Logical pre- and post-selection paradoxes are proofs of contextuality

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arXiv:1506.07850

Joint work with Matt Pusey

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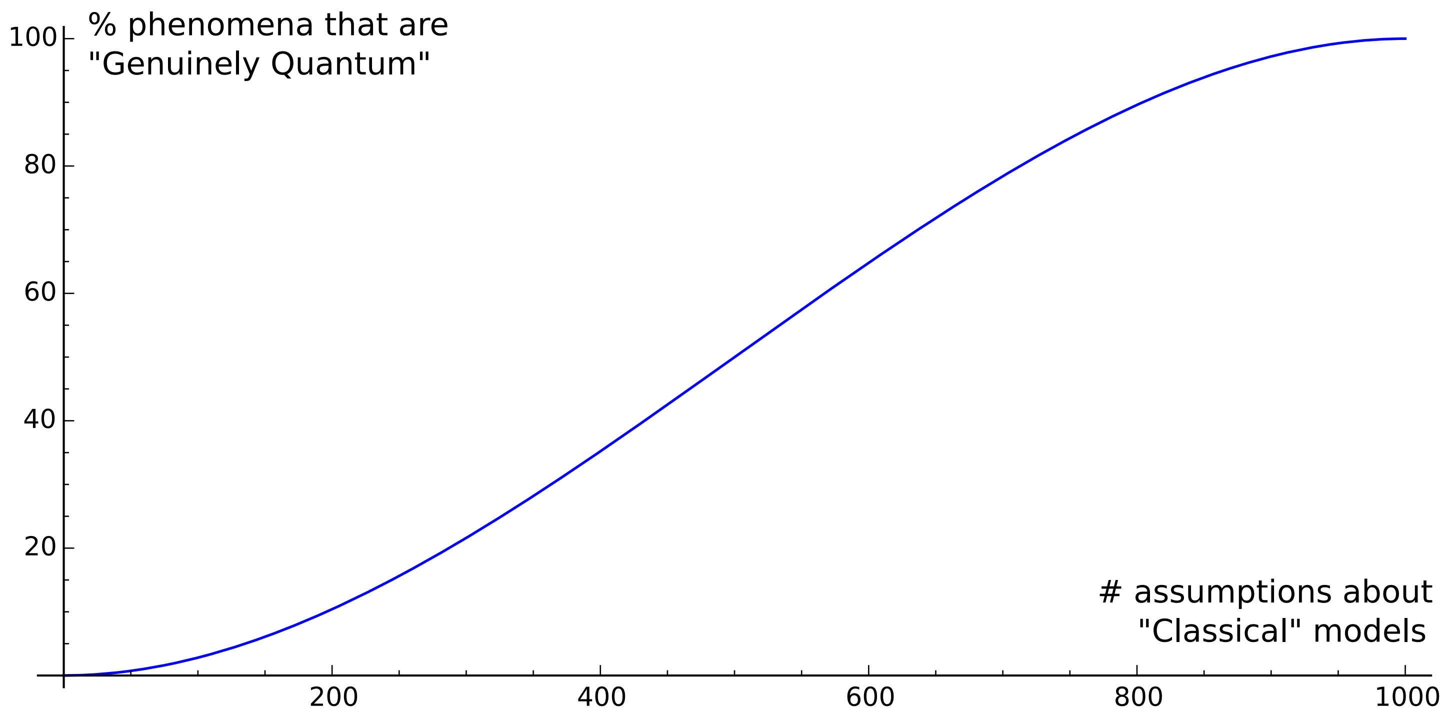
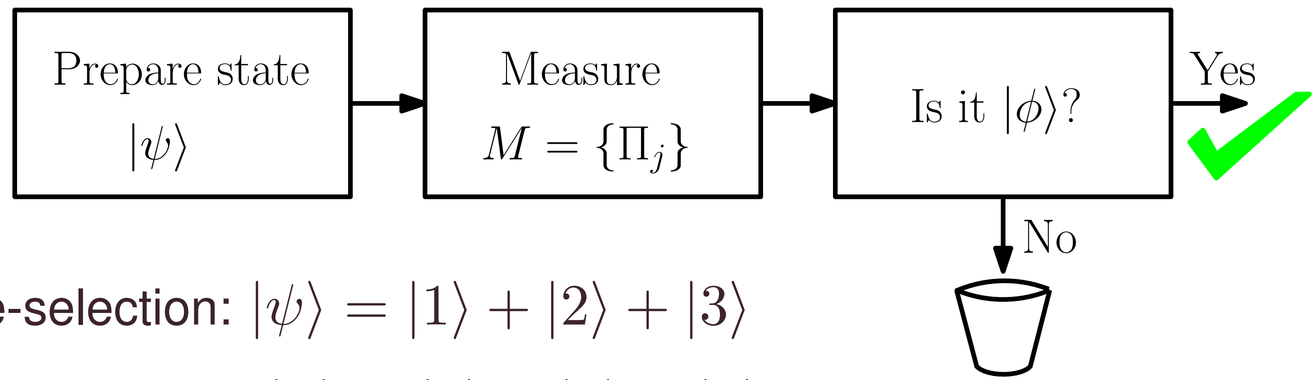
The two most meaningless words in physics

