Quantum Foundations Lecture 10

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HSC112

6.iii) Orthodoxy and the Measurement Problem

- Macroscopic superpositions and the measurement problem are often thought to be the most pressing problems in the foundations of quantum theory.
- But they have been solved multiple times. They are not problems with quantum theory per se, but rather with the interpretation of quantum theory usually given in textbooks.
  - This is why I prefer the three-point definition of the problem given in lecture 1.
- This is known as the Orthodox, Textbook, or Dirac-von Neumann interpretation.
- It is often mislabeled as the Copenhagen interpretation, but it differs so drastically from the views of Bohr, Heisenberg, etc. that it is not even in the same category.
  - The orthodox interpretation is realist, straightforward, and obviously wrong.
  - The Copenhagen interpretation is anti-realist, subtle, and wrong for interesting reasons (see lecture 13).

The Orthodox Interpretation

1. Physical systems have objective properties (this makes it realist):
   - The possible properties of a system are its observables. The possible values of those properties are the corresponding eigenvalues.
2. The eigenvalue-eigenstate link:
   - When the system is in an eigenstate of an observable M with eigenvalue m then M is a property of the system and it takes value m.
   - The system has no objective physical properties other than these.

The Orthodox Interpretation

- The eigenvalue-eigenstate link is equivalent to saying that the quantum state $|\psi\rangle$ is an objective property of an individual quantum system and that it is the only objective property of the system.
- Why?
  - By e-e link $|\psi\rangle\langle\psi|$ is a property of the system with value 1.
  - This uniquely determines $|\psi\rangle$ (up to global phase), so $|\psi\rangle$ is a property.
  - All other objective physical properties are uniquely determined by $|\psi\rangle$.

Some terminology

- ψ-ontic:
  - A theory in which the quantum state $|\psi\rangle$ is an objective physical property of an individual quantum system.
- ψ-complete:
  - A theory that is ψ-ontic and in which $|\psi\rangle$ is the only objective physical property, e.g. orthodox interpretation.
- ψ-epistemic:
  - A theory in which the quantum state has a similar status to a probability distribution, which you might call an epistemic, ensemble, or statistical state depending on how you think about probabilities.

Schrödinger's Cat

"One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter, there is a tiny bit of radioactive substance, so small, that perhaps in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer that shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The first atomic decay would have poisoned it. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.

It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be resolved by direct observation. That prevents us from so naively accepting as valid a "blurred model" for representing reality. In itself, it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks." – J. Trimmer, "The Present Situation in Quantum Mechanics: A Translation of Schrödinger's 'Cat Paradox' Paper" Proc. Am. Phil. Soc. vol. 124 pp. 323-338 (1980).

Schrödinger's Cat

- If we interact a macroscopic system with a microscopic system in a superposition, then we can generate superpositions of macroscopically distinct states, e.g.

$$\frac{1}{\sqrt{2}}\left(|\text{Trump is president}\rangle + |\text{Clinton is president}\rangle\right)$$

- In orthodox interpretation this is physically distinct from

$$|\text{Trump is president}\rangle \quad \text{or} \quad |\text{Clinton is president}\rangle$$

- This corresponds to nothing in our experience, so it does not "save the phenomena".

The Measurement Problem

- A related problem is that there are two ways of handling measurements in quantum theory.
    1. The measurement postulates.
    2. A measurement device is a physical system, made of atoms, so we ought to be able to describe it as a quantum system, which interacts unitarily with the system being measured.
- As an example, consider a qubit in state

$$\alpha|0\rangle + \beta|1\rangle$$

upon which we perform the projective measurement

$$|0\rangle\langle 0| \text{vs. } |1\rangle\langle 1|$$

The Measurement Problem

- According to the measurement postulates, the system will either collapse to

  $|0\rangle$ with probability $|\alpha|^2$

  or $|1\rangle$ with probability $|\beta|^2$.
- Now consider the measurement device as a physical system. Let $|R\rangle$ be the state in which it is ready to perform the measurement, i.e. initial state is

