Quantum Foundations Lecture 9

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HSC112

6) Phenomenology Of Quantum Mechanics

i. Interference

ii. Tensor Products and Entanglement

iii. Orthodoxy and the Measurement Problem

iv. The Einstein-Podolsky-Rosen Argument

v. The No-Cloning Theorem

2.i) Interference

- Feynman on the double slit experiment:

"We choose to examine a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery. We cannot make the mystery go away by "explaining" how it works. We will just *tell* you how it works. In telling you how it works we will have told you about the basic peculiarities of all quantum mechanics." – Feynman Lectures on Physics Vol. III 1-1

- I completely disagree with this quote, but quantum interference is one of the things we shall have to explain. Let's simplify and look at a photon in an interferometer.

Single Photon Interferometry

Single photon source 0 ──── mode 0
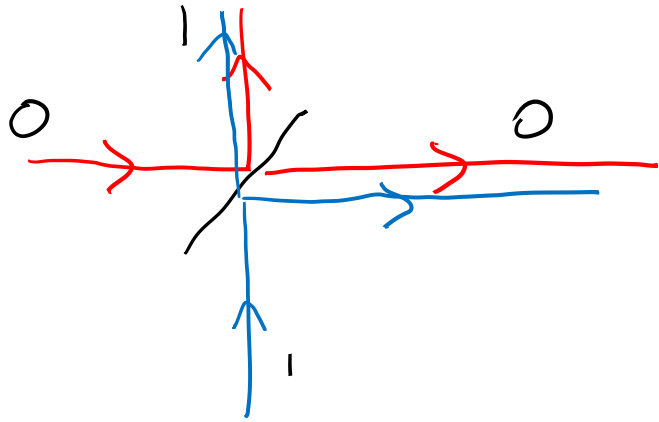
Single photon source 1 ──── mode 1

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ photon in mode 0}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ photon in mode 1}$$

○ We only consider single photon states, so we have a qubit.

○ We always take the two path lengths through the interferometer to be the same, so we only keep track of phase differences due to optical elements

Beam Splitters

○ We always give the transmitted output mode the same label as the input mode, and opposite for the reflected mode

○ Action of beamsplitter is given by a 2×2 unitary matrix
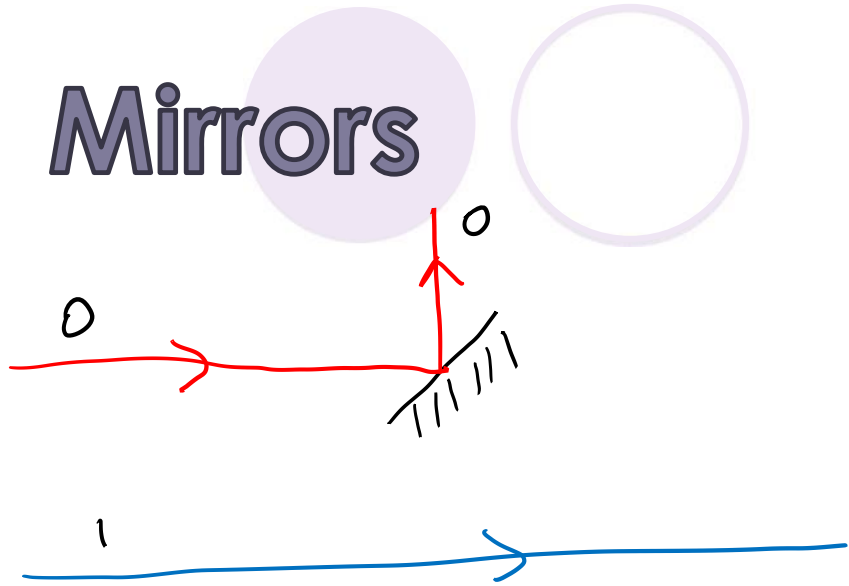
$$B = \begin{pmatrix} t & r \\ r & t \end{pmatrix}$$

