Does time-symmetry in quantum theory imply retrocausality?

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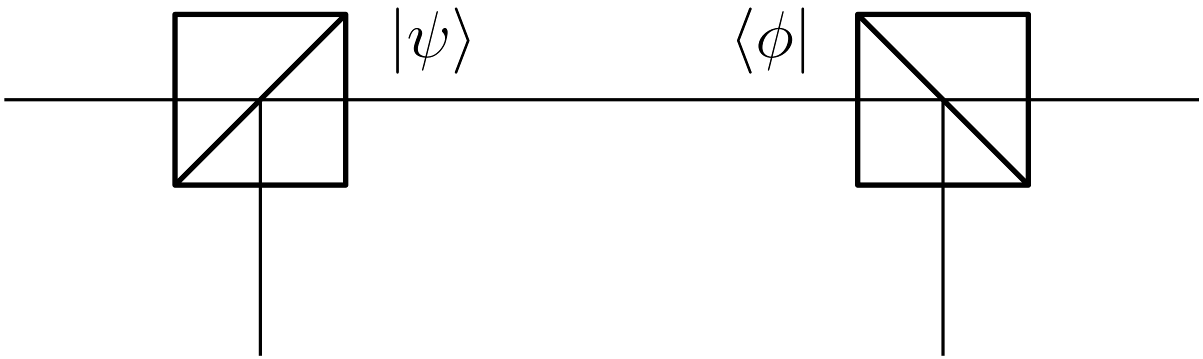
Introduction

Price's Argument for Retrocausality from Time Symmetry

- Huw Price has argued that a time-symmetric realist account of quantum theory should be retrocausal[1].
- His argument is based on an experiment in which a single photon passes through two polarizing beam-splitters.



- Assuming that $|\psi\rangle$ is a beable, he argues that $|\phi\rangle$ must also be real.
- This is an assumption of the reality of the quantum state (ψ-ontology).

[1] H. Price, Stud. Hist. Phil. Mod. Phys. 43:75–83 (2012).

Price's Argument for Retrocausality from Time Symmetry

- Our first goal is to remove the assumption of ψ-ontology, c.f. EPR vs. Bell's theorem.
- Since theories like Bohmian mechanics and Everett/many-worlds are manifestly time symmetric (in the usual physicists sense) and non-retrocausal, Price's argument is based on a stronger notion of time symmetry that these theories do not satisfy.
- Our second goal is to explain the notion of time symmetry that is at play, and extend it beyond polarization experiments. We want to make it independent of the details of quantum theory, like the definition of locality used in Bell's theorem.

Operational Framework

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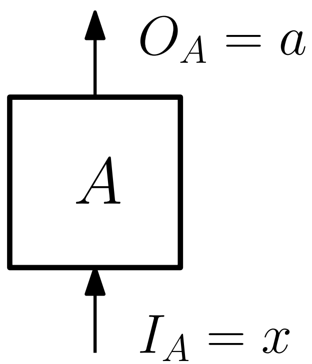
Black Box Operations

- A *Black Box* A has an input I_A and an output O_A, each of which take a finite number of values.



- The experimenter is assumed to have full control over I_A.
- She does not have any further control over O_A.
- The output of the box is described by a conditional probability distribution $p_A(O_A = a|I_A = x)$ (abbreviated $p_A(a|x)$).

Black Box Operations: Examples

- Xander (the experimenter) chooses $I_A = x$ and feeds it into the box, Alice (the person hiding inside the box) learns x and generates a random variable O_A (by tossing coins, rolling dice, etc.) with probability distribution $p_A(a|x)$ then outputs O_A.

- I_A is the setting on a quantum measurement device, O_A is the measurement outcome.

$$p_{a|x} = \mathrm{Tr}\left(E_{a|x}\rho\right)$$

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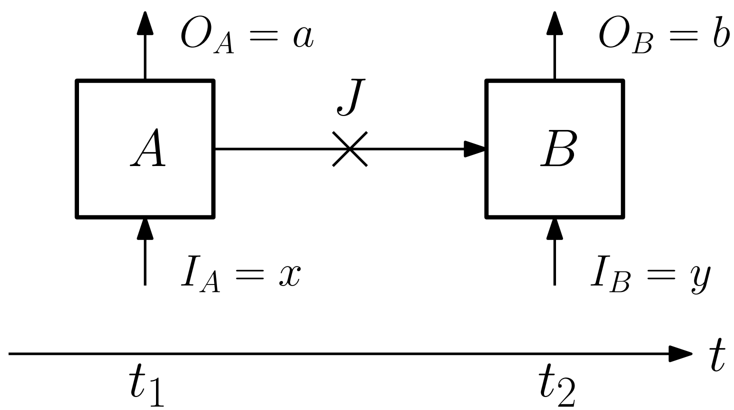
- We will be interested in experiments involving two boxes with a definite time order.