$$(\alpha|0\rangle + \beta|1\rangle) \otimes |R\rangle$$

The Measurement Problem

- The measurement is an interaction between the system and the measuring device, described by a unitary operator U.
- Let $|M_0\rangle$ be the state in which the measuring device registers 0.
- Let $|M_1\rangle$ be the state in which the measuring device registers 1.
- Then,

$$U|0\rangle \otimes |R\rangle = |0\rangle \otimes |M_0\rangle$$
$$U|1\rangle \otimes |R\rangle = |1\rangle \otimes |M_1\rangle$$

The Measurement Problem

- By the superposition principle, we should then have:

$$U[(\alpha|0\rangle + \beta|1\rangle) \otimes |R\rangle] = \alpha|0\rangle \otimes |M_0\rangle + \beta|1\rangle \otimes |M_1\rangle.$$

- On the orthodox interpretation, this is physically distinct from

$|0\rangle$ with probability $|\alpha|^2$

or $|1\rangle$ with probability $|\beta|^2$.

- So this is a flat out contradiction. The orthodox interpretation is straightforwardly wrong.

Comments on the Measurement Problem

- The measurement problem is a problem for ψ-complete theories.
  - For a ψ-ontic, but not ψ-complete, theory, additional variables may determine which branch of the superposition describes reality. The measurement postulates could be a mathematical shortcut to avoid tracking the true, but mostly irrelevant, joint state of the system and measuring device.
  - For a ψ-epistemic theory, the measurement postulates may be viewed as no different from updating a probability distribution on the acquisition of new information.
- But the problems of ontology, saving the phenomena, and making progress in physics still apply.

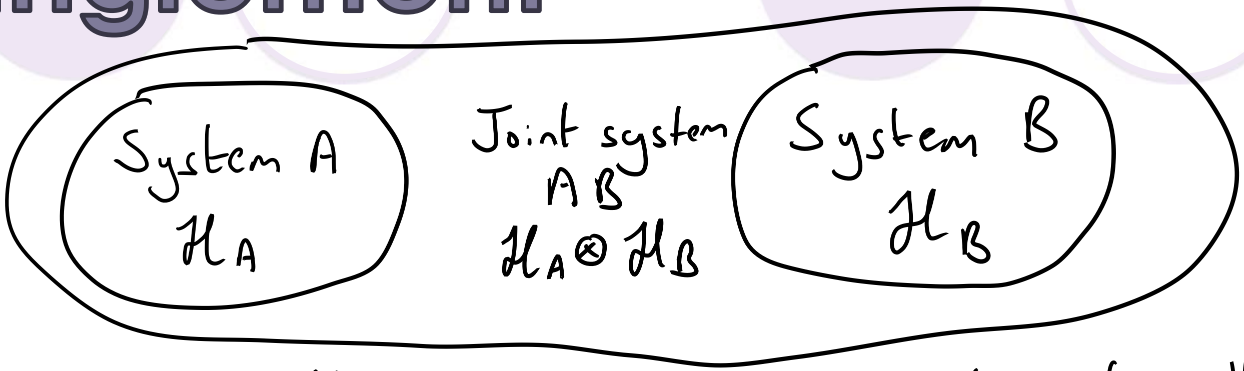
Comments on the Measurement Problem

- For a purely operational interpretation, i.e. the only things that exist are the outcomes of measurements, the problem, as we have stated it, does not apply.
- However, measurement is an undefined primitive on this approach, and we still have the problem that there is no principle that tells us when to apply the measurement postulates and when to apply unitary dynamics.
- In practice, we usually know when to apply the measurement postulates, but a fundamental theory that has measurement as an undefined primitive is arguably incomplete.

