Quantum Foundations Lecture 16

April 4, 2018

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HSC112

Announcements

- Schmid College Academic Advising:
  - Wednesday April 4, 4:30pm-6:30pm Henley Hall Basement (Prof. Dressel)
- Adam Becker is returning to Chapman:
  - Book event and signing at 1888 center: Monday April 16. RSVP required https://bit.ly/AdamBecker
- Assignments
  - First Draft due on Blackboard April 11.
  - Peer review until April 16.
  - Discussion in class April 16.
  - Final Version due May 2.
- Homework 3 due April 11.

9) Ontological Models

- The aim of this section is to investigate the possibility of constructing a realist theory (known as an *ontological model*) that can reproduce the predictions of quantum theory.
- We start with a simple toy-model that reproduces many of the apparently puzzling phenomena we have studied so far: The Spekkens' toy theory.
- These phenomena are naturally explained if there is a restriction on the amount of information we can have about the ontic state (an "epistemic restriction" or "epistriction") and the quantum state is epistemic.
- After this we will present the general definition of an ontological model and prove a number of no-go theorems that imply that a realist theory underlying quantum theory cannot be like this.

9) Ontological Models

- Good references for this section include:
  - David Jennings and Matthew Leifer, "No Return to Classical Reality", Contemporary Physics, vol. 57, iss. 1, pp. 60-82 (2015) https://doi.org/10.1080/00107514.2015.1063233 preprint: https://arxiv.org/abs/1501.03202
  - Robert W. Spekkens, "Quasi-Quantization: Classical Statistical Theories with an Epistemic Restriction", in "Quantum Theory: Informational Foundations and Foils", Giulio Chiribella and Robert W. Spekkens (eds.), pp. 83-135, Springer (2015) preprint: https://arxiv.org/abs/1409.5041
  - Robert W. Spekkens, "Contextuality for preparations, transformations, and unsharp measurements", Physical Review A, vol. 71 052108 (2005). Preprint: https://arxiv.org/abs/quant-ph/0406166
  - J. S. Bell, "Speakable and Unspeakable in Quantum Mechanics", 2nd edition, Cambridge University Press (2004).
  - Matthew Leifer, "Is the Quantum State Real? An Extended Review of ψ-ontology Theorems", Quanta, vol. 3, no. 1, pp. 67-155 (2014). http://dx.doi.org/10.12743/quanta.v3i1.22

9) Ontological Models

9. Ontological Models
    i. Epistricted Theories
    ii. Definitions
    iii. Examples
    iv. Excess Baggage
    v. Contextuality
    vi. Ψ-ontology
    vii. Bell's Theorem
    viii. The Colbeck-Renner Theorem

9.i) Epistricted Theories

- Rob Spekkens (Phys. Rev. A 75, 032110 (2007)) devised a toy theory designed to show how many apparently weird quantum phenomena could be accounted for via a ψ-epistemic, local, noncontextual model.
- Some versions of the theory accurately reproduce subtheories of quantum theory
  - Quantum theory with Gaussian states, measurements and operations (Bartlett et. al. Phys. Rev. A 86, 012103 (2012)).
  - Stabilizer quantum theory in odd prime dimensions (arXiv:1409.5041).
- The simplest version of the theory for "toy bits" (the analogue of qubits) does not exactly reproduce part of quantum theory, but is qualitatively similar.

The Knowledge-Balance Principle

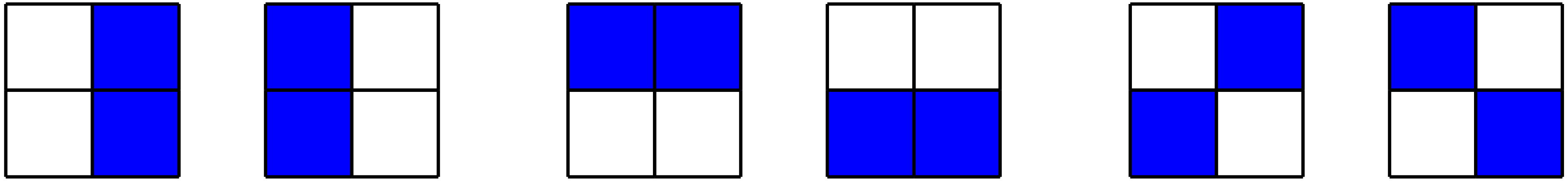
- Impose an "epistemic restriction" on how much we can know about a physical system.
  - In a state of maximal knowledge, we know as much about the system as we don't know.
- Simplest case: A system that has 4 possible ontic states called a toy bit.

$(-,+)$	$(+,+)$
$(-,-)$	$(+,-)$

- It takes a minimum of two binary questions to determine the ontic state:
  - e.g. Is the first entry + or -? Is the second entry + or -?
- We can know the exact answer to one such question in a pair, but then must be completely uncertain about the answer to the other one.

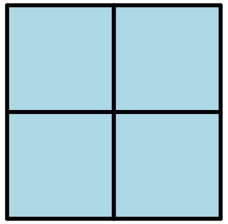
Epistemic States of a toy bit

- There are six epistemic states (probability distributions) compatible with the knowledge-balance principle.



$|+x\rangle$ $|-x\rangle$ $|+y\rangle$ $|-y\rangle$ $|+z\rangle$ $|-z\rangle$

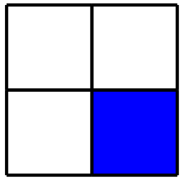
- We can also have a state of non-maximal knowledge:



$\frac{I}{2}$

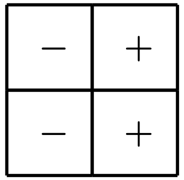
Measurements

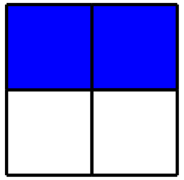
- We demand that measurements on toy bits must:
    1. Be repeatable, i.e. yield the same result if performed twice in a row.
    2. Not violate the knowledge-balance principle, i.e. they should leave the system in a valid epistemic state.
- This immediately implies that there cannot be a measurement that reveals the exact ontic state because this would have to leave us in an epistemic state like:



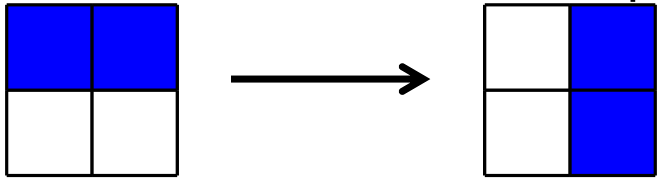
- But we can have measurements that reveal coarse grained information, provided they disturb the ontic state.