“Classical” “Quantum”



Three box paradox



- Pre-selection: $|\psi\rangle = |1\rangle + |2\rangle + |3\rangle$
- Post-selection: $|\phi\rangle = |1\rangle + |2\rangle - |3\rangle$
- Two possible intermediate measurements:
  - M_1: Is ball in box 1? $\quad \Pi_1 = |1\rangle\langle 1|, \quad \Pi_{2\vee 3} = |2\rangle\langle 2| + |3\rangle\langle 3|$

$$\mathbb{P}(\Pi_1 | \psi, M_1, \phi) = 1$$

  - M_2: Is ball in box 2? $\quad \Pi_2 = |2\rangle\langle 2|, \quad \Pi_{1\vee 3} = |1\rangle\langle 1| + |3\rangle\langle 3|$

$$\mathbb{P}(\Pi_2 | \psi, M_2, \phi) = 1$$

Y. Aharonov and L. Vaidman, *J. Phys. A* 24 pp. 2315–2328 (1991).

Kochen-Specker (KS) Noncontextuality

- *Outcome determinism*: At any given time, the system has a definite value for every observable.
  - For every projective measurement $\{\Pi_j\}$, precisely one projector is asigned the value 1, the rest 0.
- *Noncontextuality*: The outcome assigned to an observable does not depend on which other (commuting) observables it is measured with.
  - The value assigned to a projector does not depend on which other projectors are measured with it , e.g.

