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Does time-symmetry in quantum theory imply retrocausality?

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- Is the collapse of the wavefunction ia fundamental time-asymmetry?
- Yakir showed[1] that we can restore time-symmetry by introducing a backwards evolving state that depends on a future measurement choice in addition to the usual forwards evolving state-vector.

$$p(a|\psi,\phi) = \frac{|\langle\phi| P_a |\psi\rangle|^2}{\sum_{a'} |\langle\phi| P_{a'} |\psi\rangle|^2}.$$

- Does this mean we have to accept that there is retrocausality in nature if we want time-symmetry?
- Huw Price has argued that a time-symmetric realist account of quantum theory should be retrocausal[2], but his argument assumes that quantum states are real.

[1] Y. Aharonov, P. Bergmann and J. Lebowitz, Phys. Rev. B 134, 141016 (1964).
[2] H. Price, Stud. Hist. Phil. Mod. Phys. 43:75–83 (2012).

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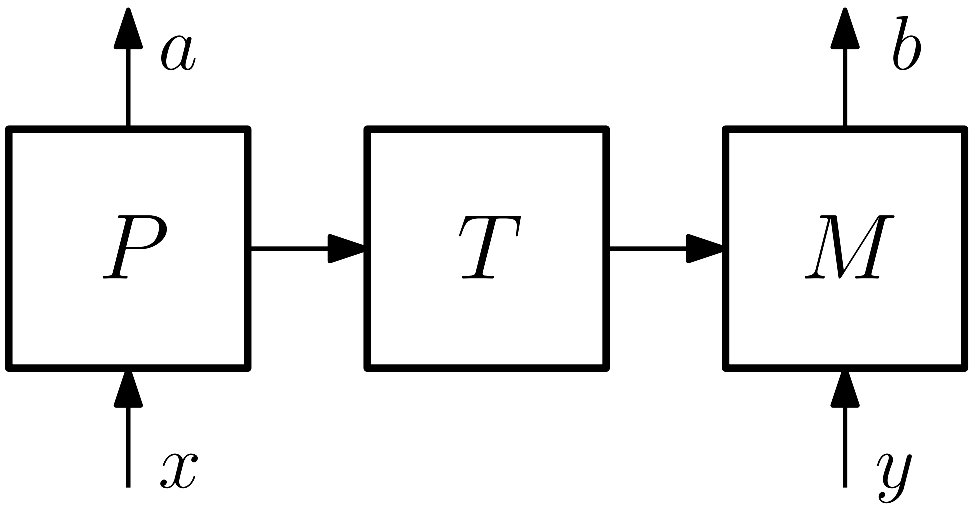
PTM Experiments

- Consider experiments involving a preparation P, a transformation T, and a measurement M in a definite time order.



- An *operational theory* consists of a set of possible *experiments* (P, T, M) and, for each experiment, a prediction

$$p_{PTM}(a, b|x, y).$$

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- A preparation P is associated with:
  - A Hilbert space $\mathcal{H}_A$.
  - A set of subnormalized density operators $\rho_{aA|x}$ on $\mathcal{H}_A$.
- A transformation T is associated with:
  - An input Hilbert space $\mathcal{H}_{\text{in}}$ and an output Hilbert space $\mathcal{H}_{\text{out}}$.
  - A Completely-Positive, Trace-Preserving map $\mathcal{E}_{\text{out}|\text{in}}$ from the density operators on $\mathcal{H}_{\text{in}}$ to the density operators on $\mathcal{H}_{\text{out}}$.
- A measurement M is associated with:
  - A Hilbert space $\mathcal{H}_B$.
  - A set of POVMs $E_{b|yB}$ on $\mathcal{H}_B$.
- (P, T, M) is an experiment if $\mathcal{H}_{\text{in}} = \mathcal{H}_A$ and $\mathcal{H}_{\text{out}} = \mathcal{H}_B$
- Quantum theory then predicts:

$$p_{PTM}(a, b|x, y) = \text{Tr}\left(E_{b|yB}\mathcal{E}_{B|A}\left(\rho_{aA|x}\right)\right).$$