- Each box now has a *type*:

$$T(A) = \alpha,\ T(B) = \beta,\ \cdots .$$

- For each pair of types (α, β), there is a set of possible *joining rules*:

$$\mathcal{J}_{\alpha,\beta} = \{J_1, J_2, \ldots\}$$

- By an *operational theory* we mean
  - A set of possible boxes for t_1:
  $$\mathcal{S}_1 = \{A_1, A_2, \ldots\}.$$
  - A set of possible boxes for t_2:
  $$\mathcal{S}_2 = \{B_1, B_2 \ldots\}.$$
  - For each pair of types (α, β), a set of possible *joining rules*:
  $$\mathcal{J}_{\alpha,\beta} = \{J_1, J_2, \ldots\}.$$
  - For each $A \in \mathcal{S}_1$, $B \in \mathcal{S}_2$, $J \in \mathcal{J}_{\alpha,\beta}$, a joint probability distribution:
  $$p_{ABJ}(a, b|x, y).$$

Two Box experiments: Quantum Experiments

- A type α is associated with a Hilbert space $\mathcal{H}_\alpha$.
- A box $A \in \mathcal{S}_1$ is associated with a probability distribution $p_A(a|x)$ and a set of density operators $\rho_\alpha^{a|x}$ on $\mathcal{H}_\alpha$.
- A joining rule $J \in \mathcal{J}_{\alpha,\beta}$ is associated with a dynamical (CPT) map:

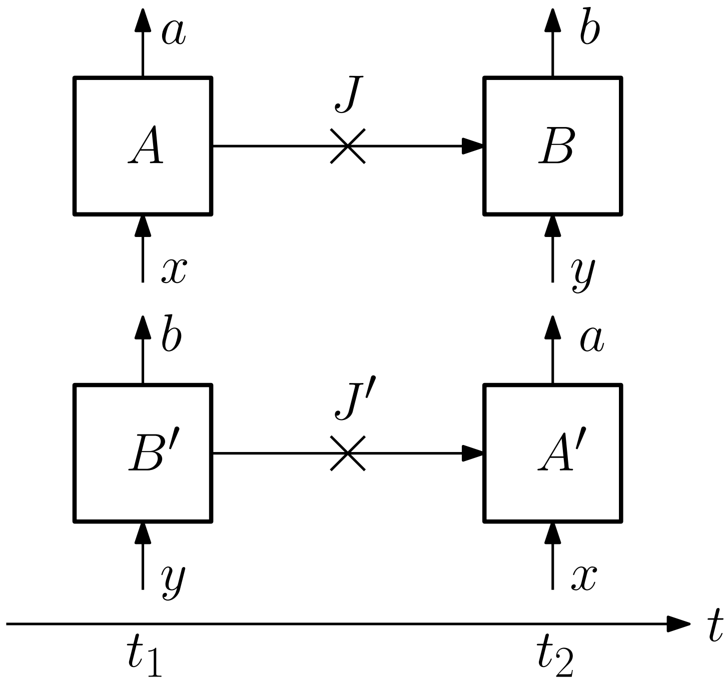
$$\mathcal{E}_{\beta|\alpha} : \mathfrak{L}(\mathcal{H}_\alpha) \to \mathfrak{L}(\mathcal{H}_\beta).$$

- A box $B \in \mathcal{S}_2$ is associated with a set of POVMs $E_\beta^{b|y}$ on $\mathcal{H}_\beta$.
- The joint probability distribution is given by:

$$p_{ABJ}(a, b|x, y) = \mathrm{Tr}_\beta \left(E_\beta^{b|y} \mathcal{E}_{\beta|\alpha} \left(\rho_\alpha^{a|x} \right) \right) p_A(a|x).$$