$$|1\rangle\langle 1| \, , |2\rangle\langle 2| \, , |3\rangle\langle 3|$$

$$|1\rangle\langle 1| \, , |2\rangle\langle 2| + |3\rangle\langle 3|$$

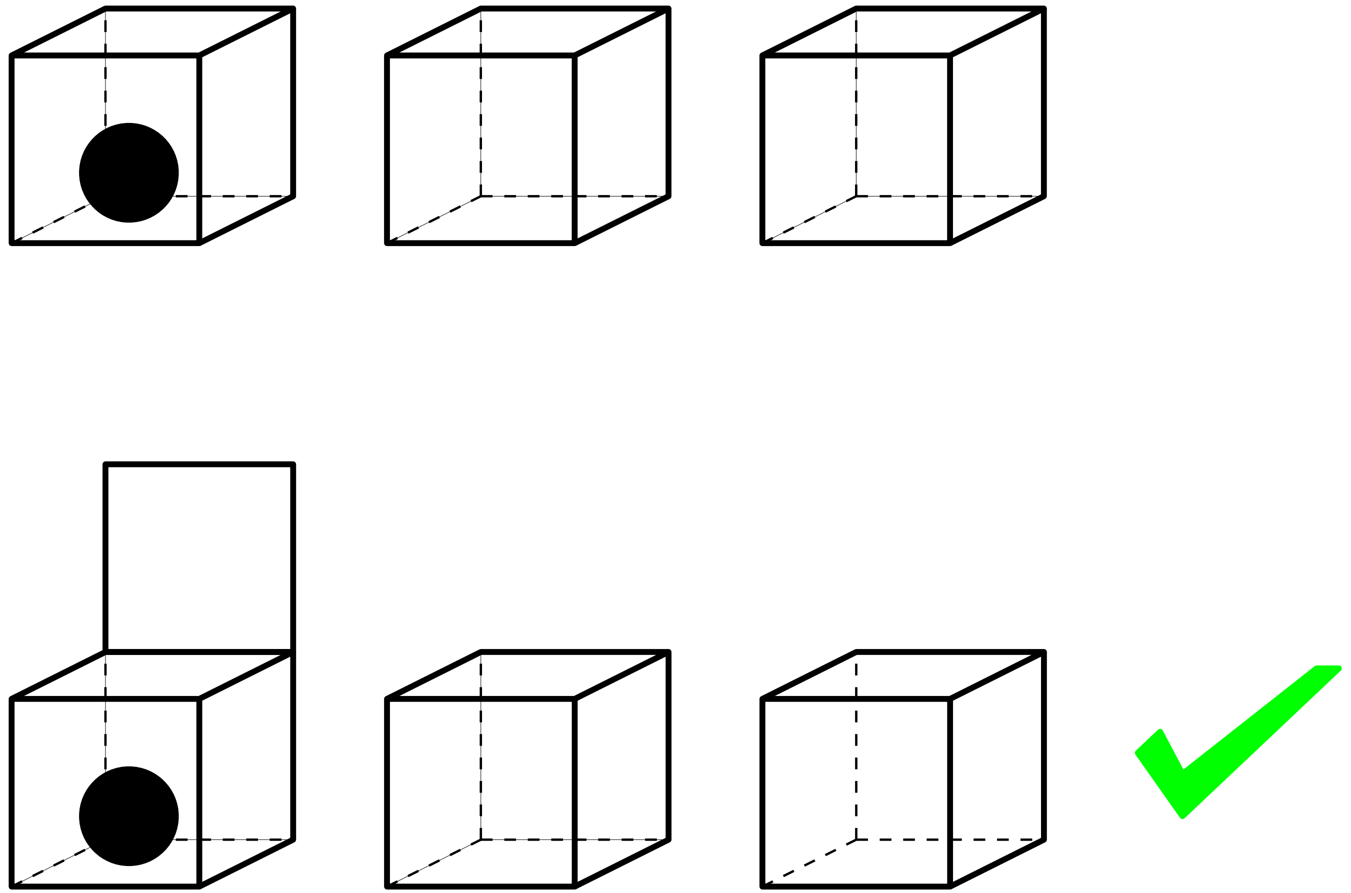
$$|2\rangle\langle 2| \, , |1\rangle\langle 1| + |3\rangle\langle 3|$$