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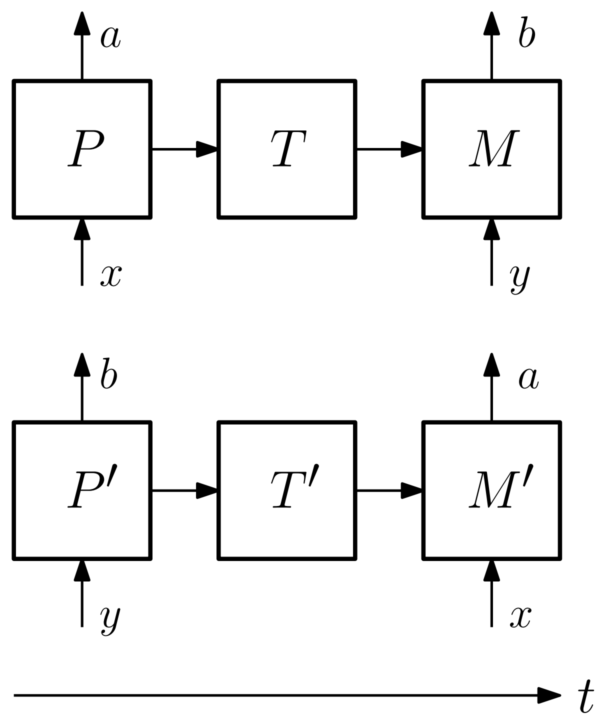
- An experiment (P, T, M) has an *operational time reverse* if there exists P', T', M' such that

$$p_{PTM}(a, b|x, y) = p_{P'T'M'}(b, a|y, x).$$



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- A theory is *operationally time symmetric* if every experiment has an operational time reverse.
- Most operational theories are not operationally time-symmetric because we can signal into the future but not into the past.

$$p_{PTM}(a|x,y) = p_{PTM}(a|x,y')$$
$$p_{PTM}(b|x,y) \neq p_{PTM}(b|x',y)$$

- We can, however, artificially restrict attention to the *no-signaling sector* of a theory i.e. only consider experiments for which

$$p_{ABJ}(a|x,y) = p_{ABJ}(a|x,y')$$
$$p_{ABJ}(b|x,y) = p_{ABJ}(b|x',y)$$

Operational Time Symmetry: Quantum Case

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- In quantum theory, no-signaling into the future corresponds to

$$\sum_a \rho_{aA|x} = \sum_a \rho_{aA|x'} = \rho_A,$$

i.e. x is the choice of an ensemble decomposition of a fixed density operator.

- The no-signaling sector of quantum theory is operationally time symmetric[3].

$$E_{a|xA} = \rho_A^{-\frac{1}{2}} \rho_{aA|x} \rho_A^{-\frac{1}{2}}$$
$$\rho_B = \mathcal{E}_{B|A}(\rho_A)$$
$$\rho_{bB|y} = \rho_B^{\frac{1}{2}} E_{b|yB} \rho_B^{\frac{1}{2}}$$
$$\mathcal{E}_{A|B}(\cdot) = \rho_A^{\frac{1}{2}} \mathcal{E}_{B|A}^{\dagger} \left(\rho_B^{-\frac{1}{2}} (\cdot) \rho_B^{-\frac{1}{2}} \right) \rho_A^{\frac{1}{2}}.$$

[3] M. Leifer and R. Spekkens, Phys. Rev. A 88:052130 (2013).

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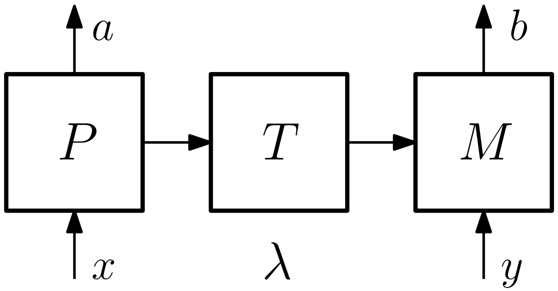
Ontic Extensions

- We now assume that the system has some ontological properties between P and M, denoted by λ, known as the system's *ontic state*.



- An *ontic extension* of an experiment is a joint distribution $p_{PTM}(a, b, \lambda | x, y)$ such that

$$\sum_{\lambda} p_{PTM}(a, b, \lambda | x, y) = p_{PTM}(a, b | x, y).$$

- A *ontic extension* of a theory is an assignment of such a distribution to every experiment.