Example of a Valid Measurement

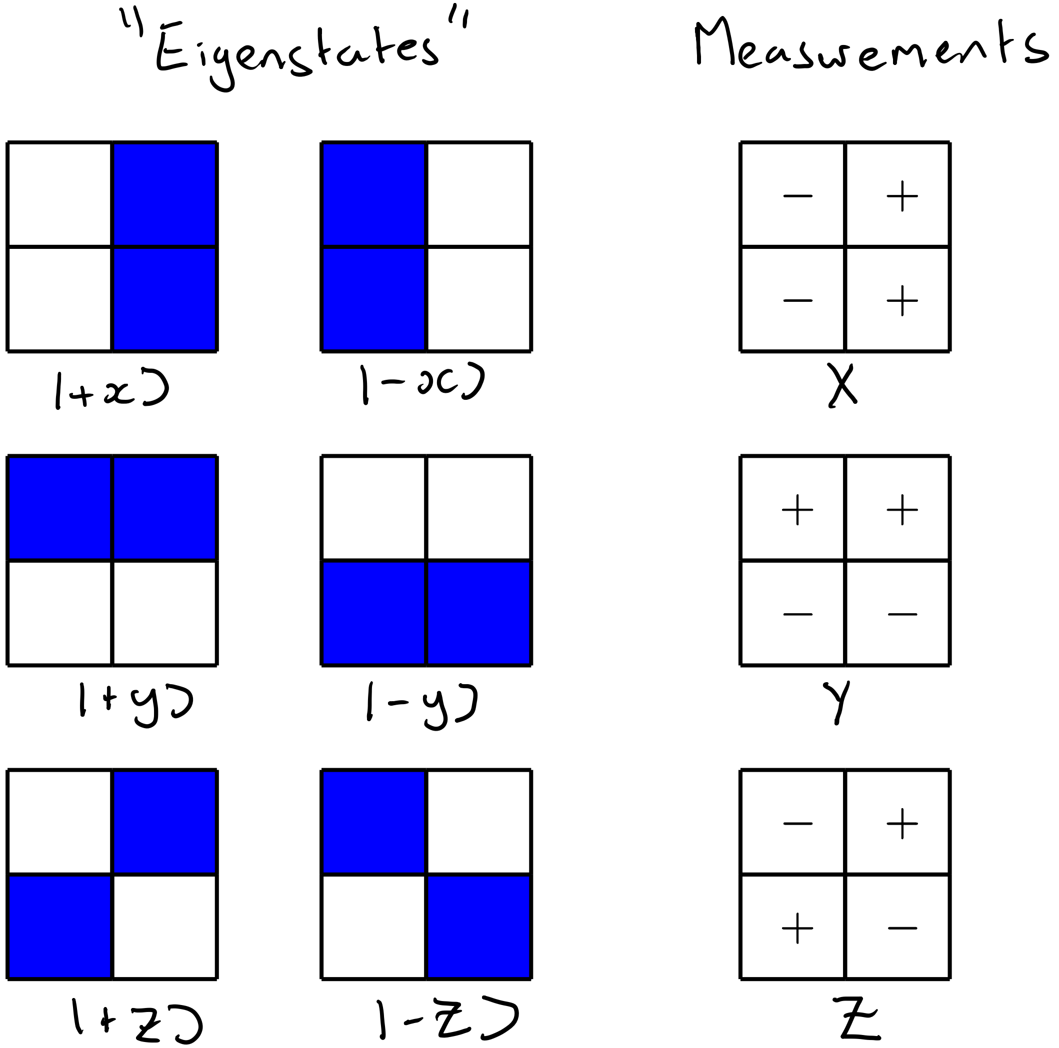
- An X measurement gives outcomes ± 1 as follows: 

- If we apply it to the $|+y\rangle$ state 

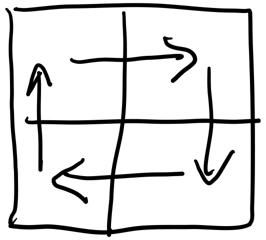
  and get the $+1$ outcome, then we will know that the ontic state must have been $(+,+)$ before the measurement.
- To preserve the knowledge-balance principle and maintain repeatability (+,+) and (+,-) must get swapped with probability ½ during the measurement.
- Thus, after the measurement, the updated epistemic state will be $|+x\rangle$.

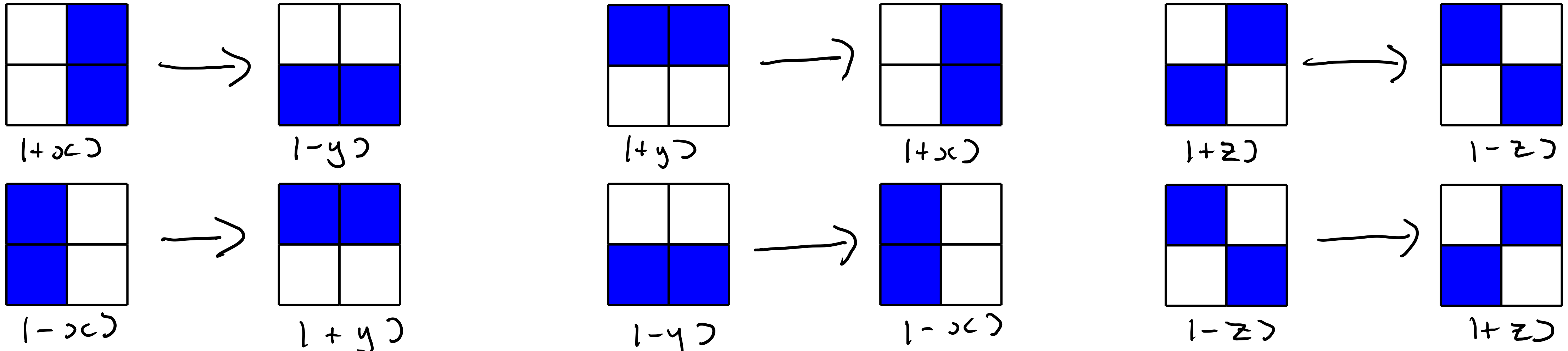


Valid Measurements on a toy bit and their "eigenstates"