- An experiment (A, B, J) has an *operational time reverse* if there exists $B' \in \mathcal{S}_1$, $A' \in \mathcal{S}_2$, and $J' \in \mathcal{J}_{\beta',\alpha'}$ such that

$$p_{ABJ}(a, b|x, y) = p_{B'A'J'}(b, a|y, x).$$



Operational Time Symmetry

- A theory is *operationally time symmetric* if every experiment has an operational time reverse.
- Most operational theories are not expected to have operational time symmetry because we can signal into the future but not into the past.

$$p_{ABJ}(a|x,y) = p_{ABJ}(a|x,y')$$
$$p_{ABJ}(b|x,y) \neq p_{ABJ}(b|x',y)$$

- We can, however, artificially restrict attention to experiments that do not allow signalling into the future, i.e. only consider experiments for which

$$p_{ABJ}(a|x,y) = p_{ABJ}(a|x,y')$$
$$p_{ABJ}(b|x,y) = p_{ABJ}(b|x',y)$$

Operational Time Symmetry: Quantum Case

- In quantum theory, no-signalling into the future corresponds to

$$\sum_a p_A(a|x)\rho_\alpha^{a|x} = \rho_\alpha,$$

i.e. $I_A = x$ corresponds to choosing an ensemble decomposition of a fixed density operator.

- The theory of quantum experiments that satisfy this condition is operationally time symmetric[2].

$$\begin{aligned} E_\alpha^{a|x} &= p_A(a|x)\rho_\alpha^{-\frac{1}{2}}\rho_\alpha^{a|x}\rho_\alpha^{-\frac{1}{2}} \\ \rho_\beta &= \mathcal{E}_{\beta|\alpha}(\rho_\alpha) \\ \rho_\beta^{b|y} &= \rho_\beta^{\frac{1}{2}} E_\beta^{b|y} \rho_\beta^{\frac{1}{2}} \\ \mathcal{E}_{\alpha|\beta}(\sigma_\beta) &= \rho_\alpha^{\frac{1}{2}}\mathcal{E}_{\beta|\alpha}^{\dagger}\left(\rho_\beta^{-\frac{1}{2}}\sigma_\beta\rho_\beta^{-\frac{1}{2}}\right)\rho_\alpha^{\frac{1}{2}}. \end{aligned}$$

[2] M. Leifer and R. Spekkens, Phys. Rev. A 88:052130 (2013).

Ontological Framework

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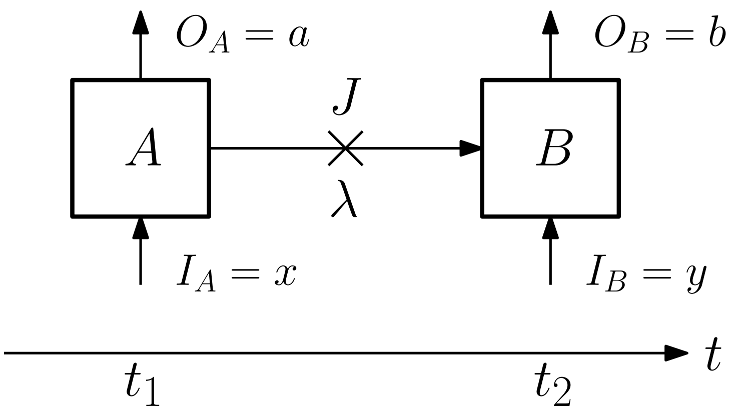
- We now assume that the system has some ontological properties between t_1 and t_2, denoted by λ, known as the system's *ontic state*.



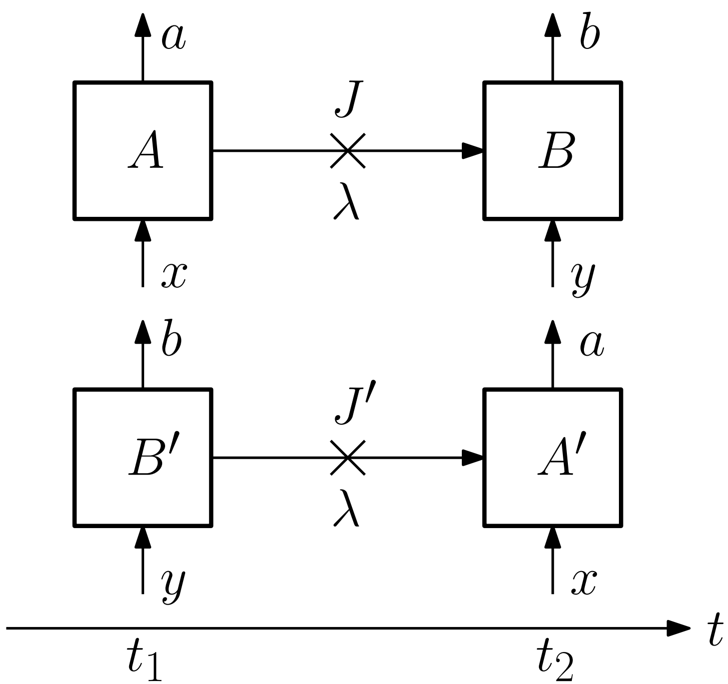
- These will be correlated with the box settings and outcomes, so we will have a joint distribution $p_{ABJ}(a, b, \lambda | x, y)$ such that