t = coefficient of transmission
r = coefficient of reflection

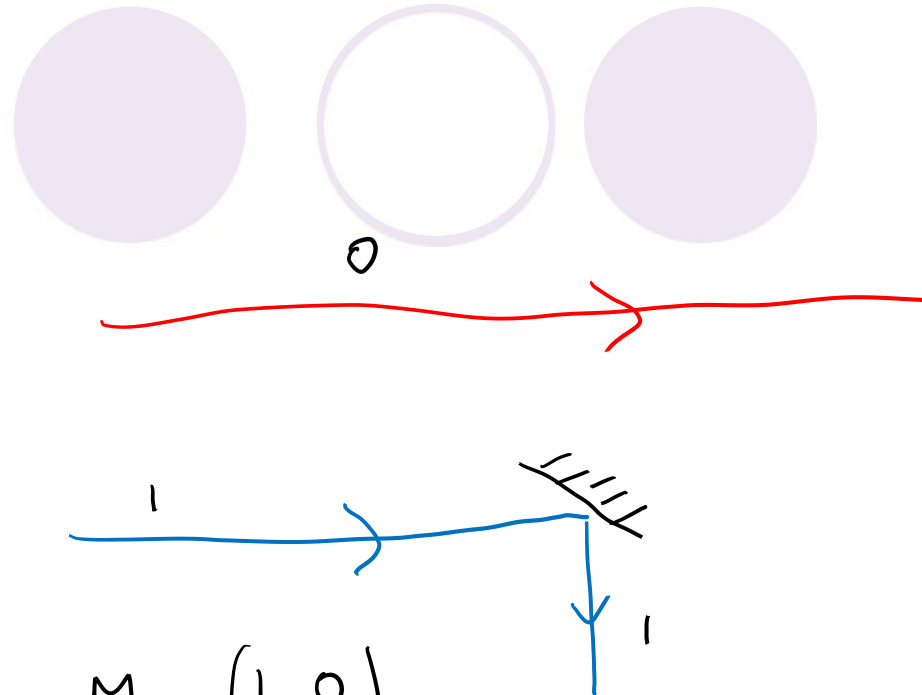○ $T = |t|^2$ is the transmittivity $R = |r|^2$ is the reflectivity

$$T + R = 1 \quad \text{by unitarity}$$

○ A 50/50 beamsplitter has $B = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$
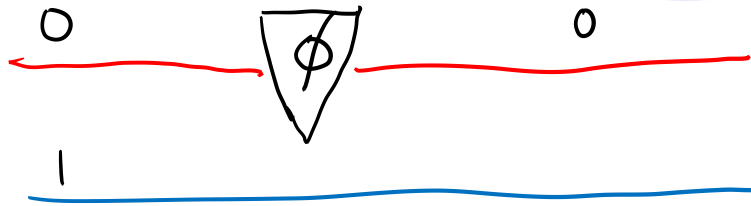
Mirrors

O

O

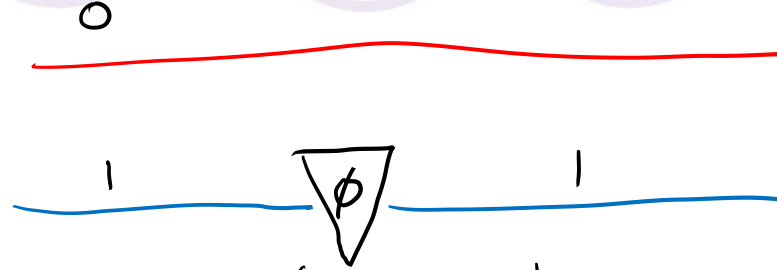$M_0 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

O

1

1

$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

- Reflection at a mirror induces a π-phase change, so action is given by M_0 or M_1

- Remember, we will always make total path length of 0 and 1 the same, so we don't have to track more than this.