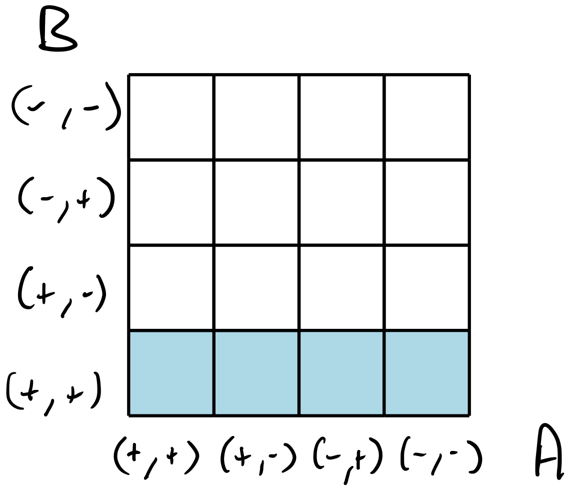
Reversible Dynamics

- Reversible dynamics (the analogue of unitary dynamics) on a toy bit is just a permutation on the underlying ontic states. We can then compute the action on the epistemic states.
- Example: 



Composite systems

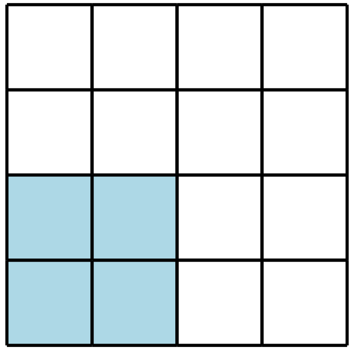
- When we have two toy bits, each toy bit has its own ontic state $(\pm,\pm)_A, (\pm,\pm)_B$.
- There are $4 \times 4 = 16$ possible ontic states, so it takes 4 binary questions to specify the exact ontic state.
- By the knowledge-balance principle, we can only know the answer to 2 of them.
- Subtlety: We not only apply the knowledge-balance principle to the global system, but also to the individual subsystems.



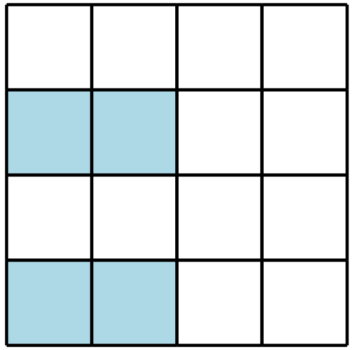
This is not a valid epistemic state because we know the exact ontic state (+,+) of system B.

Product and Correlated States

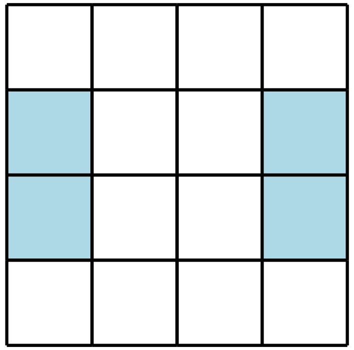
- Of the valid epistemic states, some of them are products of independent distributions of the two toy bits.



$|+x\rangle \otimes |+x\rangle$

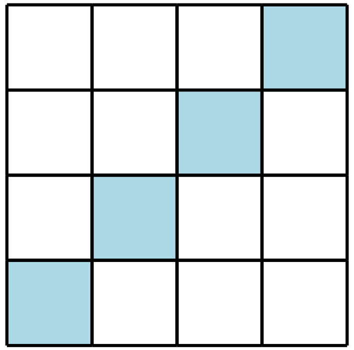


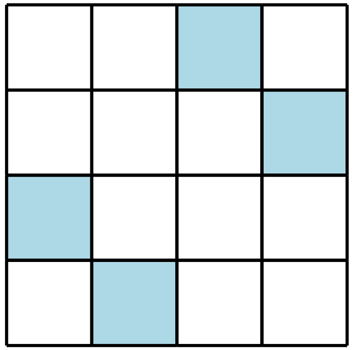
$|+x\rangle \otimes |+y\rangle$

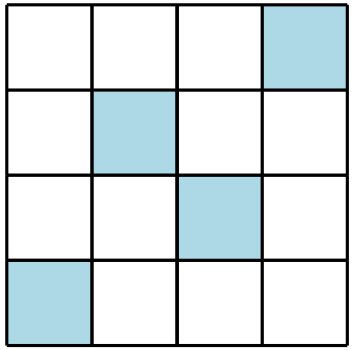


$|+z\rangle \otimes |-z\rangle$

- And some of them are correlated ("entangled")



