Quantum Foundations Lecture 7

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Dr. Matthew Leifer
leifer@chapman.edu
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4) Postulates of Quantum Mechanics (As in Undergrad QM)

1. **State Space**: A physical system A is associated with a Hilbert space $\mathcal{H}_A$.
   - A Hilbert space is just an inner product space with extra properties needed to make the infinite dimensional case work correctly (which we won't worry about).
2. **States**: A (pure) state of a physical system is a unit vector

   $|\psi\rangle_A \in \mathcal{H}_A$ (up to a global phase).
3. **Dynamics**: An isolated (not interacting with the environment or being measured) system evolves according to the Schrödinger equation

$$i\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle.$$

Equivalently, dynamics is unitary $|\psi(t)\rangle = U(t)|\psi(0)\rangle, \quad U^{\dagger}(t)U(t) = I$

4) Postulates of Quantum Mechanics

4. **Observables**: Measurable physical quantities correspond to self-adjoint matrices $M^\dagger = M$, which can be written in spectral form

$$M = \sum_j \lambda_j P_j.$$

The possible outcomes of a measurement of M are the eigenvalues λ_j.

4. **The Born rule**: When M is measured on a system assigned the state $|\psi\rangle$, the outcome λ_j occurs with probability $\mathrm{Prob}(\lambda_j|\psi) = \langle\psi|P_j|\psi\rangle$.

5. **Composite systems**: A system AB composed of two subsystems, A with Hilbert space $\mathcal{H}_A$ and B with Hilbert space $\mathcal{H}_B$ has a Hilbert space

$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$$

where $\otimes$ denotes the tensor product.

Dynamics of Quantum Systems

- Quantum systems obey the *superposition principle*: If $|\psi_1(t)\rangle$ and $|\psi_2(t)\rangle$ are solutions to the equation of motion then so is
$$\alpha|\psi_1(t)\rangle + \beta|\psi_2(t)\rangle$$
- This implies that time evolution can be described by a linear operator $U(t)$, such that
$$|\psi(t)\rangle = U(t)|\psi(0)\rangle$$
- In order to conserve probabilities, dynamics should preserve normalization. To see this, consider an orthonormal basis $\{|j\rangle\}$. The Born rule probability of getting outcome $|j\rangle$ at time $t = 0$ is
$$\mathrm{Prob}(j, 0) = |\langle j|\psi\rangle|^2 = \langle\psi|j\rangle\langle j|\psi\rangle$$
- so the probability of getting any outcome is
$$\sum_j \mathrm{Prob}(j, 0) = \sum_j \langle\psi|j\rangle\langle j|\psi\rangle = \langle\psi|\psi\rangle = 1$$

Dynamics of Quantum Systems

- If we want this to also be true at time t then
$$\text{Prob}(j,t) = |\langle j|\psi(t)\rangle|^2 = |\langle j|U(t)|\psi\rangle|^2 = \langle\psi|U^\dagger(t)|j\rangle\langle j|U(t)|\psi\rangle$$
and $\sum_j \text{Prob}(j,t) = \langle\psi|U^\dagger(t)U(t)|\psi\rangle$.
- If we want this to equal 1 then we must have
$$\langle\psi|U^\dagger(t)U(t)|\psi\rangle = \langle\psi|\psi\rangle = \langle\psi|I|\psi\rangle$$
- This implies that $U^\dagger(t)U(t) = I$, which is the definition of a *unitary* matrix.
- We also want $U(0) = I$, and we want evolution to be continuous, so suppose, for small Δt,
$$U(t) = I + \Delta t A$$
to first order for some matrix A.