Phase Shifters

0 —————— ϕ —————— 0

1 ——————————————

$$A_{\phi,0} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$$

0 ——————————————

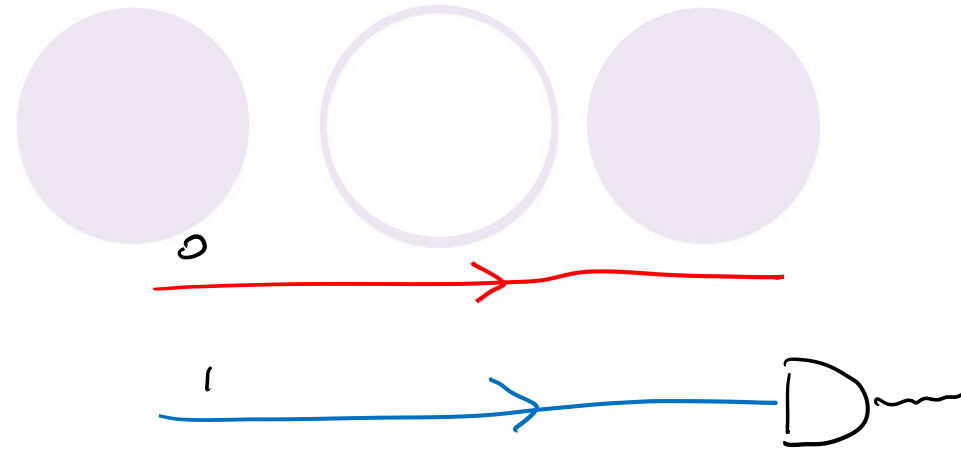1 —————— ϕ —————— 1

$$A_{\phi,1} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

○ We can induce a relative phase between modes 0 and 1 by inserting a block of refractive material in one of the paths, reducing the wavelength

○ By altering the length/refractive index of the material, we can induce any relative phase difference ϕ we like.

Detectors

0

1

$$Prob(0) = |\langle 0 | \psi \rangle|^2$$

0

1
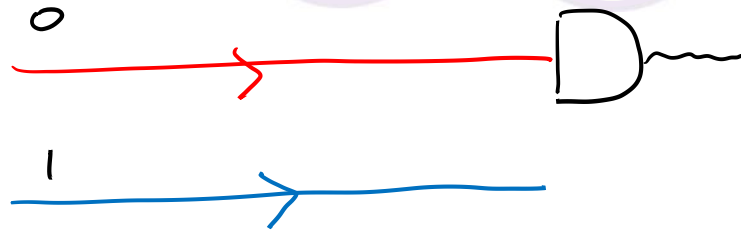
$$Prob(1) = |\langle 1 | \psi \rangle|^2$$