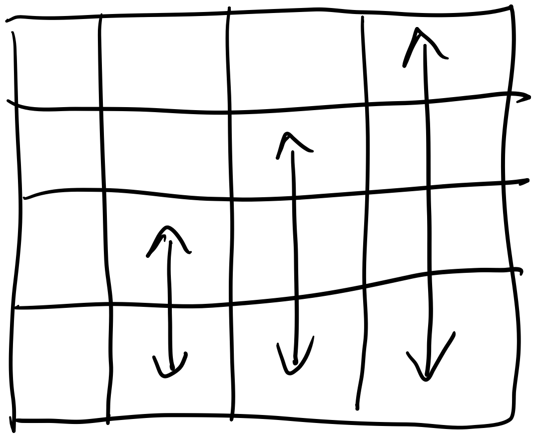


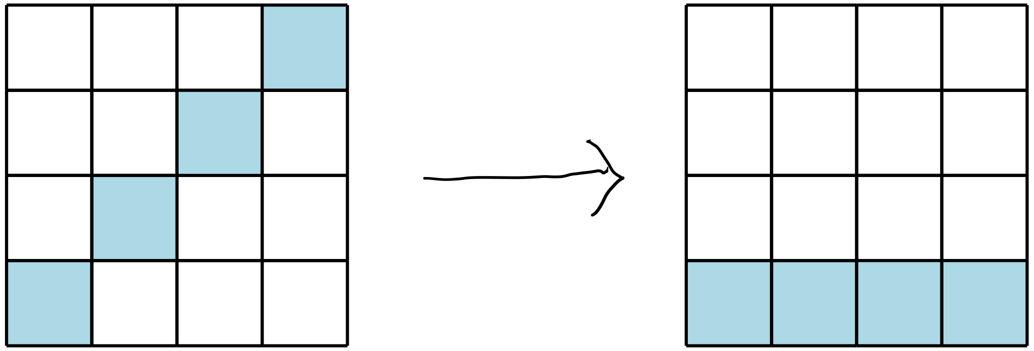


These have uniform marginal distributions

Reversible Dynamics on Composites

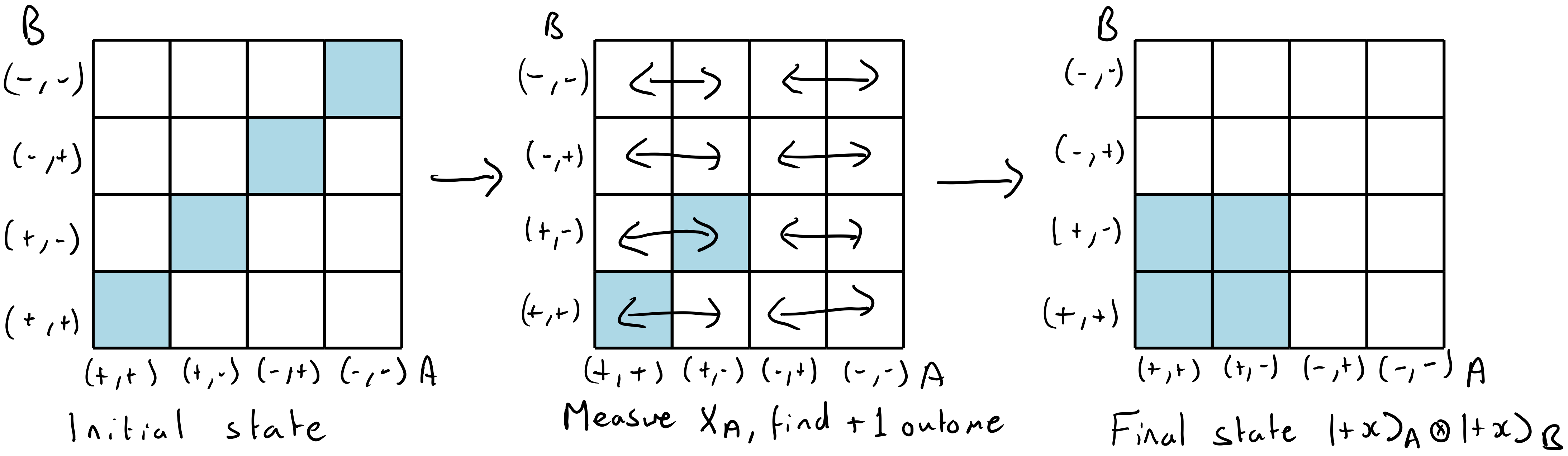
- Because we need to preserve the knowledge-balance principle for subsystems, not all permutations represent valid dynamics for a composite system.
- Example:



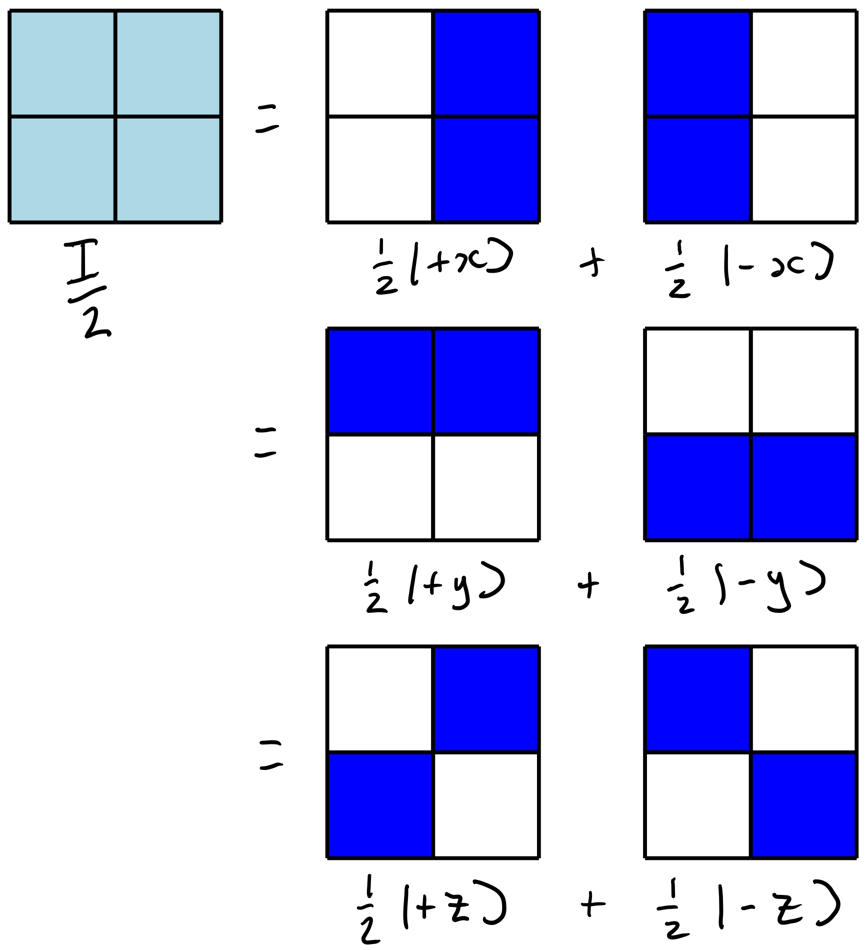


EPR

- The EPR experiment works as EPR expected in this theory:
  - The outcomes of all measurements are predetermined.
  - The two systems are initially in a correlated probability distribution.
  - The collapse is just updating information, followed by a local randomization of the system being measured.

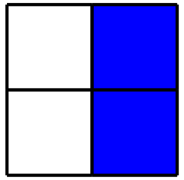


Non-Uniqueness of Mixed State Decompositions

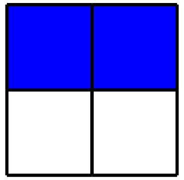


Because pure states correspond to overlapping probability distributions, mixtures of different sets of pure states can give the same probability distribution.