6.iv) The Einstein-Podolsky-Rosen (EPR) Argument

- In 1935, Einstein, Podolsky and Rosen pointed out a conflict between orthodox quantum mechanics and locality. – A. Einstein, B. Podolsky, N. Rosen, "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?," *Phys. Rev.*, vol. 47 pp. 777–780 (1935).
- "When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become entangled." – E. Schrödinger "Discussion of Probability Relations Between Separated Systems," *Proc. Cambridge Phil. Soc.*, 31, pp. 555–563 (1935).

Entanglement



- The Hilbert space $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$ consists of all states of the form

$$|\psi\rangle_{AB} = \sum_{jk} \alpha_{jk} |j\rangle_A \otimes |k\rangle_B$$

- A state $|\psi\rangle_{AB}$ is a product state if it can be written as

$$|\psi\rangle_{AB} = |\phi\rangle_A \otimes |\chi\rangle_B \quad \text{for some} \quad |\phi\rangle_A \in \mathcal{H}_A, \ |\chi\rangle_B \in \mathcal{H}_B$$

- Otherwise it is an entangled state
- According to the orthodox interpretation A and B have no individual properties when AB is entangled.

Entanglement

- For 2-qubits it is straightforward to prove that

$$|\psi\rangle_{AB} = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

is a product state iff

$$\alpha_{00}\alpha_{11} = \alpha_{01}\alpha_{10}$$

- So, in particular,

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)$$

is an entangled state.

- Note: If $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $\vec{n}\cdot\vec{\sigma} = n_1\sigma_1 + n_3\sigma_3$ with $\vec{n} = \begin{pmatrix} n_1 \\ n_3 \end{pmatrix}$ a unit vector in the x-z plane then

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|\vec{n}_+\rangle_A|\vec{n}_+\rangle_B + |\vec{n}_-\rangle_A|\vec{n}_-\rangle_B\right)$$

with $\vec{n}\cdot\vec{\sigma}\,|\vec{n}_\pm\rangle = \pm|\vec{n}_\pm\rangle$

Partial Measurement

- If we measure one of the subsystems of a joint system in a complete orthonormal basis, then after the measurement the state gets updated to a product state.

- Joint system starts in state

$$|\psi\rangle_{AB} = \sum_{jk} \alpha_{jk} |j\rangle_A \otimes |k\rangle_B$$

- A is measured in basis $\{|\phi_m\rangle\}$, outcome $|\phi_n\rangle$ is obtained.
- B gets updated to

$$\frac{{}_A\langle \phi_n|\psi\rangle_{AB}}{\| {}_A\langle\phi_n|\psi\rangle_{AB}\|^2} = \frac{\sum_{jk} \alpha_{jk} \langle\phi_n|j\rangle\ |k\rangle_B}{\sum_k |\alpha_{jk}\langle\phi_n|j\rangle|^2}$$

Partial Measurement

- In particular, if $|\Phi^+\rangle_{AB}$ is measured in the basis $\{|0\rangle_A, |1\rangle_A\}$ then system B ends up in the state

$|0\rangle_B$ if $|0\rangle_A$ is found

or $|1\rangle_B$ if $|1\rangle_A$ is found

- More generally, if $|\Phi^+\rangle_{AB}$ is measured in the basis $\{|\vec{n}+\rangle_A, |\vec{n}-\rangle_A\}$ then B ends up in