⊙ If we place a detector in path 0/1, it will click with the probabilities indicated.
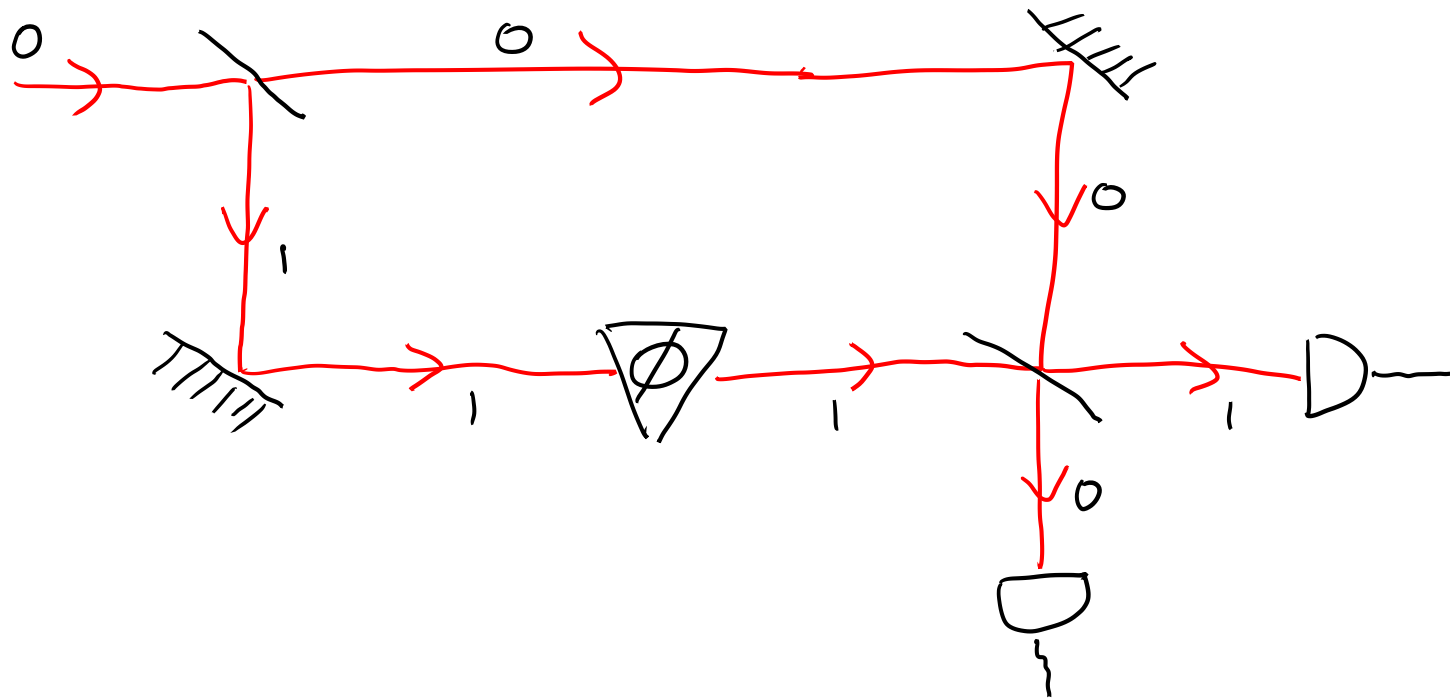
⊙ If the measurement is ideal/nondemolition/nondestructive, then after detection, the state is updated to

     Found in path 0: $|\psi\rangle \longrightarrow |0\rangle$

     Found in path 1: $|\psi\rangle \longrightarrow |1\rangle$

Hard to do in practice.

Mach-Zehnder Interferometer

○ A general Mach-Zehnder interferometer has 50/50 beamsplitters and a phase shift ϕ on path 1.

○ The state just before detection is

$$|\psi\rangle = B \, A_{\phi,1} \, M_1 \, M_0 \, B \, |0\rangle$$

⊙ You will show that $\quad \text{Prob}(0) = \sin^2\left(\frac{\phi}{2}\right) \qquad \text{Prob}(1) = \cos^2\left(\frac{\phi}{2}\right)$

⊙ With $\phi = 0$, the photon is always detected on path 1 due to constructive interference on path 1 destructive " " " " 0.

Feynman's Interference "Paradox"

- Classically, particles and waves are mutually exclusive.
- The interference at the second beamsplitter indicates that the photon is behaving like a wave.
- But the fact that only one detector clicks at a time, indicates that it is behaving like a particle.
- If it is a particle, it ought to travel along either path 0 or path 1.
- So what happens if we try to check which one is the case?

Feynman's Interference "Paradox"