Dynamics of Quantum Systems

- Now, if $U(t)$ is unitary, then A must be *anti-Hermitian*, i.e. $A^\dagger = -A$.
- Proof: $U^\dagger(\Delta t)U(\Delta t) = I \qquad \Rightarrow \qquad \left(I + \Delta t A^\dagger\right)(I + \Delta t A) = I$

$$I + \Delta t\left(A^\dagger + A\right) + O\left(\Delta t^2\right) = I$$

Therefore, all the terms of order Δt and higher must be zero, so

$$A^\dagger + A = 0 \quad \text{or} \quad A^\dagger = -A.$$

- Therefore, we have

$$\frac{\partial|\psi\rangle}{\partial t} = \lim_{\Delta t \to 0}\left(\frac{|\psi(\Delta t)\rangle - |\psi(0)\rangle}{\Delta t}\right) = A|\psi(0)\rangle$$

- If we now define $H = iA$ then H will be Hermitian $H^\dagger = H$ and

$$\boxed{i\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle}$$

Which is the Schrödinger equation.

5) Philosophical Background

i. Realism vs. Anti-Realism
ii. Making the Problem Precise

5) Philosophical Background

- We have seen an approach to quantum theory (test spaces and GPTs) that is *operational*, i.e. it refers only to the outcomes of experimental procedures, and not properties of systems that exist independently of that.
- Our goal in this section is to make the central question in the foundations of quantum mechanics precise. To do so, we shall have to delve a little more deeply into the philosophy of science.
- See Ladyman on supplementaty reading for more details

5.i) Realism vs. Anti-Realism

- Scientific Realism is the idea that:
  - There exists an objectively real physical world, independent of observers.
  - The job of a physical theory is to attempt to describe it.
  - Successful physical theories are approximately correct descriptions of the objectively real physical world.
- iIt is more accurate to think of theoretical entities, e.g. electrons, quarks, as referring to things that actually exist than to do otherwise.

What realism is not

- Realism does not necessarily mean:
  - All quantum observables must have definite values at all times.
  - The quantum state is a description of reality.
  - Nature must be deterministic.
  - Our theories are literally true.
- Warning: “Realism” is often used in these ways in the literature on quantum foundations. It is much harder to deny “real” realism.

Anti-Realism

- Varieties: idealism, logical positivism, empiricism, instrumentalism, operationalism.
- The only things we have direct access to are our own perceptions and/or the records of results from our experimental apparatuses.
- Theories are simply systems for organizing/predicting regularities in those perceptions/results.
- Theoretical entities, e.g. electrons, are a convenient fiction used in our calculations.
- Operationalism: Every statement of a theory should boil down to a list of instructions for what to do in the lab and what will be seen as a result.

Putnam's "No Miracles" Argument

When they argue for their position, realists typically argue against some version of idealism - in our time, this would be positivism or operationalism. (…) And the typical realist argument against idealism is that it makes the success of science a miracle

(…)The modern positivist has to leave it without explanation (the realist charges) that'electron calculi' and 'space-time calculi' and 'DNA calculi' correctly predict observable phenomena if, in reality, there are no electrons, no curved space-time, and no DNA molecules. If there are such things, then a natural explanation of the success of theories is that they are partially true accounts of how they behave. And a natural account of the way scientific theories succeed each other (…) is that a partially correct/incorrect account of a theoretical object (…) is replaced by a better account of the same object or objects. But if those objects don't really exist at all, then it is a miracle that a theory which speaks of gravitational action at a distance successfully predicts phenomena; it is a miracle that a theory which speaks of curved space-time successfully predicts phenomena; and the fact that the laws of the former theory are derivable 'in the limit' from the laws of the latter theory has no methodological significance

H. Putnam, Meaning and the Moral Sciences, Routledge (1978)

Eddington's Fishy Story

Let us suppose that an ichthyologist is exploring the life of the ocean. He casts a net into the water and brings up a fishy assortment. Surveying his catch, he proceeds in the usual manner of a scientist to systematise what it reveals. He arrives at two generalisations: (1) No sea-creature is less than two inches long. (2) All sea-creatures have gills. These are both true of his catch, and he assumes tentatively that they will remain true however often he repeats it.

In applying this analogy, the catch stands for the body of knowledge which constitutes physical science, and the net for the sensory and intellectual equipment which we use in obtaining it. The casting of the net corresponds to observation; for knowledge which has not been or could not be obtained by observation is not admitted into physical science.