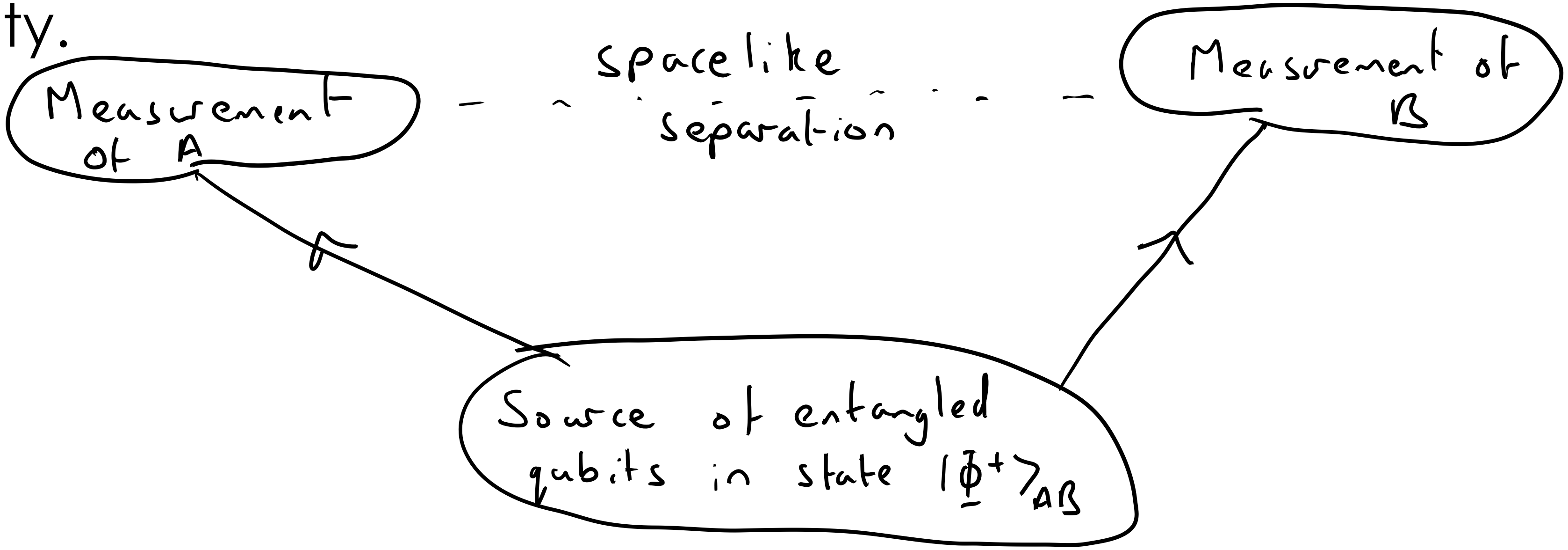
$|\vec{n}+\rangle_B$ if $|\vec{n}+\rangle_A$ is found

$|\vec{n}-\rangle_B$ if $|\vec{n}-\rangle_A$ is found

- We will be able to predict the result of a $\vec{n}\cdot\vec{\sigma}$ measurement on system B with certainty.

The EPR Criterion of Reality

- "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity." – A. Einstein, B. Podolsky, N. Rosen, "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?," *Phys. Rev.*, vol. 47 pp. 777–780 (1935).
- We can ensure that a measurement of A "does not disturb" B by locality.



The EPR Argument

- By the EPR criterion and locality, system B must have an element of reality that determines the outcome of a $\{|0\rangle_B, |1\rangle_B\}$ measurement before A is measured.
- The orthodox interpretation is nonlocal, because this "pops into existence" from nothing when A is measured.
- But note: Any interpretation in which measurement of $\{|0\rangle_B, |1\rangle_B\}$ is undetermined before A is measured would also be nonlocal by the EPR criterion.
- Note that, because of the perfect correlations in all $\{|\vec{n}+\rangle, |\vec{n}-\rangle\}$ measurements, the same is true for all possible measurement directions. Having all of these elements of reality would violate the uncertainty principle for B.
  - This is irrelevant to the main argument, which holds for just one measurement.
  - However, one can use this to show that a local theory must also be ψ-epistemic – N. Harrigan, R. Spekkens, Found. Phys. 40, 125 (2010).
- Bell's Theorem will show that no completion of quantum theory can restore locality.

