\psi(\boldsymbol{q},t)|^2}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{J}(\boldsymbol{q},t) = 0$$

where $\boldsymbol{J}(\boldsymbol{q},t)$ is the probability current:

$$\boldsymbol{J} = (\vec{J}_1, \vec{J}_2, \dots, \vec{J}_N) \qquad \vec{J}_k(\boldsymbol{q}) = \frac{\hbar}{m_k}\mathrm{Im}(\psi^*\vec{\nabla}_k\psi)(\boldsymbol{q})$$

- If we consider a preparation of $|\psi\rangle \otimes |\psi\rangle \otimes \cdots$ we want to consider $\boldsymbol{J}$ as a flow of particle density rather than probability.

- If we assume this is generated by a velocity field $\boldsymbol{v}(\boldsymbol{q})$, e.g. as in hydrodynamics, then $\boldsymbol{J} = \rho\boldsymbol{v}$, so the equation for the velocity field should be:

$$\boldsymbol{v}(\boldsymbol{q}) = \frac{\boldsymbol{J}(\boldsymbol{q})}{\rho(\boldsymbol{q})} \qquad \vec{v}_k(\boldsymbol{q}) = \frac{\hbar}{m_k}\frac{\mathrm{Im}(\psi^*\vec{\nabla}_k\psi)}{\rho}(\boldsymbol{q})$$

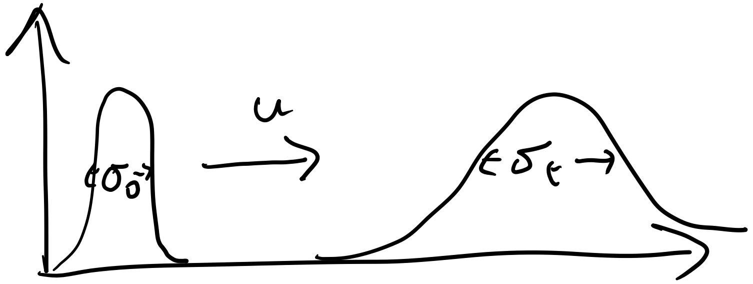
which gives the dBB velocities if we set $\rho(\boldsymbol{Q}) = |\psi(\boldsymbol{Q})|^2$.

Trajectories for a 1D Gaussian Wavepacket

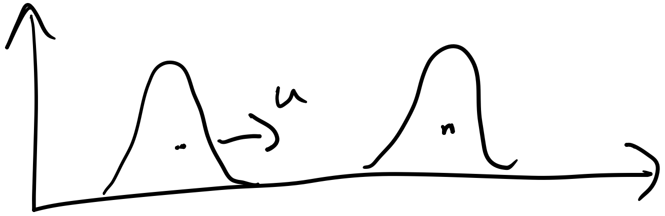
- Consider an initial Gaussian wavepacket moving towards the right

$$\psi(x,0) = \frac{1}{(2\pi\sigma_0^2)^{1/4}} \exp\left[-\frac{x^2}{4\sigma_0^2} + ikx\right]$$

- Under free-particle evolution this moves with group velocity $u = \frac{\hbar k}{m}$ and spreads $\sigma_t = \sigma_0 \sqrt{1 + \frac{\hbar^2 t^2}{4m^2\sigma_0^4}}$



- If we consider a timescale s.t. spreading is negligible $t^2 \ll \frac{2m\sigma_0^2}{\hbar}$