$$\sum_{\lambda} p_{ABJ}(a, b, \lambda | x, y) = p_{ABJ}(a, b | x, y).$$

- A realist *model* of a theory is an assignment of such a distribution to every experiment.

- An experiment (A, B, J) has an *ontological time reverse* if there exists $B' \in \mathcal{S}_1$, $A' \in \mathcal{S}_2$, and $J' \in \mathcal{J}_{T(B'),T(A')}$ such that

$$p_{ABJ}(a, b, \lambda | x, y) = p_{B'A'J'}(b, a, \lambda | y, x).$$



- A model is *ontologically time symmetric* if every experiment has an ontological time reverse.

Our Assumptions

The Time Symmetry Assumption

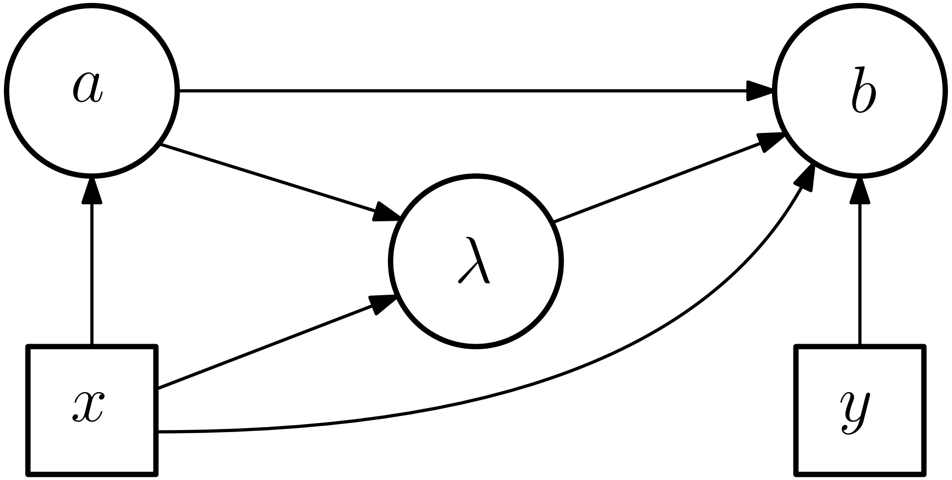
- If a theory is operationally time symmetric then it should have a model that is ontologically time symmetric.

$$p_{ABJ}(a,b|x,y) = p_{B'A'J'}(b,a|y,x)$$

$$\Rightarrow \quad p_{ABJ}(a,b,\lambda|x,y) = p_{B'A'J'}(b,a,\lambda|y,x)$$

No Retrocausality

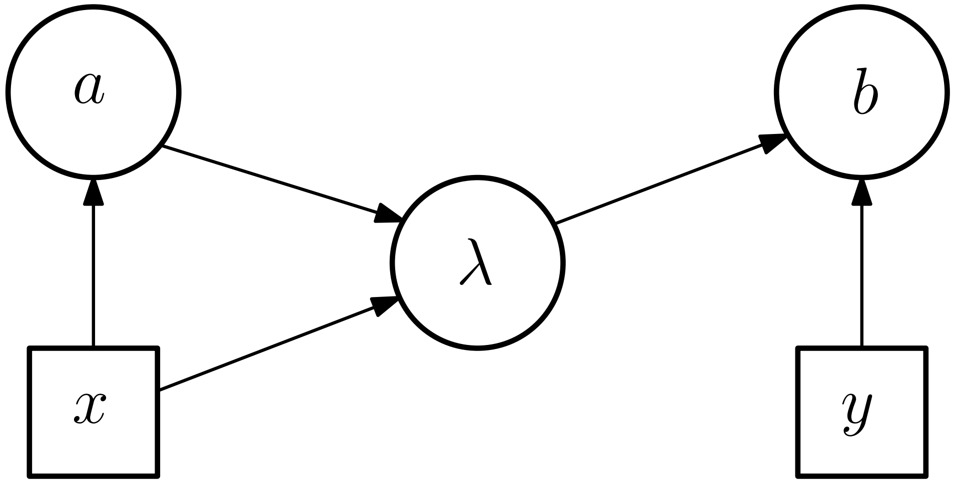
- x and y are free choices and the model has the following causal structure:



$$p(a, b, \lambda | x, y) = p(b|\lambda, a, x, y)p(\lambda|a, x)p(a|x)$$

Correlations are Mediated by Ontic States

- All correlations between the two boxes are mediated by λ.



$$p(a, b, \lambda | x, y) = p(b|\lambda, y)p(\lambda|a, x)p(a|x)$$

- Taken together, the last two assumptions are equivalent to saying that the model is an *ontological model*[3].

[3] N. Harrigan and R. Spekkens, Found. Phys. 40:125 (2010).

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Main Results

- Theorem: Any model satisfying our three assumptions must satisfy Bell's local causality

$$p(a, b|x, y) = \sum_\lambda p(a|\lambda, x)p(b|\lambda, y)p(\lambda).$$

- *Proof*:
- By time symmetry

$$\begin{aligned} p(a, b, \lambda|x, y) &= p(b|\lambda, y)p(\lambda|a, x)p(a|x) \\ &= p(a|\lambda, x)p(\lambda|b, y)p(b|y). \end{aligned}$$

- Use Bayes' rule to rewrite

$$p(\lambda|a, x)p(a|x) = p(a|\lambda, x)p(\lambda|x)$$
$$p(\lambda|b, y)p(b|y) = p(b|\lambda, y)p(\lambda|y).$$

- So,

$$p(a, b, \lambda|x, y) = p(b|\lambda, y)p(a|\lambda, x)p(\lambda|x)$$
$$= p(a|\lambda, x)p(b|\lambda, y)p(\lambda|y).$$