Indistinguishability of Pure States



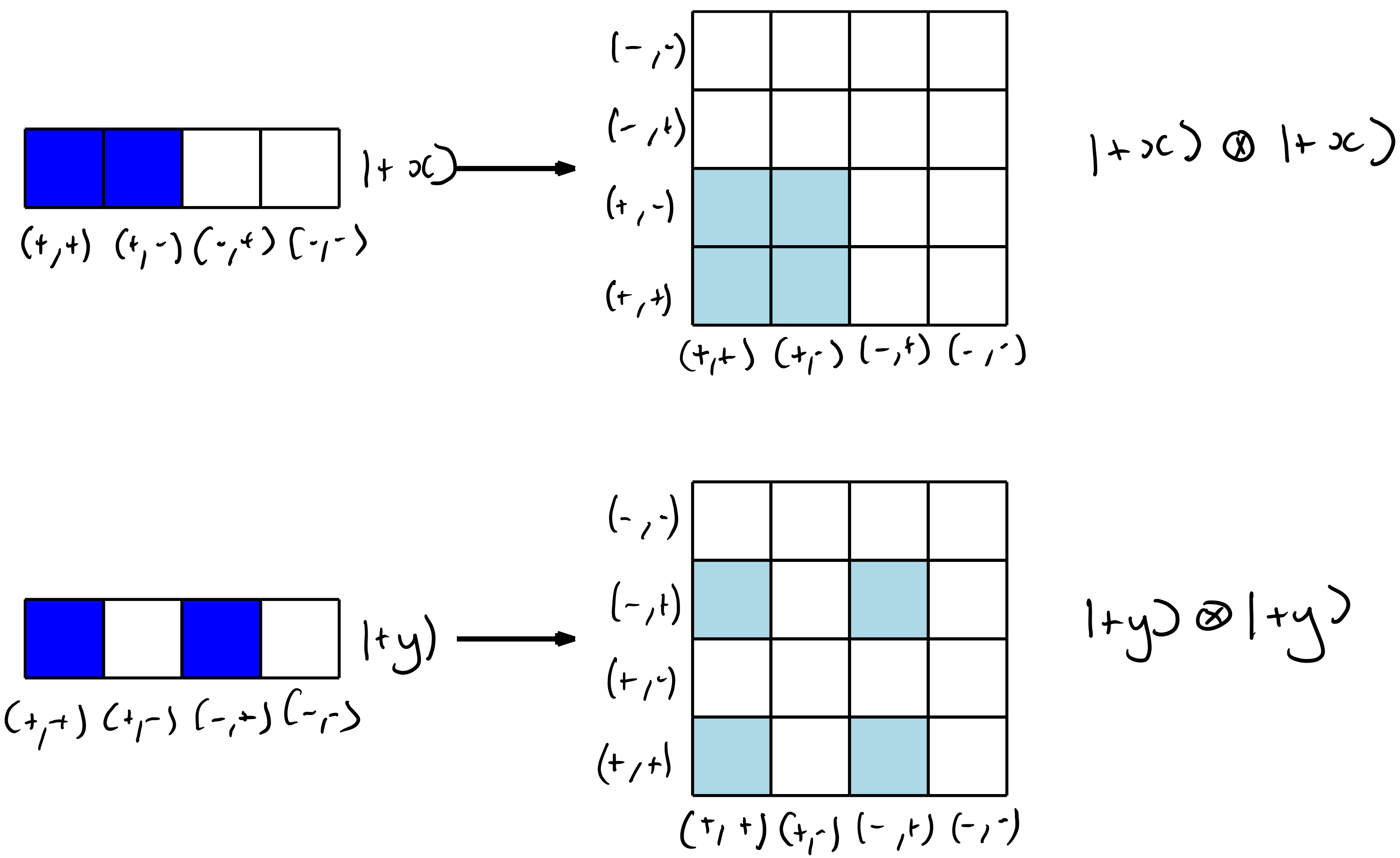
$|+x\rangle$



$|+y\rangle$

- It is impossible to perform a measurement that distinguishes $|+x\rangle$ from $|+y\rangle$ with certainty because, in both cases, the ontic state is $(+,+)$ with probability $\frac{1}{2}$

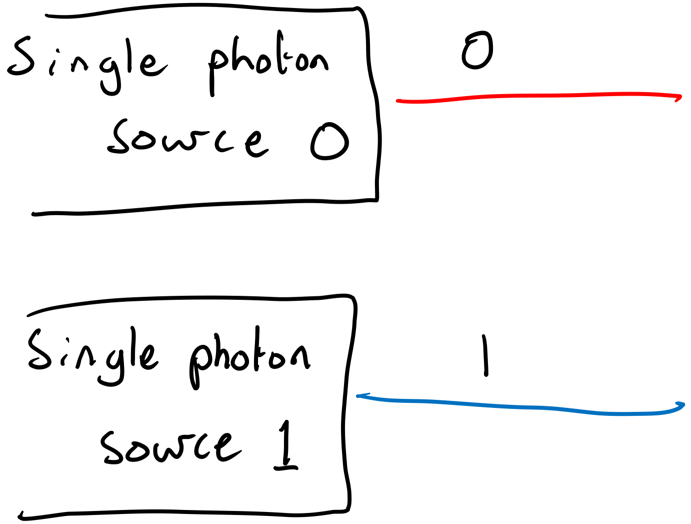
No-Cloning Theorem



This transition is impossible because initial states overlap with probability $\frac{1}{2}$ but final states overlap with probability $\frac{1}{4}$

Interference

- We can get an exact analogy with the Feynman interference paradox and the Elitzur-Vaidman Bomb.
- In the quantum mechanical version, we first switch to a Fock Space (optical mode) description.



We previously used $|0\rangle$: photon in mode 0

$|1\rangle$: photon in mode 1

Now use: $|0\rangle_0$ = vacuum of mode 0

$|1\rangle_0$ = 1 photon in mode 0

$|0\rangle_1$ = vacuum of mode 1

$|1\rangle_1$ = 1 photon in mode 1

$$\left.\begin{aligned} |0\rangle &\rightarrow |1\rangle_0 |0\rangle_1 \\ |1\rangle &\rightarrow |0\rangle_0 |1\rangle_1 \end{aligned}\right\}$$

We now have 2 qubits if we stick to states with at most one photon in each mode

Beamsplitters and Dectectors

- Action of a 50/50 beam splitter is

OLD

$$|0\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right)$$

$$|1\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(i|0\rangle + |1\rangle\right)$$

NEW

$$|1\rangle_0|0\rangle_1 \longrightarrow \frac{1}{\sqrt{2}}\left(|1\rangle_0|0\rangle_1 + i|0\rangle_0|1\rangle_1\right)$$

$$|0\rangle_0|1\rangle_1 \longrightarrow \frac{1}{\sqrt{2}}\left(i|1\rangle_0|0\rangle_1 + |0\rangle_0|1\rangle_1\right)$$

In new description, the beamsplitter entangles the two mode qubits.