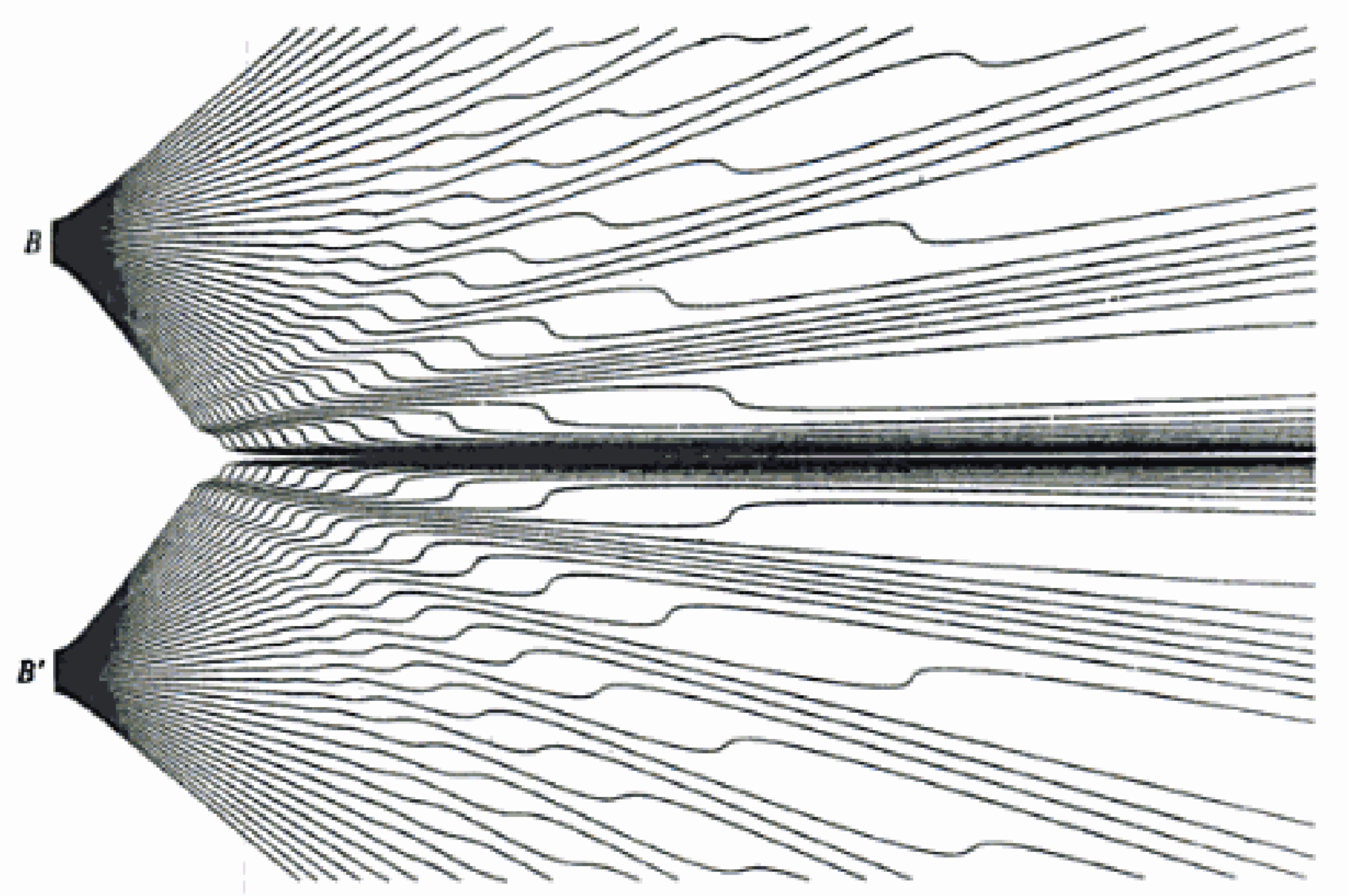


then the dBB velocity $\frac{dX}{dt} \simeq u$

Particle is dragged along with wavepacket at group velocity

See e.g. A. Pan, Pramana J. Phys. 74:867 (2010)

Double-Slit Trajectories



C.Philippidis et. al. Il Nuovo Cimento, vol.52B, No.1 (1979)

Model with Gaussian slits.

$$\psi(\vec{q},t) = \psi_B(\vec{q},t) + \psi_{B'}(\vec{q},t)$$

- When ψ_B, ψ_B have approximately no overlap (close to slits)

$$\vec{J} \simeq \vec{J}_B + \vec{J}_{B'}$$

$$\vec{J}_B = \frac{1}{m} \operatorname{Im} \psi_B^* \vec{\nabla} \psi_B$$

$$\vec{J}_{B'} = \frac{1}{m} \operatorname{Im} \psi_{B'}^* \vec{\nabla} \psi_B$$

The trajectories are as in geometric optics, i.e. perpendicular to wavefronts

- When they overlap there are cross-terms (interference) in the current, causing deflections which give the charateristic double-slit pattern.

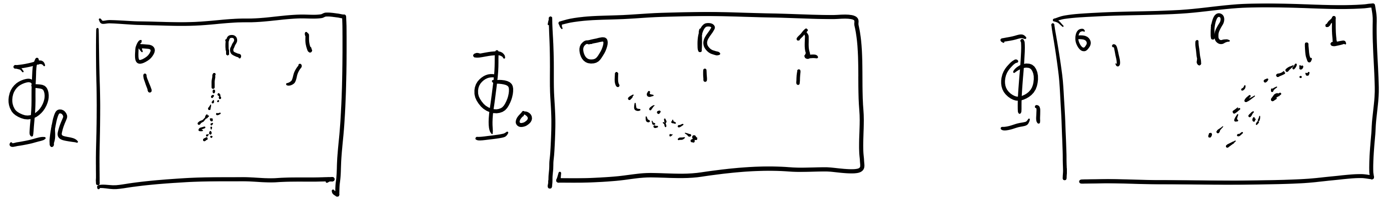
Measurements in de Broglie-Bohm Theory

- Dividing the universe into system S and environment E allows us to define a pure state for the system called the conditional quantum state.

$$\left|\psi_{\boldsymbol{Q}_E}\right\rangle_S = \ {}_E\langle \boldsymbol{Q}_E|\psi\rangle_{SE}$$

  where $\boldsymbol{Q}_E$ is the actual configuration point of the environment.