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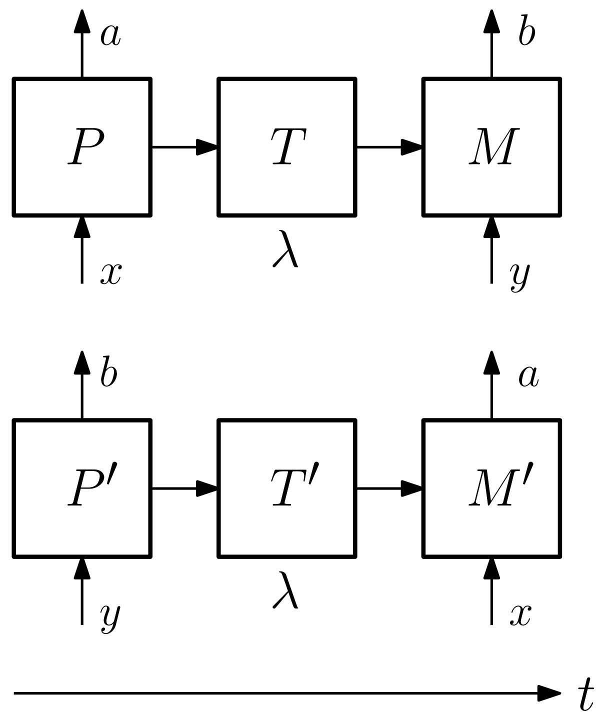
- An experiment (P, T, M) has an *ontological time reverse* if there exists P', T', and M' such that

$$p_{PTM}(a, b, \lambda | x, y) = p_{P'T'M'}(b, a, \lambda | y, x).$$



- An ontic extension of a theory is *ontologically time symmetric* if every experiment has an ontological time reverse.

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- If a theory is operationally time symmetric then it should have an extension that is ontologically time symmetric.

$$p_{PTM}(a, b|x, y) = p_{P'T'M'}(b, a|y, x)$$

$$\Rightarrow \qquad p_{PTM}(a, b, \lambda|x, y) = p_{P'T'M'}(b, a, \lambda|y, x)$$

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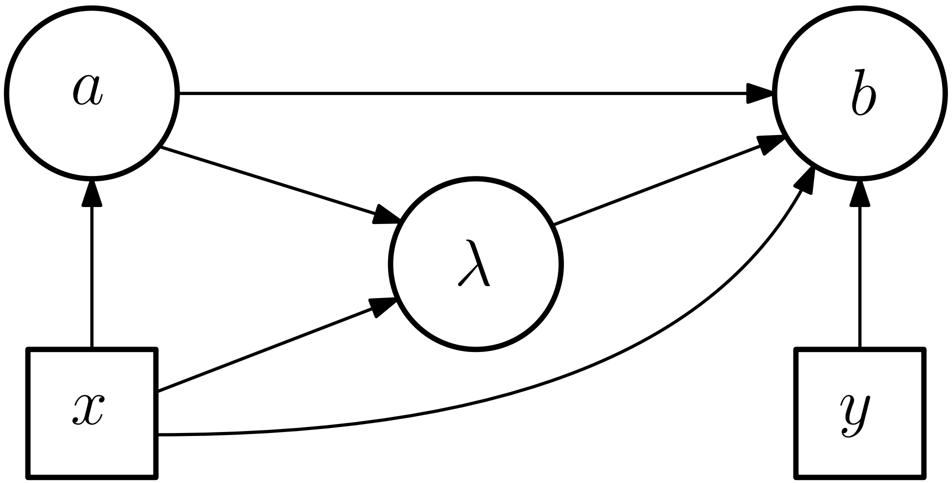
- x and y are free choices and the model has the following causal structure:



$$p(a, b, \lambda | x, y) = p(b | \lambda, a, x, y) p(\lambda | a, x) p(a | x)$$

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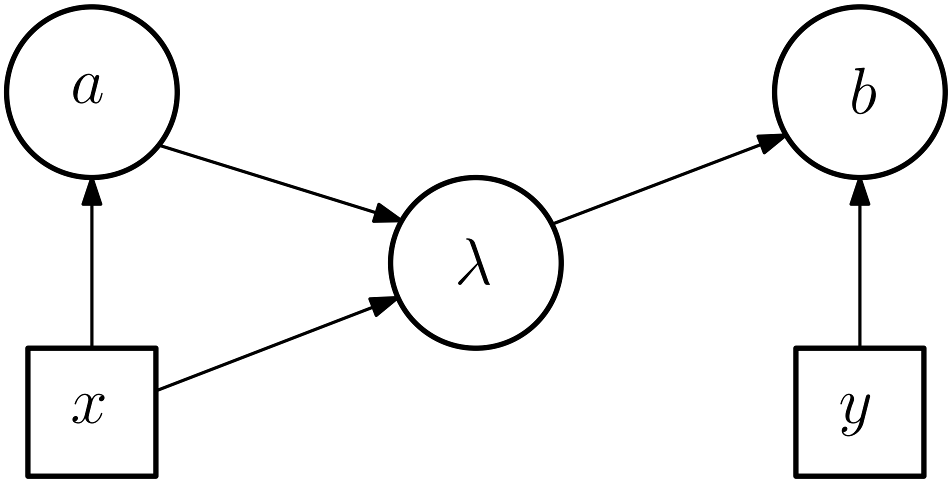
- All correlations between P and M are mediated by λ.



$$p(a, b, \lambda | x, y) = p(b|\lambda, y)p(\lambda|a, x)p(a|x)$$

- Taken together, the last two assumptions are equivalent to saying that the model is an *ontological model*[4].

[4] N. Harrigan and R. Spekkens, Found. Phys. 40:125 (2010).

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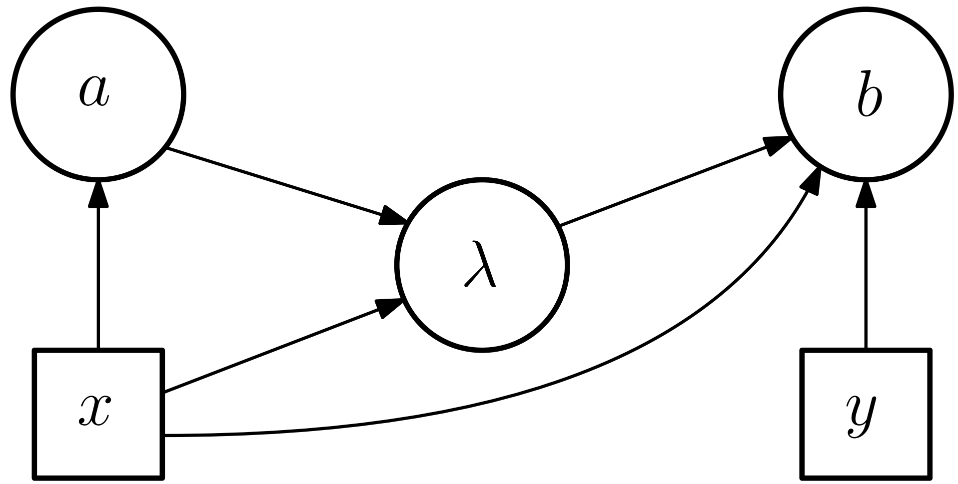

$$p(a, b, \lambda | x, y) = p(b|\lambda, x, y)p(\lambda|a, x)p(a|x)$$

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- Theorem: An ontic extension of an operationally time symmetric experiment that satisfies *Time Symmetry* and *No Retrocausality* must satisfy

$$
\begin{aligned}
p(\lambda|x, y) &= p(\lambda) \\
p(a|\lambda, x, y) &= p(a|\lambda, x) \\
p(b|\lambda, x, y) &= p(b|\lambda, y).
\end{aligned}
$$