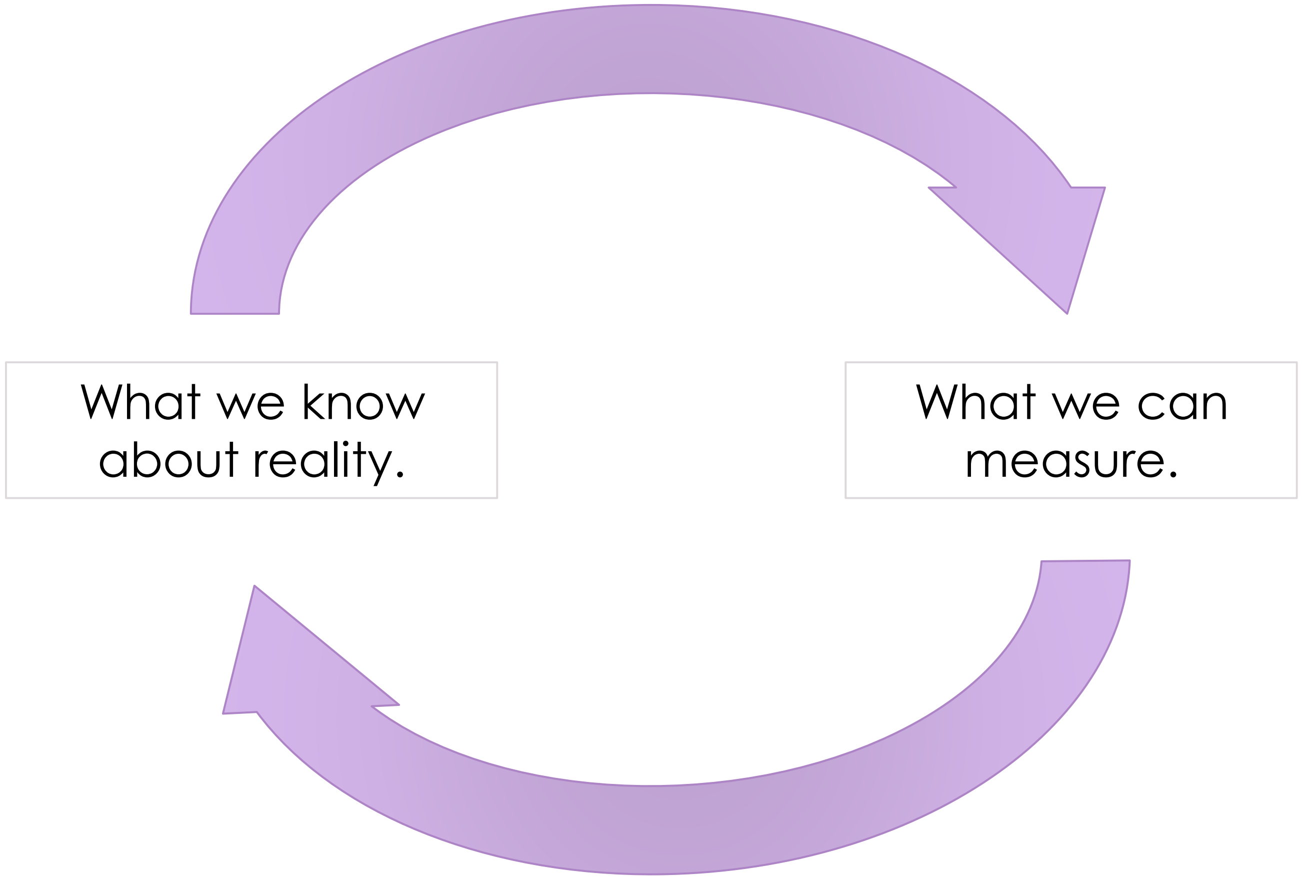
An onlooker may object that the first generalisation is wrong. "There are plenty of sea-creatures under two inches long, only your net is not adapted to catch them." The icthyologist dismisses this objection contemptuously. "Anything uncatchable by my net is *ipso facto* outside the scope of icthyological knowledge. In short, "what my net can't catch isn't fish." Or — to translate the analogy — "If you are not simply guessing, you are claiming a knowledge of the physical universe discovered in some other way than by the methods of physical science, and admittedly unverifiable by such methods. You are a metaphysician. Bah!"