- Generally, these do not evolve according to the Schrödinger equation, but they do if there is decoherence into localized environment states.
  - For example, if $\boldsymbol{Q}_E$ is the pointer variable after a von Neumann measurement interaction.
- Model the measurement device as a large number of particles, with outcomes represented by macroscopically distinct states with very small overlap:



with $\Phi_0(q_E)\Phi_1(q_E) \simeq 0$

- In a measurement interaction:

$$[\alpha\psi_0(\boldsymbol{q}_S) + \beta\psi_1(\boldsymbol{q}_S)]\Phi_R(\boldsymbol{q}_E) \quad \rightarrow \quad \alpha\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E) + \beta\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$$

Measurements in de Broglie-Bohm Theory

$$\alpha\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E) + \beta\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$$

- If the lack of position overlap between $\Phi_0(\boldsymbol{q}_E)$ and $\Phi_1(\boldsymbol{q}_E)$ persists in time then:
  - The actual configuration of the environment $\boldsymbol{Q}_E$ is either in the support of $\Phi_0(\boldsymbol{q}_E)$ or the support of $\Phi_1(\boldsymbol{q}_E)$.
  - By equivariance, it will be in the support of $\Phi_0(\boldsymbol{q}_E)$ with probability $|\alpha|^2$ and in the support of $\Phi_1(\boldsymbol{q}_E)$ with probability $|\beta|^2$.
  - The conditional state of the system will either be $\propto \psi_0(\boldsymbol{q}_S)$ or $\propto \psi_1(\boldsymbol{q}_S)$.
  - $\psi_0(\boldsymbol{q}_S)$ and $\psi_1(\boldsymbol{q}_S)$ each evolve according to the Schrödinger equation.
  - The current breaks into two terms $\boldsymbol{J} = \boldsymbol{J}_0 + \boldsymbol{J}_1$, with $\boldsymbol{J}_0 = 0$ in the support of $\Phi_1(\boldsymbol{q}_E)$ and vice versa, i.e. no cross terms in the guidance equation.
- We get an effective collapse into either $\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E)$ or $\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$ and we can use the corresponding current $\boldsymbol{J}_0$ or $\boldsymbol{J}_1$ in the guidance equation to compute subsequent evolution.

Measurements in de Broglie-Bohm Theory

- If the measurement is an (approximate) position measurement then also $\psi_0(\boldsymbol{q}_S)\psi_1(\boldsymbol{q}_S) \approx 0$.
- The initial configuration $\boldsymbol{Q}_S$ of the system is either in the support of $\psi_0(\boldsymbol{q}_S)$ with probability $|\alpha|^2$ or in the support of $\psi_1(\boldsymbol{q}_S)$ with probability $|\beta|^2$.
- The measurement outcome is a deterministic function of $\boldsymbol{Q}_S$: position measurements simply reveal the pre-existing position.
- However, for other observables, e.g. momentum, $\psi_0(\boldsymbol{q}_S)\psi_1(\boldsymbol{q}_S) \neq 0$, i.e. the initial configuration does not necessarily "belong" to one of the two eigenstates.
- Which measurement outcome occurs is a function of *both* $\boldsymbol{Q}_S$ and $\boldsymbol{Q}_E$.
- Momentum measurement does not measure the dBB momentum $m_k \frac{d\vec{Q}_k}{dt}$.
- The theory is deterministic: outcome uniquely determined by ontic states of system and measuring device.
- But not outcome deterministic: outcome uniquely determined by ontic state of system on its own.

Treatment of Spin

- In the minimalist Bell approach to dBB, no observables apart from position are part of the primitive ontology.
- Spin only appears in the wavefunction.
- We can write a wavefunction including spin as a spinor, e.g. for a single particle:

$$\psi_0(\vec{q}) \otimes |\uparrow\rangle + \psi_1(\vec{q}) \otimes |\downarrow\rangle \quad \rightarrow \quad \bar{\psi}(\vec{q}) = \begin{pmatrix} \psi_0(\vec{q}) \\ \psi_1(\vec{q}) \end{pmatrix}$$

- For N spin-1/2 particles, we would have a 2^N dimensional spinor vector.
- The guidance equation is now:

$$\frac{d\vec{Q}_k}{dt} = \frac{\hbar}{m_k} \frac{\mathrm{Im}(\bar{\psi}^* \cdot \vec{\nabla}_k \bar{\psi})}{\bar{\psi}^* \cdot \bar{\psi}} (\boldsymbol{Q}),$$

  where $\cdot$ is spinor inner product.
- It is possible instead to have primitive ontic states for any complete orthonormal basis, but discrete bases require a stochastic guidance equation.

Counterintuitive Features of dBB Trajectories

- dBB trajectories display several features that violate classical intuitions about particle trajectories.
- It is important to note that, if decoherence occurs in an environmental basis that is localized in position, dBB trajectories of the system will approximately follow classical trajectories.
- dBB doesn't *owe* us anything more than that. So long as:
  - It reproduces the predictions of quantum theory in measurements.
  - Macroscopic systems typically have approximately classical trajectories.

  then the theory saves the phenomena.
- Since quantum and classical predictions are different, dBB trajectories *must* differ from classical ones in some situations.
- The question is only if they are weirder than absolutely necessary to reproduce quantum theory, and whether that is a bad thing.