S. Kochen and E. Specker, *J. Math. Mech.* 1 pp. 59–87 (1967).

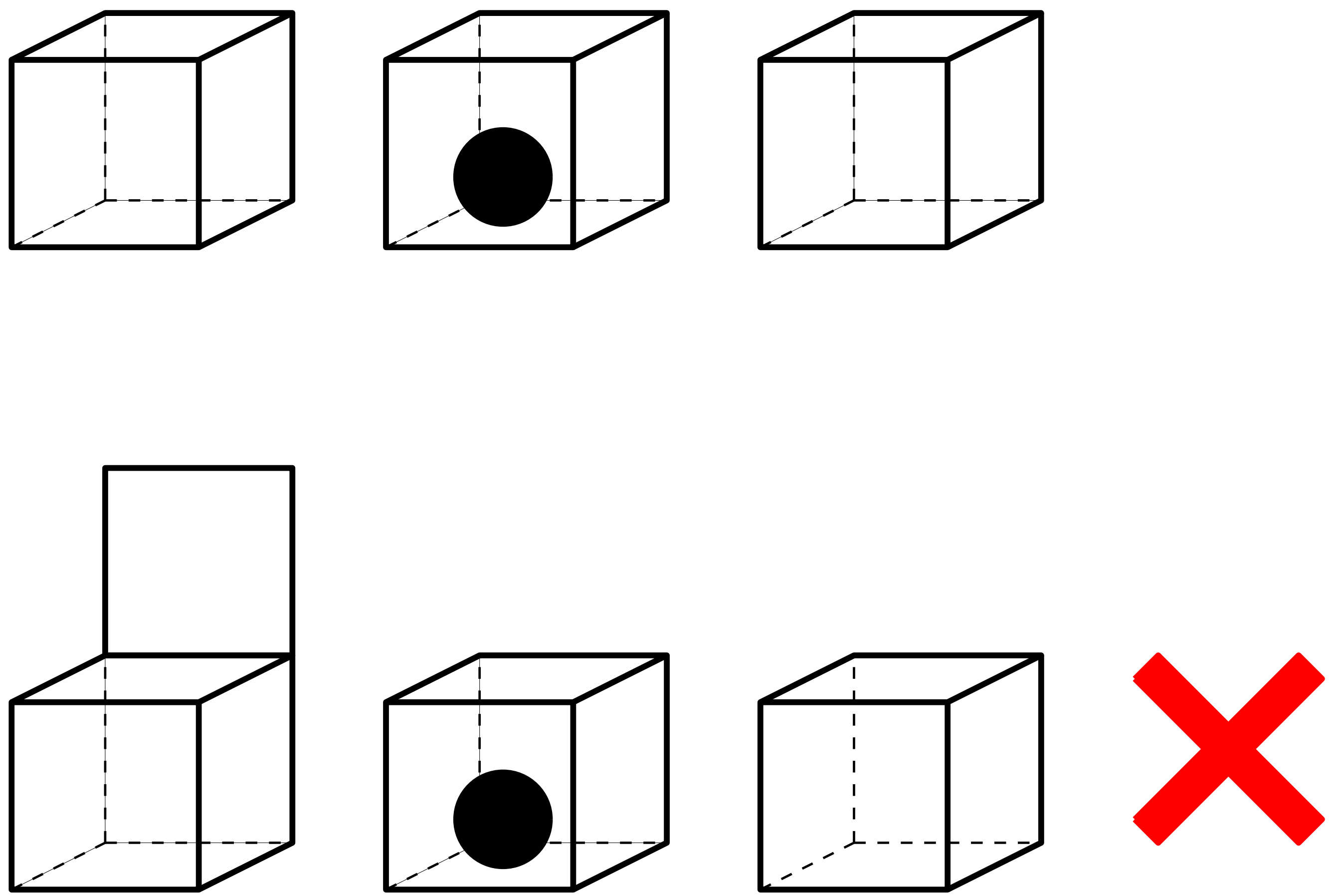
A Kochen-Specker noncontextual model

- Pre-selection:
  - Place ball in box 1, 2 or 3 at random.
- Intermediate measurement:
  - Open box j.
  - Observe whether ball is present.
  - Leave lid open.
- Post selection:
  - Is there a ball in the box with an open lid?