⊙ Suppose we put non-destructive detectors on both of the paths
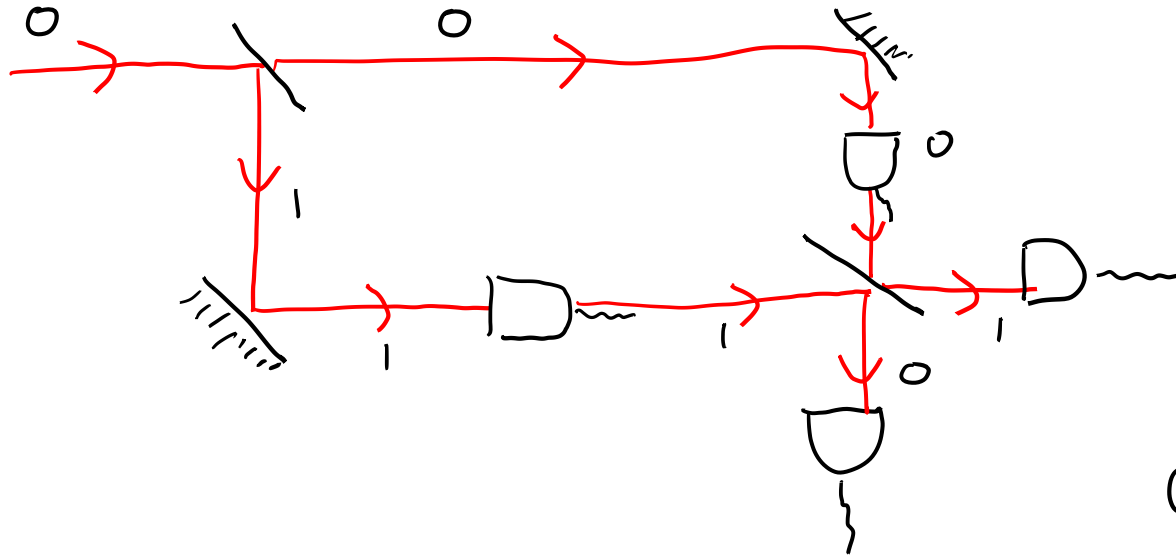
⊙ Before path detectors, state is

$$|\psi\rangle = M_1 M_0 B |0\rangle$$
$$= -\frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right)$$

⊙ ∴ Each detector clicks with 50/50 probability

⊙ Afterwards, the state will be $|0\rangle$ or $|1\rangle$ with 50/50 probability

⊙ Before final detectors, state will be either

$$B|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right) \quad \text{or} \quad B|1\rangle = \frac{1}{\sqrt{2}}\left(i|0\rangle + |1\rangle\right)$$

⊙ ∴ Final detectors will each click with 50/50 probability

⊙ Attempting to check which path the photon travels along destroys any interference.

Traditional Resolution

- Wave-particle duality: A photon will behave either as a particle or as a wave, depending on how the experiment is set up.
- If we set up an experiment to detect particle paths, there will be no interference.
- If we set up an experiment to detect interference, we necessarily cannot say which path the photon travels along.
- The question of which path a photon travels along during an interference experiment has no meaning.
  - People sometimes say "it travels along both paths" or "neither path", but "has no meaning" is closer to Copenhagen and textbook accounts.

Criticism of Traditional Resolution

- It assumes that photons must behave either like waves or particles.
  - Why not both at the same time (c.f. de Broglie-Bohm theory)?
  - Why not something else entirely?
- It assumes that having a property is synonymous with being able to measure that property without disturbing anything else.
  - This may be true if you are an operationalist.
  - In a realist account, why can't the photon have a trajectory that is either unmeasurable or not measurable without disturbing some other property that is responsible for interference.
- Interference is rather weak evidence for "quantum weirdness".
  - We shall have to explain it, but there are much more difficult problems.

Elitzur-Vaidman Bomb Test

- This is a wrinkle on the Feynman paradox that makes it much more dramatic.

- Consider a very sensitive bomb that explodes as soon as even the tiniest amount of electromagnetic radiation is incident on it.

- Can we detect whether a bomb is good or a dud without blowing ourselves up?

  - Classically: No. The only way to tell is to shine light on it and see if it blows up.

  - Quantumly: Yes. We can use Mach-Zehnder.