- Detectors: These now act as local measurements on a tensor product

Detector on mode 0 : $\{|0\rangle_0\langle 0|,\ |1\rangle_0\langle 1|\}$

Detector on mode 1 : $\{|0\rangle_1\langle 0|,\ |1\rangle_0\langle 1|\}$

Click if $|1\rangle\langle 1|$ outcome

No click if $|0\rangle\langle 0|$ outcome

- We won't worry about the two mirrors in a Mach-Zehnder because they just give a global phase factor.

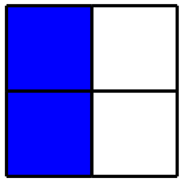
Toy Theory Model

- Since we have 2 qubits, we will use a model with 2 toy bits. One toy bit travels along mode 0 and the other along mode 1.
- We make the following correspondence:

Vacuum states:

$|0\rangle_0 = |-x)_0$

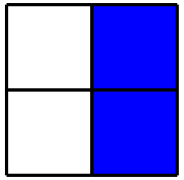
$|0\rangle_1 = |-x)_1$

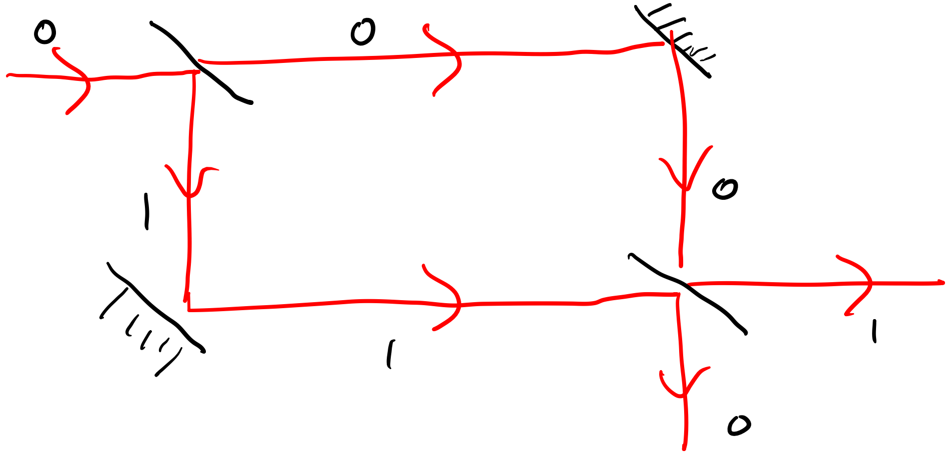


One-photon states:

$|1\rangle_0 = |+x)_0$

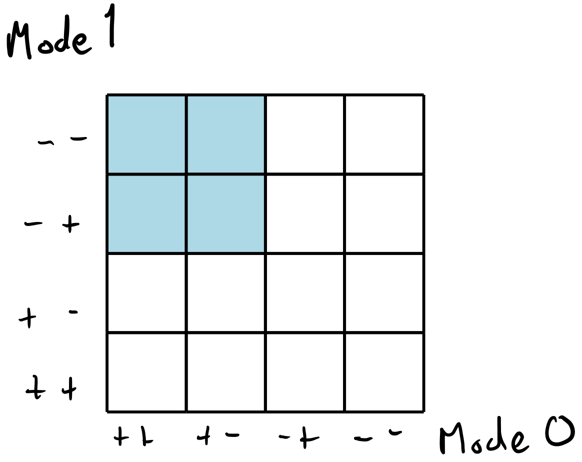
$|1\rangle_1 = |+x)_1$





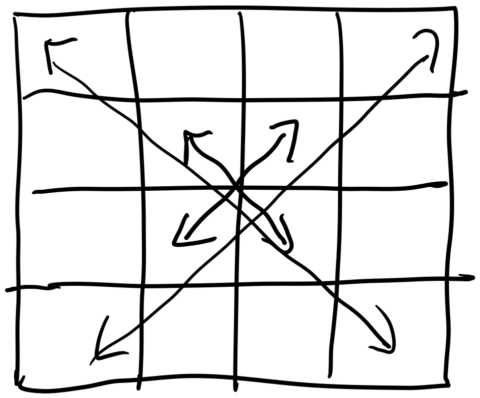
Initial state of a Mach-Zehnder

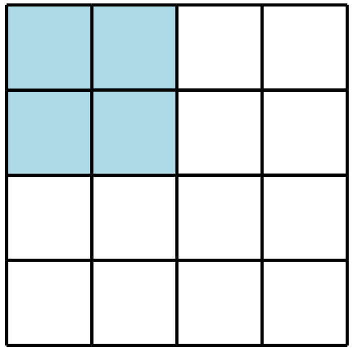
$|1\rangle_0 |0\rangle_1 \rightarrow |+x)_0 \, |-x)_1$



Toy Theory Beamsplitter

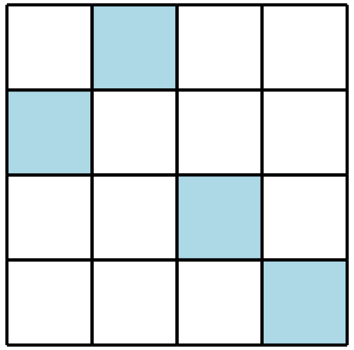
- To model the entangling 50/50 beamsplitter in the toy theory, we use the following permutation





$|+x\rangle_0 |-x\rangle_1$

analogue of $|1\rangle_0 |0\rangle_1$

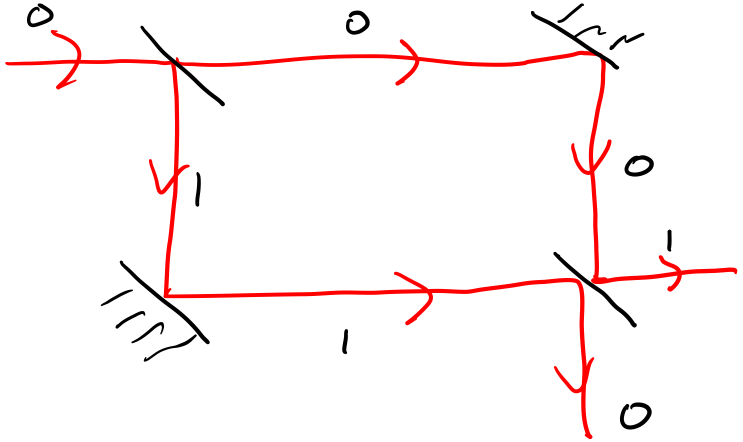


analogue of $\frac{1}{\sqrt{2}}\left(|1\rangle_0|0\rangle_1 + i|0\rangle_0|1\rangle_1\right)$