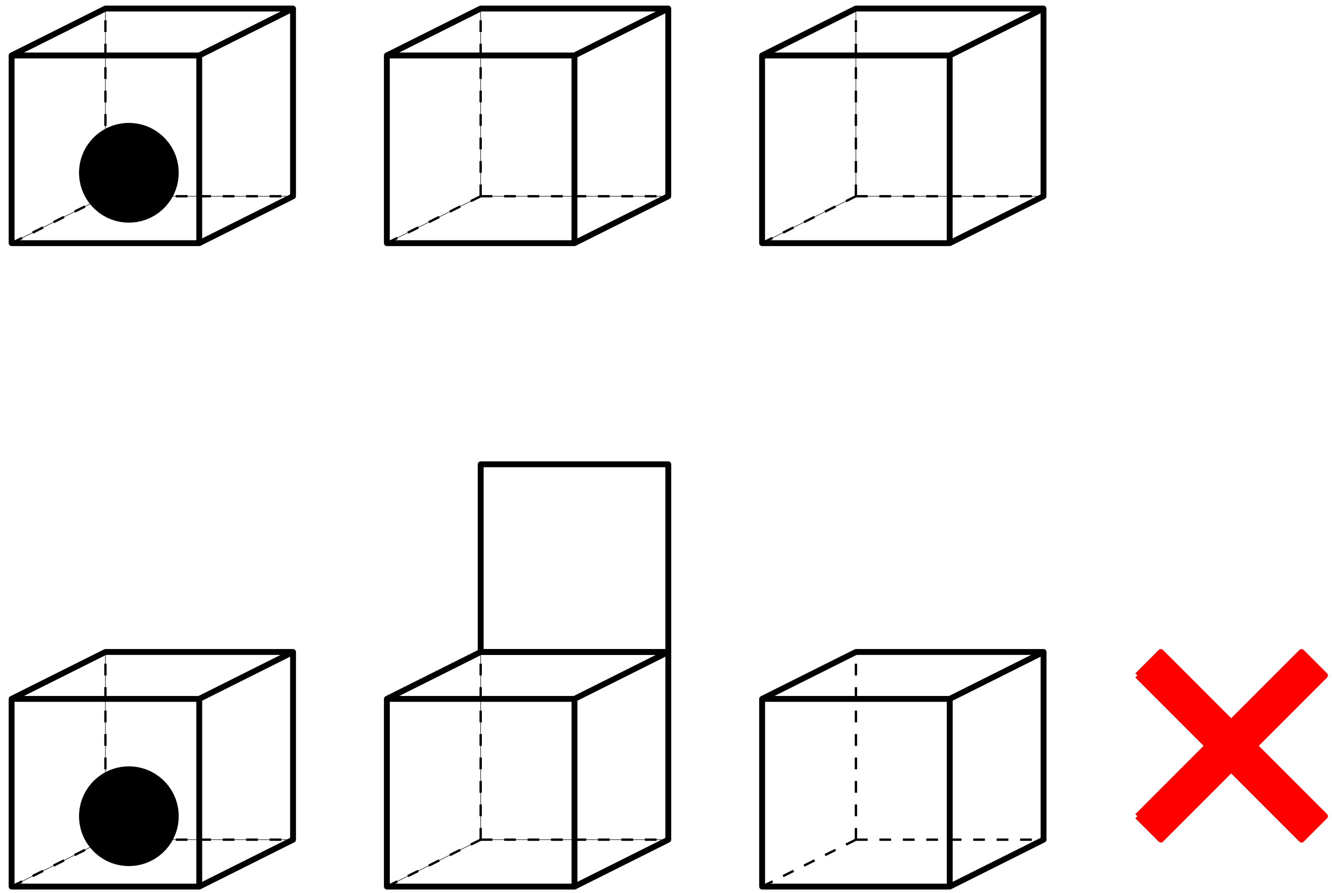
A Kochen-Specker noncontextual model



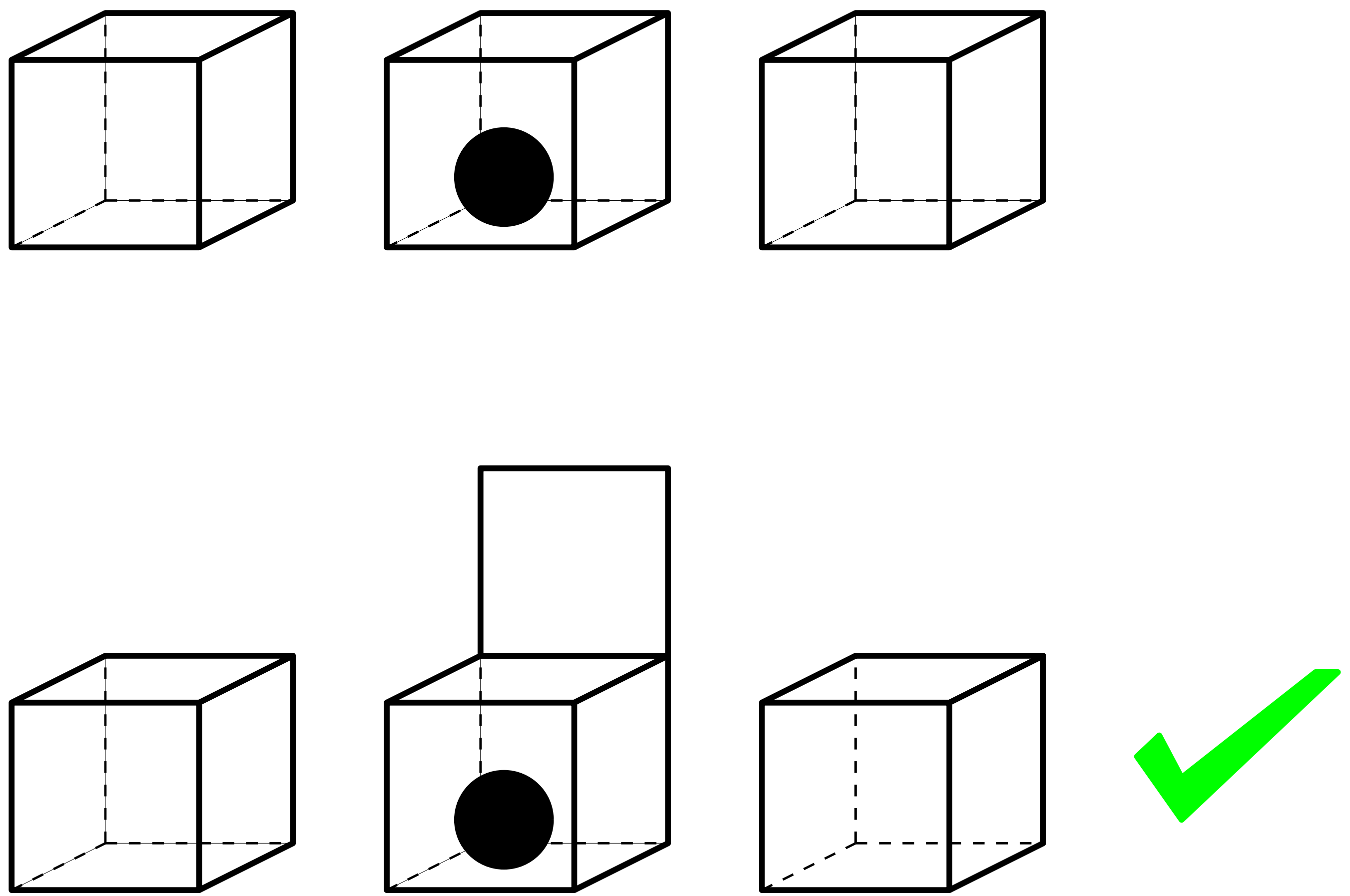
A Kochen-Specker noncontextual model



A Kochen-Specker noncontextual model

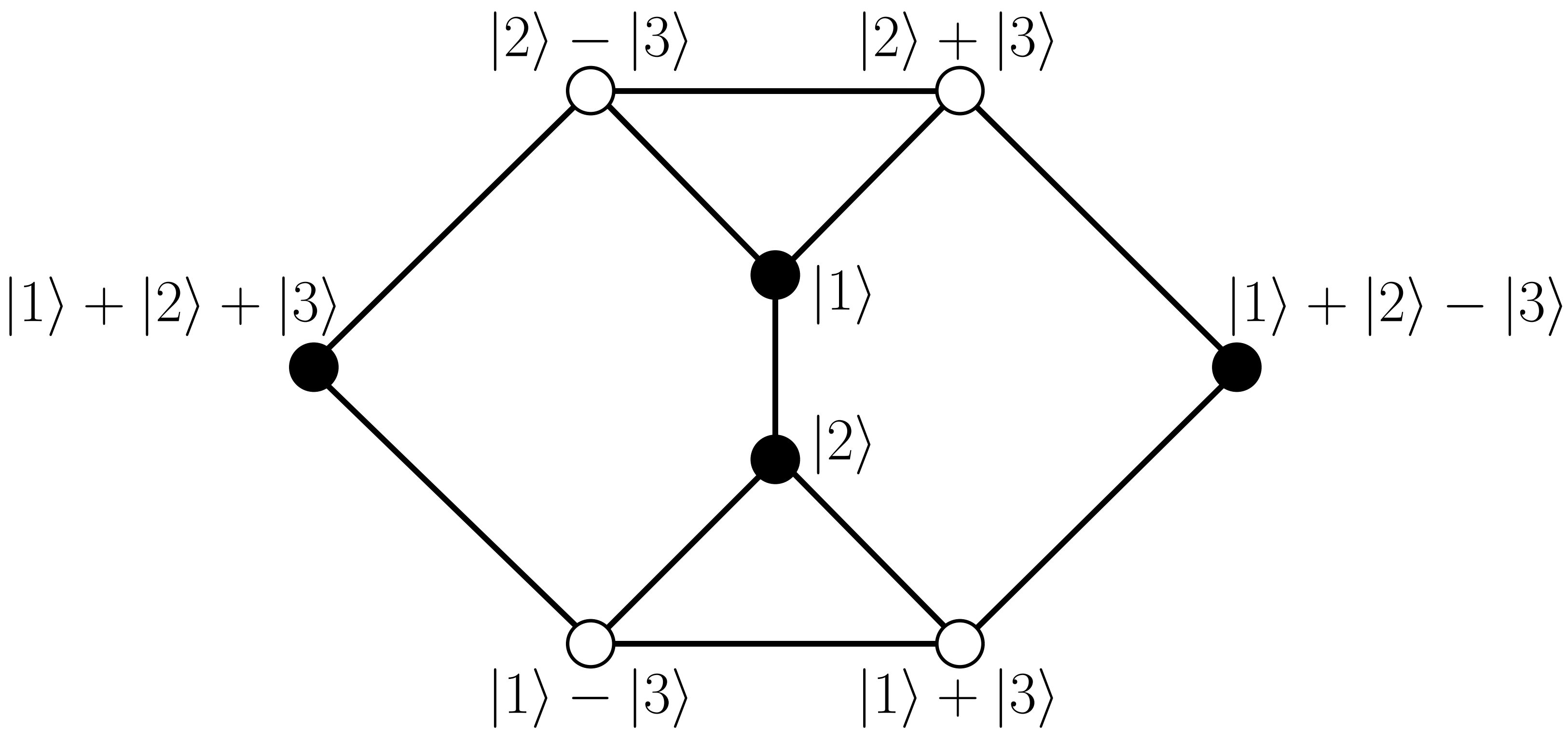


A Kochen-Specker noncontextual model



A Kochen-Specker noncontextual model

- See arXiv:1207.3114 for a model that completely reproduces the quantum predictions.



- All logical pre- and post-selection paradoxes are related to a proof of (BS) contextuality in the same way[1].

R. Clifton, *Am. J. Phys.* 61 443 (1993).

[1] M. Leifer and R. Spekkens, *Phys. Rev. Lett.* 95 200405 (2005).

- Spekkens proposed a more general and operational definition of noncontextuality[2].
  - The reason why projectors receive the same value is because they are always assigned the same probability in quantum theory.
  - General principle: Operationally indistinguishable experimental procedures should be represented the same way in the underlying model.