- Then sum over a and b to get

$$p(\lambda|x) = p(\lambda|y) = p(\lambda).$$

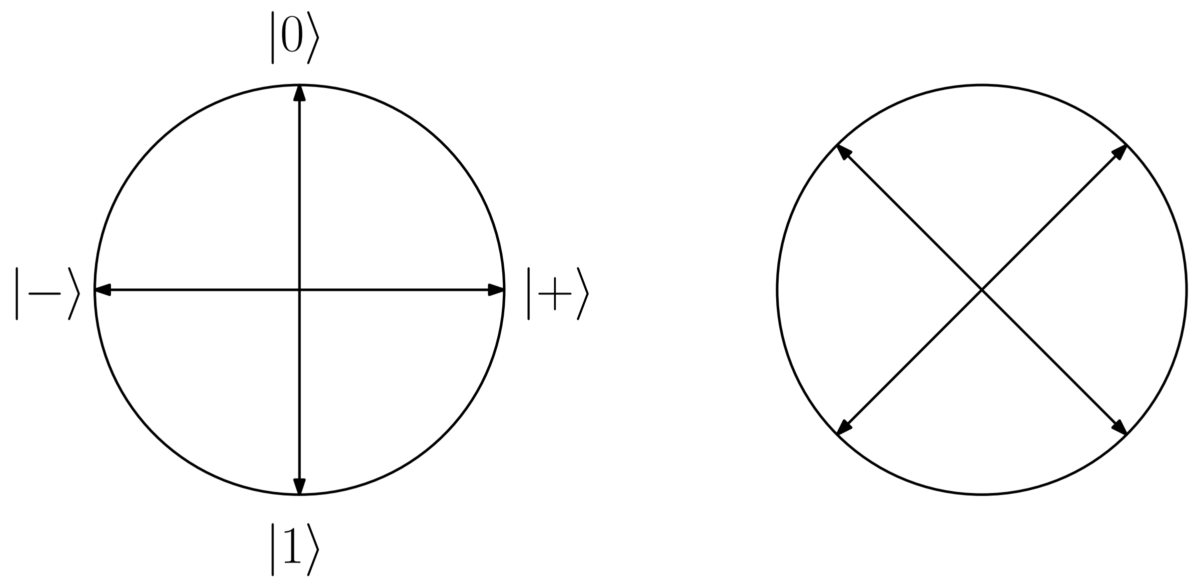
- Substituting gives,

$$p(a, b, \lambda|x, y) = p(a|x, \lambda)p(b|y, \lambda)p(\lambda).$$

- Sum over λ to get

$$p(a, b|x, y) = \sum_{\lambda} p(a|x, \lambda)p(b|y, \lambda)p(\lambda).$$

- Does quantum theory violate this for timelike experiments with no signalling into the future?
- Qubit Example:



- Prepare and measure in the optimal bases for CHSH violation, with identity dynamics in between.

- Theorem: An ensemble average state ρ_α and CPT map $\mathcal{E}_{\beta|\alpha}$ can be used to violate local causality in a timelike experiement iff the isomorphic bipartite state

$$\rho_{\alpha\beta} = \rho_\alpha^{\frac{1}{2}} \mathcal{E}_{\beta|\alpha'} \left(\left|\Phi^+\right\rangle \left\langle \Phi^+\right|_{\alpha\alpha'} \right) \rho_\alpha^{\frac{1}{2}},$$

can be used to violate local causality in a spacelike experiment.

M. Leifer and R. Spekkens, Phys. Rev. A 88:052130 (2013).

Discussion of Assumptions

Time Symmetry

- Qualitative notion of time symmetry: If we watch a video of a process we cannot tell whether it is playing forwards or in reverse.
- In quantum theory it matters if:
  - We really mean a video (operational time symmetry).
  - We actually mean a record of everything that exists (ontological time symmetry).
- Our principle states that if you cannot tell from a video then you cannot tell from a record.

- Ususal notion of time symmetry: If a trajectory is *possible* in the forward direction then the time reverse of that trajectory is also *possible*.

- Our notion: If a joint probability distribution is predicted in the forward direction then there is an experiment with the same probabilities in the reverse direction.

- Our notion is violated in general due to the thermodynamic arrow of time. It would hold, for example, for a classical system in thermodynamic equilibrium.

- We do not assume that the universe satisfies our notion of time symmetry, only that if it already holds operationally then it should hold ontologically as well.

- Spekkens' noncontextuality[4]: If two experimental procedures are operationally equivalent then they ought to be ontologically equivalent.

- More general principle: If the operational predictions of a theory have a symmetry then the ontological model ought to have the same symmetry.

- Why? Otherwise there is a fine-tuning — correlations with λ have to be just right so that marginalizing over λ washes out the asymmetry.

- Any ψ-ontic ontological model, such as Bohmian mechanics, violates our assumption. This should not be a big surprise as such models are already contextual.
  - A Bohmian would deny the significance of operational time-symmetry.

[4] R. Spekkens, Phys. Rev. A 71:052108 (2005).

The Other Two Assumptions

- No-Retrocausality is a common assumption in all no-go theorems. However, the symmetry of the argument is designed to make it the most plausible assumption to give up here.
- However, allowing retrocausality also gives rise to a fine tuning: If there are influences that travel backwards in time then why can't they be used to signal?
- Having ontic states mediate the correlations is really the definition of what we mean by an ontic state. We assume that the boxes are not otherwise causally connected, e.g. by a telephone wire, so that the experiment is investigating properties of the system. If there are other influences that give rise to correlations, they should be included as part of the ontic state.
- Note that we cannot even formulate the mediation assumption without no-retrocausality, so if we give up the latter, we need an entirely new framework for this.

Conclusions

Conclusions

- There is no model of quantum theory in our framework that satisfies our time symmetry assumption, has no retrocausality, and has ontic states mediate the correlations.
- Whether we give up time symmetry or no-retrocausality, there is a fine-tuning in the theory. How should we respond to fine-tunings?
  1. Accept them as brute facts.
  2. Look for a theory that does not have them.
  3. Explain them as emergent (c.f. thermalization).
- 2 or 3 seem preferable, but note that there might be other grounds for preferring theories where certain symmetries (e.g. time-symmetry or Lorentz invariance) are fundamental.
- It does not seem completely impluasible that the same processes that are responsible for the thermodynamic arrow of time might explain why retrocausality does not lead to signalling.