- You can check that this permutation satisfies the knowledge-balance principle

Toy Theory Mach-Zehnder

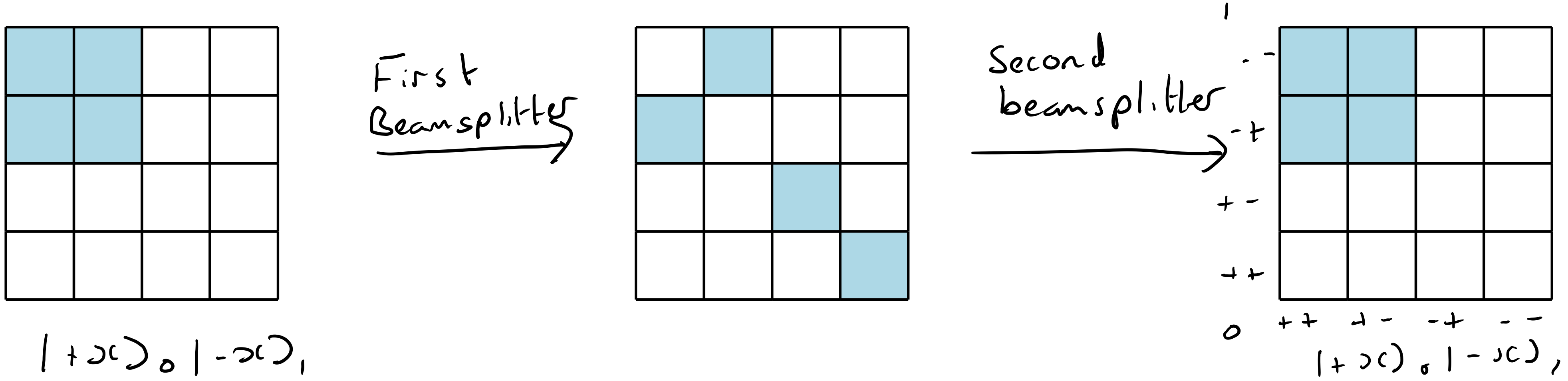
○ Mach-Zehnder without detectors on the paths



An X measurement of mode 0 always yields +1

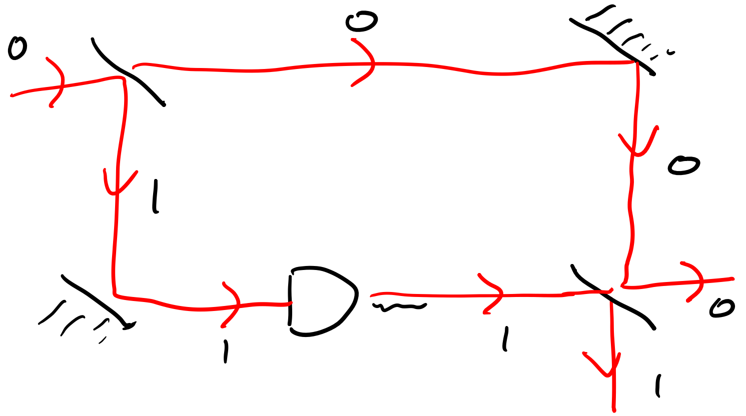
An X measurement of mode 1 always yields -1

$\Rightarrow$ Photon is always found in mode 0

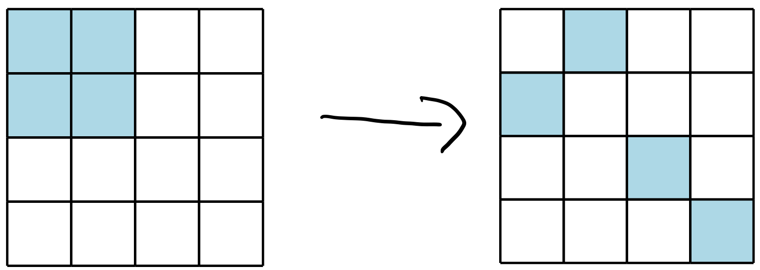


Toy Theory Mach-Zehnder

- Suppose we now put a detector in mode 1, which could be an Elitzur-Vaidman bomb