Elitzur-Vaidman Bomb Test
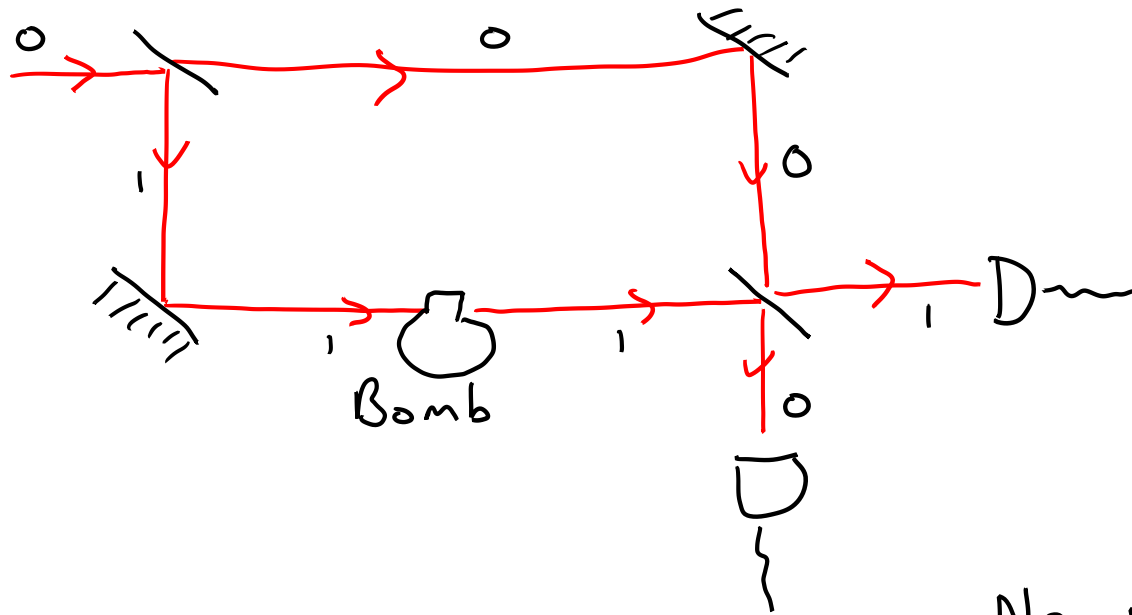
⊙ If bomb is a dud, it does nothing, so we have an ordinary M-Z interferometer
  - Photon will always be detected on output path 1.

⊙ If bomb is good it will act as a detector

KABOOM ⟹ Photon is on path 1

No KABOOM ⟹ Photon is on path 0

⊙ But if we learn that photon is on path 0, it will be detected on output path 0 or 1 with 50/50 probability.

⊙ Conclusion: If detector 0 clicks, we know the bomb is good <u>and</u> that the photon did not touch the bomb. This happens with prob $\frac{1}{2}$.

Elitzur-Vaidman Bomb Test

- Note: We can make the probability of detection without kaboom $1 - \epsilon$ for any $\epsilon > 0$ by using a more sophisticated interferometer (see Hwk2).

- It is sometimes claimed that EV bomb test is evidence for nonlocality. We know for sure that the photon was nowhere near the bomb, but the presence of the bomb still influences what happens to it.

  - This assumes that if the photon goes along path 0 then there is literally nothing that goes along path 1 that could mediate the influence.

  - QFT should make us skeptical of this, as the quantum vacuum has substructure (see epistricted theories later in course).

2.ii) Tensor Products and Entanglement

- Given a system A with Hilbert space $\mathcal{H}_A$ and another with Hilbert space $\mathcal{H}_B$, we need a way of constructing the state space $\mathcal{H}_{AB}$ of the composite system AB.

- Assuming our systems are distinguishable, $\mathcal{H}_{AB}$ is the *tensor product* of $\mathcal{H}_A$ and $\mathcal{H}_B$, denoted

$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B.$$

- Clearly, if $|\psi\rangle_A$ is a state of A and $|\phi\rangle_B$ is a state of B, then we want there to be a state of AB, where A has state $|\psi\rangle_A$ and B has state $|\phi\rangle_B$. This state is called a *product* state and is denoted

$$|\psi\rangle_A \otimes |\phi\rangle_B \quad \text{OR} \quad |\psi\rangle_A|\phi\rangle_B \quad \text{OR} \quad |\psi\phi\rangle_{AB}$$

- But this is not yet a vector space because it is not closed under linear combinations.