6.v) The No-Cloning Theorem

- If $0 < |\langle\phi|\psi\rangle| < 1$ then there is no physical operation that outputs $|\psi\rangle \otimes |\psi\rangle$ when $|\psi\rangle$ is input and also $|\phi\rangle \otimes |\phi\rangle$ when $|\phi\rangle$ is input.

Proof:

- Physical operations must be unitary, so let $|\chi\rangle$ be a fixed state on the same Hilbert space as $|\psi\rangle$ and $|\phi\rangle$.
- A cloning unitary would satisfy

$$U|\psi\rangle\otimes|\chi\rangle = |\psi\rangle\otimes|\psi\rangle \qquad U|\phi\rangle\otimes|\chi\rangle = |\phi\rangle\otimes|\phi\rangle$$

- Unitaries preserve inner products, so

$$\langle\phi|\otimes\langle\chi|U^\dagger U|\psi\rangle\otimes|\chi\rangle = \left(\langle\phi|\otimes\langle\phi|\right)\left(|\psi\rangle\otimes|\psi\rangle\right)$$

$$\Rightarrow \quad \langle\phi|\psi\rangle\langle\chi|\chi\rangle = \langle\phi|\psi\rangle^2$$

$$\Rightarrow \quad \langle\phi|\psi\rangle = \langle\phi|\psi\rangle^2 \qquad \Rightarrow \quad |\langle\phi|\psi\rangle| = 0 \text{ or } \underline{1}$$

Comments on No-Cloning

- No-cloning is related to a number of other key features of quantum theory:
  - If we could perfectly clone, we could create an arbitrarily large number of copies of the initial state. Would allow us to determine the state exactly from just one initial copy.
  - This would allow us to signal superluminally in the EPR experiment (consider what would happen if we could clone state of B after measurement of A).
  - Could measure any observable without disturbing the state of the system (just clone first and put one copy to the side).
- So its good that no-cloning holds, but we should explain why. In particular, if the quantum state really exists then why should it be uncopiable? (suggests ψ-epistemic interpretation).

7) Abstract Tensor Systems

i. Vectors, Dual Vectors, Inner Products, and Tensor Products
ii. Abstract Index Notation
iii. More Interesting Tensor Products
iv. Diagrammatic Notation
v. Raising and Lowering Indices
vi. Transpose, Conjugate, and Duals
vii. The Space of Linear Operators
viii. Application: Quantum Teleportation

7.i) Vectors, Dual Vectors, Inner Products, and Tensor Products

- In quantum mechanics, the (pure) states of a quantum system are vectors ("kets") $|\psi\rangle \in V$ in a vector space V.
- We can also define dual vectors $\langle g| \in V^\dagger$ as linear functions from V to $\mathbb{C}$.

$$\langle g| : V \to \mathbb{C}$$
$$\langle g|(a|\psi\rangle + b|\phi\rangle) = a\langle g|(|\psi\rangle) + b\langle g|(|\phi\rangle)$$

- If we define $(a\langle f| + b\langle g|)(|\psi\rangle) = a\langle f|(|\psi\rangle) + b\langle g|(|\psi)$ then $V^\dagger$ is a vector space called the *dual vector space*.

Inner Products

- An inner product on a vector space V is a functional $V \times V \to \mathbb{C}$ satisfying
  - Conjugate symmetry $(|\phi\rangle, |\psi\rangle) = (|\psi\rangle, |\phi\rangle)^*$
  - Linearity: $(|\chi\rangle, \alpha|\psi\rangle + \beta|\phi\rangle) = \alpha(|\chi\rangle, |\psi\rangle) + \beta(|\chi\rangle, |\phi\rangle)$
  - Positive definiteness: $(|\psi\rangle, |\psi\rangle) \geq 0$

    with $(|\psi\rangle, |\psi\rangle) = 0$ iff $|\psi\rangle =$ the zero vector
- A vector space with an inner product is called a Hilbert space (we are in finite dimensions, so ignoring ∞-dimensional complications)
- We usually use $\mathcal{H}$ to denote a Hilbert space.