A. Eddington, The Philosophy of Physical Science (1938)

Realism vs. Anti-Realism

- It is undeniable that what we can measure affects what we can theorize about.
  - Anti-Realists insist that this is primary.
- It is also undeniable that we rely on our theories to define/interpret what we can measure.
  - e.g. An anti-realist has a hard time explaining how we can redefine the metre.
  - Realists insist that this is primary.

The Realist/Antirealist Bootstrap



Leiferian Pragmatism: A synthesis?

- At least part of what distinguishes a scientific truth from other types of knowledge is that it is useful.
  - Allows us to make correct predictions.
  - Provides us with an explanatory framework.
  - Leads to new experiments we would not have thought of otherwise.
  - Leads to the development of new technologies.
  - Allows us to generalize a theory beyond its current scope.
- Pragmatism: truth=utility. I don't go as far as that, but it is an important part of the story.

Realism vs. Realist Explanations

- You don't have to be a realist to realize that realist explanations are often useful.
  - If I have a story about what exists and how it behaves then I have a framework for reasoning about novel physical situations and for generalizing the theory.
- It is interesting that all of our physical theories prior to quantum theory admit a realist account, even if you don't believe they are literally (approximately) true.
- We might ask why we cannot find a realist account that we all agree upon for quantum theory.

Operationalism vs. Operational Methodology

- You don’t have to be an operationalist to realize that stepping back from a realist account and temporarily defining things in terms of things we can do in the lab is often useful.
  - E.g. Einstein’s derivation of special relativity, the development of thermodynamics prior to statistical mechanics.
- Since the operational implications of quantum theory are the only part we all agree upon, it may be useful to reformulate the theory operationally and come back to the realism question later.

5.ii) Making the Problem Precise

"We always have had … a great deal of difficulty in understanding the world view that quantum mechanics represents. At least I do, because I'm an old enough man that I haven't got to the point that this stuff is obvious to me. Okay, I still get nervous with it. And therefore, some of the younger students … you know how it always is, every new idea, it takes a generation or two until it becomes obvious that there's no real problem. It has not yet become obvious to me that there's no real problem. **I cannot define the real problem, therefore I suspect there's no real problem, but I'm not sure there's no real problem.**" – R. P. Feynman, "Simulating Physics with Computers", *International Journal of Theoretical Physics*, volume 21, 1982, p. 467-488

5.ii) Making the Problem Precise

If quantum theory is an approximately correct theory of physics:

1. What kinds of things exist and how do they behave?
2. How do they save the phenomena?
3. Do so in such a way that leads to progress in physics.

The ontology question

1. What kinds of things exist and how do they behave?
   - Ontology is the study of what exists, so this is asking for the ontology underlying quantum theory.
   - We are not asking what actually exists in the world, but rather what would exist in a world where quantum theory is an exact theory of physics?
   - It is more about the internal explanatory structure of the theory than about the actual physical world.
   - This does not assume realism: only measurement outcomes exist is an acceptable answer (but you also have to answer the other two questions).

Saving the phenomena

2. How do they save the phenomena?
   - What we see in experiments should be explained in terms of the answer to 1.
   - How do the things that exist give rise to the predictions that we observe in experiments.
   - Explain the emergence of the classical world in terms of the things that exist.
   - E.g. if I say that quantum theory is about a bunch of green aliens running around on mars then that is a valid answer to 1, but will probably fail to provide a compelling answer to 2.

The pragmatic criterion

3. Do so in such a way that leads to progress in physics.
   - Most quantum foundationists will probably agree with 1 and 2. 3 will be more controversial.
   - We already have several viable answers to 1 and 2. How do we know which one is the scientific truth?
     - If it makes a prediction that conflicts with standard quantum theory and is subsequently confirmed.
     - If it allows us to generalize quantum theory beyond its current scope, e.g. to quantum gravity, in a way not suggested by standard quantum theory.
     - If it provides better explanations: i.e. leads to new developments that could have been derived from standard quantum theory, but it would have been difficult to do so.