Tensor Products

- To form a vector space, we must close under linear combinations, so the *tensor product* $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$ is defined as the set of all vectors of the form

$$|\psi\rangle_{AB} = \sum_{jk} a_{jk} |\phi_j\rangle_A \otimes |\chi_k\rangle_B,$$

$$\text{where } |\phi_j\rangle_A \in \mathcal{H}_A \text{ and } |\chi_k\rangle_B \in \mathcal{H}_B.$$

- But it is not enough to just be a vector space, we need an inner product (Hilbert) space.

- To define an inner product, we start with

$$(\langle\psi|_A \otimes \langle\phi|_B)(|\chi\rangle_A \otimes |\eta\rangle_B) = \langle\psi|\chi\rangle_A \langle\phi|\eta\rangle_B$$

$$\text{and then extend by linearity.}$$

Tensor Products

⊙ Specifically, if

$$|\psi\rangle_{AB} = \sum_{jk} a_{jk} |\phi_j\rangle_A \otimes |\chi_k\rangle_B$$

$$|\eta\rangle_{AB} = \sum_{jk} b_{jk} |\mu_j\rangle_A \otimes |\nu_k\rangle_B$$

then

$$\langle\psi|\eta\rangle_{AB} = \sum_{jklm} a_{jk}^* b_{lm} \langle\phi_j|\mu_l\rangle_A \langle\chi_k|\nu_m\rangle_B$$

Tensor Products

- **Proposition**: If $\{|j\rangle_A\}$ is an orthonormal basis for $\mathcal{H}_A$ and $\{|k\rangle_B\}$ is an orthonormal basis for $\mathcal{H}_B$ then $\{|j\rangle_A \otimes |k\rangle_B\}$ is an orthonormal basis for $\mathcal{H}_A \otimes \mathcal{H}_B$.

- Proof: To prove orthonormality, we compute

$$(\langle j|_A \otimes \langle k|_B)(|l\rangle_A \otimes |m\rangle_B) = \langle j|l\rangle_A \langle k|m\rangle_B = \delta_{jl}\delta_{km}.$$

- Now consider a product state $|\psi\rangle_A \otimes |\phi\rangle_B$. We can write these as

$$|\psi\rangle_A = \sum_j a_j |j\rangle_A \qquad |\phi\rangle_B = \sum_k b_j |k\rangle_B$$

and hence $|\psi\rangle_A \otimes |\phi\rangle_B = \sum_{jk} a_j b_k |j\rangle_A \otimes |k\rangle_B.$

- Since any state in the tensor product is a linear combination of product states, it can also be written as a linear combination of the basis states $|j\rangle_A \otimes |k\rangle_B$.

Entanglement

- A state $|\psi\rangle_{AB} \in \mathcal{H}_A \otimes \mathcal{H}_B$ is a *product state* if it can be written as
$$|\psi\rangle_{AB} = |\phi\rangle_A \otimes |\chi\rangle_B$$
for some $|\phi\rangle_A \in \mathcal{H}_A$ and $|\chi\rangle_B \in \mathcal{H}_B$.
- Otherwise it is called an *entangled state*.
- Examples:
  - Product state:
$$\frac{1}{2\sqrt{2}}|00\rangle_{AB} - \frac{1}{2\sqrt{2}}|01\rangle_{AB} + \frac{\sqrt{3}}{2\sqrt{2}}|10\rangle_{AB} - \frac{\sqrt{3}}{2\sqrt{2}}|11\rangle_{AB} = \left(\frac{1}{2}|0\rangle_A + \frac{\sqrt{3}}{2}|1\rangle_A\right) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle_B - \frac{1}{\sqrt{2}}|1\rangle_B\right)$$
  - Entangled state:
$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_A + |11\rangle_B)$$