If it also satisfies *Weak λ-mediation* then,

$$
p(a, b|x, y) = \sum_{\lambda} p(a|\lambda, x)p(b|\lambda, y)p(\lambda).
$$

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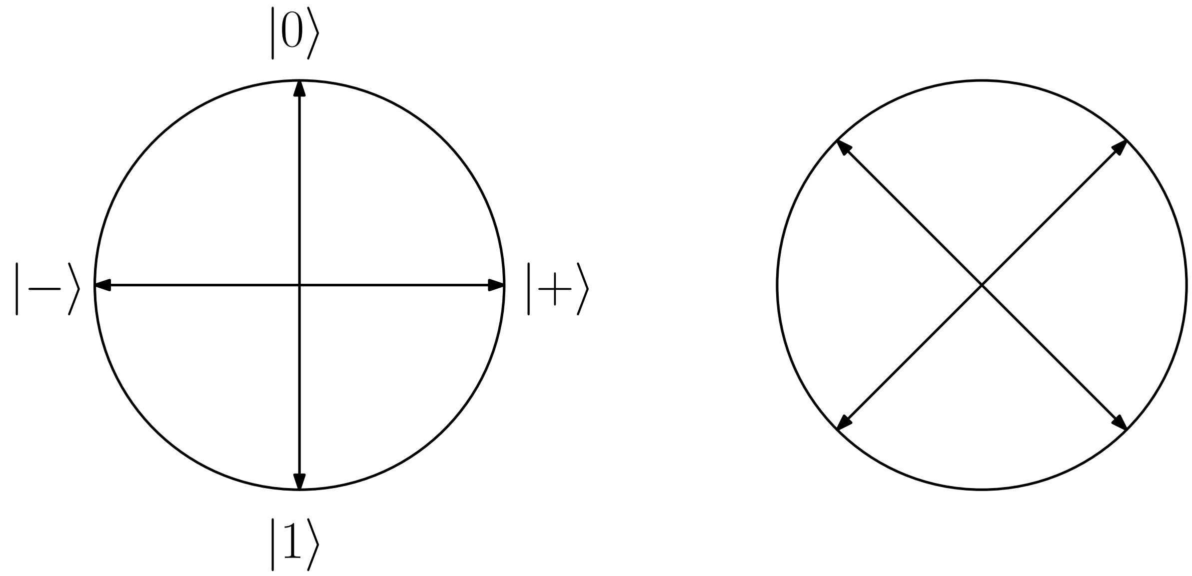
- Does quantum theory violate this for timelike experiments with no signalling into the future?
- Qubit Example:



- Prepare and measure in the optimal bases for CHSH violation, with identity dynamics in between.

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- By *No Retrocasality*:

$$p(a,b,\lambda|x,y) = p(b|a,\lambda,x,y)p(\lambda|a,x)p(a|x).$$

- Using Bayes' rule on $p(\lambda|a,x)$ gives:

$$p(a,b,\lambda|x,y) = p(b|a,\lambda,x,y)p(a|\lambda,x)p(\lambda|x).$$

- Sum over a and b to get: $p(\lambda|x,y) = p(\lambda|x)$.
- By *Time Symmetry*:

$$p(a,b,\lambda|x,y) = p(a|b,\lambda,x,y)p(\lambda|b,y)p(b|y).$$

- Applying the same argument gives: $p(\lambda|x,y) = p(\lambda|y)$.
- Both these conditions together imply: $p(\lambda|x,y) = p(\lambda)$.

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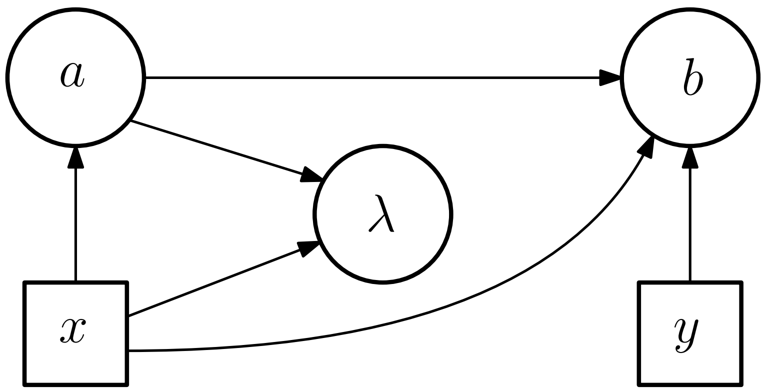
- If we drop *Weak λ-mediation* then we still have the first part of the theorem.
- We can use the Colbeck-Renner theorem[5] to prove that there is an experiment for which we must have:

$$p(b|a,\lambda,x,y) = p(b|a,x,y)$$



- This would be a fairly pointless ontic extension.

[5] R. Colbeck and R. Renner, Nature Communications 2, 411 (2011)