  - *Transformation noncontextuality*: Two procedures corresponding to the same CPT map must be represented in the same way.

[2] R. Spekkens, *Phys. Rev. A* 71:052108 (2005).

Theorem. *Let $\{\Pi_j\}$ be a projective measurement and let $\mathcal{E}$ be the nonselective state-update rule*

$$\mathcal{E}(\rho) = \sum_j \Pi_j \rho \Pi_j .$$

Then,

$$\mathcal{E}(\rho) = p\rho + (1-p)\mathcal{C}(\rho),$$

where $\mathcal{C}$ is a completely-positive, trace-preserving map and $0 < p \leq 1$.

- Proof for special case $\{\Pi_1, \Pi_2\}$:

$$U_1 = \Pi_1 + \Pi_2 = I \qquad\qquad U_2 = \Pi_1 - \Pi_2$$

$$\mathcal{E}(\rho) = \frac{1}{2}U_1\rho U_1^\dagger + \frac{1}{2}U_2\rho U_2^\dagger = \frac{1}{2}\rho + \frac{1}{2}U_2\rho U_2^\dagger .$$

The three box paradox is a proof of contextuality

Theorem. *The three box paradox is a proof of (Spekkens) contextuality.*

- Assume transformation noncontextuality.
- Since $|\langle\psi|\phi\rangle|^2 > 0$, there must be some hidden states that assign value 1 to both $|\psi\rangle\langle\psi|$ and $|\phi\rangle\langle\phi|$.
- With probability at least $1/2$, the intermediate measurement does not change the hidden state.
- Therefore, these hidden states must assign probability 1 to $|1\rangle\langle 1|$ in M_1 and probability 1 to $|2\rangle\langle 2|$ in M_2, but this is measurement contextual.

- Read arXiv:1506.07850 for:
  - Generalization to all logical pre- and post-selection paradoxes.
    - Quantum pigeonhole principle, failure of the product rule, . . .
  - Proof using measurement noncontextuality instead of transformation noncontextuality.
  - Relation to weak measurement paradoxes.
  - Importance of 0/1 probabilities and von-Neumann update rule.

Weak measurements

- Anomalous weak values have classical analogues:
  - C. Ferrie and J. Combes, *Phys. Rev. Lett.* 113 120404 (2014).
- But, if you try to simulate the quantum predictions exctly, the model must be (Spekkens) contextual:
  - M. Pusey, *Phys. Rev. Lett.* 113 200401 (2014).