Joint and Partial Measurements

- Suppose Alice measures an orthonormal basis $\{|\phi_j\rangle_A\}$ on system A and Bob measures $\{|\chi_k\rangle_B\}$ on system B. The joint probabilities for their oucomes are:

$$\text{Prob}(j,k) = {}_{AB}\langle\psi|\left(|\phi_j\rangle\langle\phi_j|_A \otimes |\chi_k\rangle\langle\chi_k|_B\right)|\psi\rangle_{AB}$$

- Suppose Alice obtains the outcome j and we would like to know the conditional probability $\text{Prob}(k|j) = \text{Prob}(j,k)/\text{Prob}(j)$ for Bob to obtain outcome k. This can be computed as follows:

$$\text{Prob}(k|j) = \frac{\text{Prob}(j,k)}{\text{Prob}(j)} = \frac{{}_{AB}\langle\psi|\left(|\phi_j\rangle\langle\phi_j|_A \otimes |\chi_k\rangle\langle\chi_k|_B\right)|\psi\rangle_{AB}}{\sum_k {}_{AB}\langle\psi|\left(|\phi_j\rangle\langle\phi_j|_A \otimes |\chi_k\rangle\langle\chi_k|_B\right)|\psi\rangle_{AB}}$$

$$= \frac{\left({}_{AB}\langle\psi|\phi_j\rangle_A\right)\left(|\chi_k\rangle\langle\chi_k|_B\right)\left({}_A\langle\phi_j|\psi\rangle_{AB}\right)}{\|{}_A\langle\phi_j|\psi\rangle_{AB}\|^2} = \langle\psi_j|\chi_k\rangle_B\langle\chi_k|\psi_j\rangle_B$$

Joint and Partial Measurement

- $|\psi_j\rangle_B = \langle\phi_j|\psi\rangle_B / \left\||\langle\phi_j|\psi\rangle_B\right\|$ is the correct state to use for Bob's system after Alice has made her measurement and we know the outcome, but before Bob has made his measurement. It is an example of *the collapse of the wavefunction*.

- Example: Suppose $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$. If Alice measures $\{|0\rangle_A, |1\rangle_A\}$ and gets the $|0\rangle_A$ outcome then

$$\langle 0|\psi\rangle_B = \langle 0|_A \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$$

$$= \frac{1}{\sqrt{2}}(\langle 0|0\rangle_A|0\rangle_B + \langle 0|1\rangle_A|1\rangle_B) = \frac{1}{\sqrt{2}}|0\rangle_B$$

and if we divide by the norm we get $|\psi_0\rangle_B = |0\rangle_B$.

2.iii) Orthodoxy and the Measurement Problem

- Macroscopic superpositions and the measurement problem are often thought to be the most pressing problems in the foundations of quantum theory.
- But they have been solved multiple times. They are not problems with quantum theory per se, but rather with the interpretation of quantum theory usually given in textbooks.
  - This is why I prefer the three-point definition of the problem given in lecture 1.
- This is known as the Orthodox, Textbook, or Dirac-von Neumann interpretation.
- It is often mislabeled as the Copenhagen interpretation, but it differs so drastically from the views of Bohr, Heisenberg, etc. that it is not even in the same category.
  - The orthodox interpretation is realist, straightforward, and obviously wrong.
  - The Copenhagen interpretation is anti-realist, subtle, and wrong for interesting reasons (see lecture 13).

The Orthodox Interpretation

1. Physical systems have objective properties (this makes it realist):
   - The possible properties of a system are its observables. The possible values of those properties are the corresponding eigenvalues.

2. The eigenvalue-eigenstate link:
   - When the system is in an eigenstate of an observable M with eigenvalue m then M is a property of the system and it takes value m.
   - The system has no objective physical properties other than these.