Inner Products and Dual Vectors

- In a Hilbert space, the inner product induces an isomorphism $\mathcal{H} \equiv \mathcal{H}^\dagger$
- Given a vector $|\psi\rangle \in \mathcal{H}$, we can define a dual vector $\langle\psi| \in \mathcal{H}^\dagger$ via

$$\langle\psi|\phi\rangle = (|\psi\rangle, |\phi\rangle)$$

Linearity of inner product ensures this is a linear functional.

- Given a dual vector $\langle g| \in \mathcal{H}^\dagger$, let $\{|j\rangle\}$ be an orthonormal basis for $\mathcal{H}$ and define $g_j = \langle g|j\rangle$

Then define $|g\rangle = \sum_j g^{\dagger j} |j\rangle$ with $g^{\dagger j} = g_j^*$

Straightforward to prove that this is an isomorphism.

Tensor Products and Partial Inner Products

- Suppose $\mathcal{H}_A$ has an orthonormal basis $|j\rangle_A$ $\quad |\psi\rangle_A = \sum_j \psi^j |j\rangle_A$

  $\mathcal{H}_B$ " " " " $|k\rangle_B$ $\quad |\phi\rangle_B = \sum_k \phi^k |k\rangle_B$

- The tensor product $\mathcal{H}_A \otimes \mathcal{H}_B$ is the vector space spanned by $|j\rangle_A \otimes |k\rangle_B$

$$|\psi\rangle_{AB} = \sum_{jk} \psi^{jk} |j\rangle_A \otimes |k\rangle_B$$

- It is a Hilbert space, inheriting its inner product from $\mathcal{H}_A$ and $\mathcal{H}_B$

  i.e. if $|\phi\rangle_{AB} = \sum_{jk} \phi^{jk} |j\rangle_A \otimes |k\rangle_B$

$$\langle\phi|\psi\rangle_{AB} = \sum_{\substack{jk \\ lm}} \phi^{\dagger}_{jk} \psi^{lm} \langle j|l\rangle_A \langle k|m\rangle_B = \sum_{jk} \phi^{\dagger}_{jk} \psi^{jk} \quad \text{where } \phi^{\dagger}_{jk} = \phi^{jk*}$$

7.ii) Abstract Index Notation

- It is cumbersome to keep track of long strings of bras/kets

e.g. $|j\rangle_A \otimes |k\rangle_B \otimes |l\rangle_C \otimes \ldots\ldots$

- We can develop an abstract index notation similar to that used in differential geometry and GR.

$$|\psi\rangle_A = \sum_j \psi^j |j\rangle_A \Rightarrow \psi^{j_A} \qquad {}_A\langle\phi| = \sum_j \phi_j \, {}_A\langle j| \Rightarrow \phi_{j_A}$$

$$\langle\phi|\psi\rangle_A = \phi_{j_A}\psi^{j_A}$$ ← summation convention for repeated indices

- It is necessary to include the label A of the Hilbert space $\mathcal{H}_A$ in the index j_A because Hilbert spaces may have different dimensions

Only upper A indices can be contracted with lower A indices.

Abstract Index Notation

- For a tensor product space $\mathcal{H}_A \otimes \mathcal{H}_B$, we would have

$$|\psi\rangle_{AB} = \sum_{jk} \psi^{jk} |j\rangle_A \otimes |k\rangle_B \implies \psi^{j_A k_B}$$

$${}_{AB}\langle\phi| = \sum_{jk} \phi_{jk} \, {}_A\langle j| \otimes {}_B\langle k| \implies \phi_{j_A k_B}$$

- The inner product is

$$\langle\phi|\psi\rangle_{AB} = \phi_{j_A k_B} \psi^{j_A k_B}$$

- However $\phi_{j_A k_B} \psi^{k_A j_B}$ is not a valid contraction