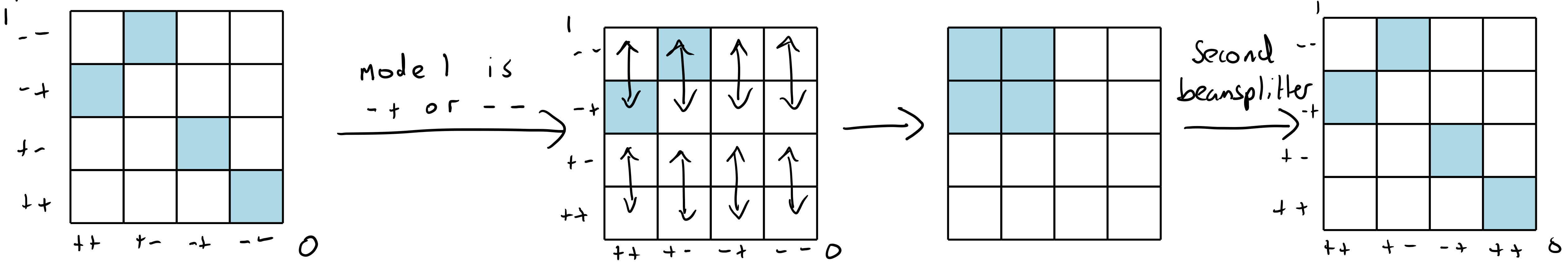


First beamsplitter works as before



Probabilities of each detector firing are now 50/50

Now we measure in mode 1: suppose the detector does not click

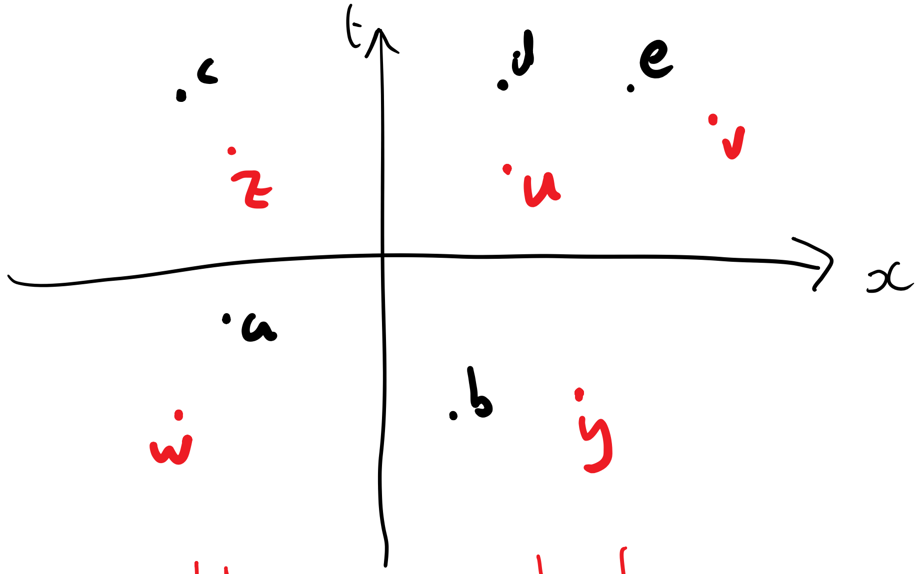


Summary

- A large number of apparently strange phenomena are accounted for (at least qualitatively) in the toy theory.
- They occur because it is ψ-epistemic:
  - Pure states are states of incomplete knowledge. Nonorthogonal pure states correspond to overlapping probability distributions.
  - This accounts for non-uniqueness of mixed state decompositions, indistinguishability, and no-cloning.
  - For interference, we should recognize that the vacuum is a quantum state, so also a state of incomplete knowledge. The degrees of freedom in the vacuum can mediate information about whether or not a detector is present, even when there is no photon in the mode.

9.ii) Ontological Model Definitions

- An operational theory looks like this:



• : variables we control

• : variables we observe

Theory predicts $\mathrm{Prob}(a,b,c,d,e \mid u,v,w,y,z)$

- Ontic states:

In addition to the variables we control and observe, there may be additional physical properties denoted λ that take values in a set Λ.

λ is called an ontic state.

Λ is called the ontic state space

λ can be anything you like: particle trajectories, field configurations, quantum states, green aliens, none of the above.

Single World Realism

- (Single world) Realism:

  On each run of the experiment, the operational variables and λ each take a definite value.

- Independence:

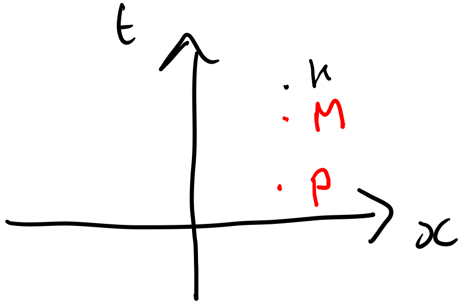
  Each run of the experiment is independent and identically distributed

$\Longleftrightarrow$ $\exists$ a joint probability distribution

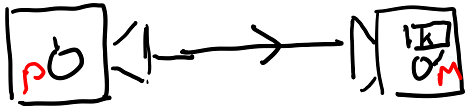
$$\Pr(a, b, c, d, e, \lambda \mid u, v, w, y, z)$$

Prepare and Measure Experiments

- For the next few lectures we will be considering a very simple kind of experiment



- P is a choice of preparation
- M is a choice of measurement
- k is the outcome of the measurement
- An operational theory predicts $\mathrm{Prob}(k \mid P, M)$



- If there are ontic states with single world realism and independence, we should have $\Pr(k, \lambda \mid P, M)$
- We reproduce the operational predictions if

$$\mathrm{Prob}(k \mid P, M) = \int_\Lambda d\lambda \, \Pr(k, \lambda \mid P, M).$$

Ontological Models

- In general, we can write

$$\mathrm{Prob}(k|P,M) = \int_\Lambda d\lambda \, \Pr(k,\lambda|P,M) = \int_\Lambda d\lambda \, \Pr(k|\lambda,P,M)\Pr(\lambda|P,M)$$

- We now want to think of λ as representing the physical properties of the system we are experimenting on between preparation and measurement. We introduce two assumptions.

1) Measurement independence: $\Pr(\lambda|P,M) = \Pr(\lambda|P)$

   Often motivated as free choice / free will / no retrocausality

2) λ-mediation: $\Pr(k|\lambda,P,M) = \Pr(k|\lambda,M)$

   If λ represents all the physical properties of the system, it ought to be soley responsible for mediating any observed correlation between preparation and measurement.

Ontological Models

- With measurement independence and λ-mediation we have

$$\mathrm{Prob}(k|P,M) = \int_\Lambda \Pr(k|M,\lambda)\Pr(\lambda|P)\,d\lambda$$

- An ontological model is a model with ontic states satisfying: Single world realism, independence, measurement independence, and λ-mediation.

- In other words:
  - There is an ontic state space Λ
  - To each preparation we assign a probability distribution $\Pr(\lambda|P)$ over Λ
  - To each measurement we assign a probability distribution $\Pr(k|M,\lambda)$ over measurement outcomes that depends on λ
  - These should reproduce the operational predictions

$$\mathrm{Prob}(k|P,M) = \int_\Lambda \Pr(k|M,\lambda)\Pr(\lambda|P)\,d\lambda$$

Quantum Models

- We are usually interested in experiments that are well-described by quantum theory. In which case:

  - We assign a Hilbert space $\mathcal{H}$ to the system.
  - To each preparation P we assign a density operator $\rho_P \in \mathcal{L}(\mathcal{H})$
  - To each measurement M we assign a POVM E_k^M
  - The operational predictions are $\mathrm{Prob}(k|P,M) = \mathrm{Tr}(E_k^M \rho_P)$

- Then, we'll have

$$\mathrm{Tr}(E_k^M \rho_P) = \int_\Lambda d\lambda \, \Pr(k|M,\lambda) \Pr(\lambda|P)$$