Aharonov Meets Spekkens: What do quantum paradoxes tell us about the nature of reality?

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arXiv:1506.07850 & arXiv:1509.08893

Joint work with Josh Combes, Chris Ferrie & Matt Pusey

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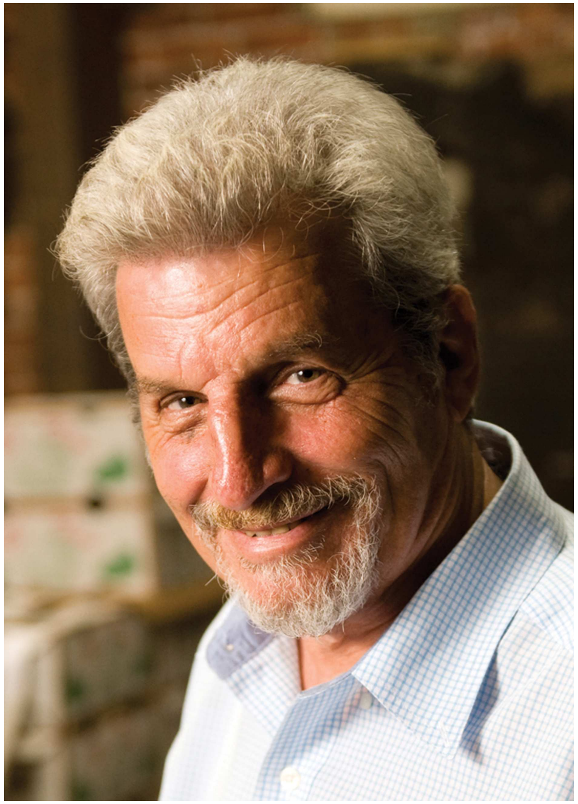
Aharonov

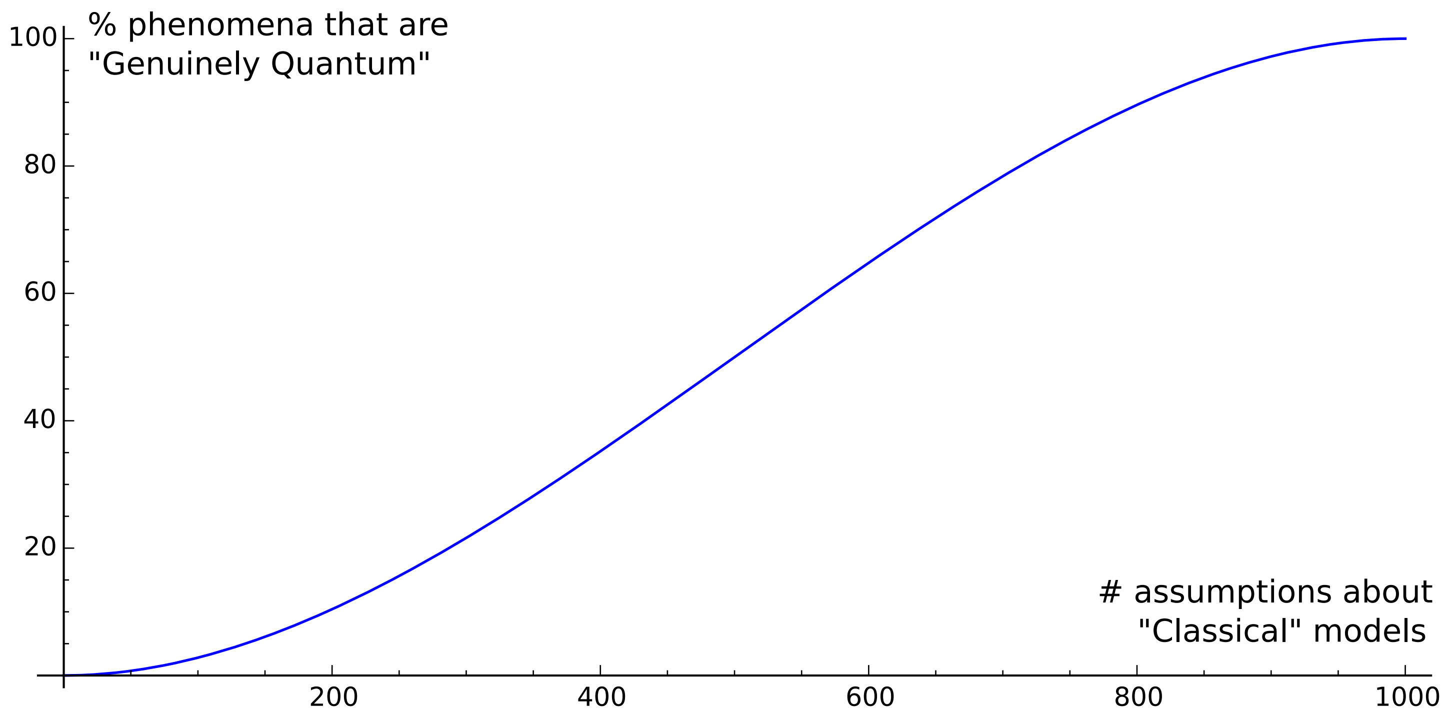


- "Progress through paradox"[a]:
  - Three box paradox
  - Quantum pigeonhole principle
  - Quantum Cheshire cats
  - Anomalous weak values
  - Protective measurement

[a] Y. Aharonov and D. Rohrlich, "Quantum Paradoxes" (Wiley, 2005).

LPPS paradoxes
Protective measurement
Discussion and Conclusions

“Classical” “Quantum”



LPPS paradoxes
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- A vast array of seemingly puzzling quantum phenomena occur in classical models with a restriction on how much you can know about the system[1].
- Those that do not, seem to fall under the rubric of Spekkens contextuality[2].

[1] R. Spekkens, *Phys. Rev. A* 75:032110 (2007).
[2] R. Spekkens, *Phys. Rev. A* 71:052108 (2005).

LPPS paradoxes
Three box paradox
KS Noncontextuality
KSNC model
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S Noncontextuality
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Three box contextuality
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Logical pre- and post-selection paradoxes

Collaborator

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Matt Pusey

Three box paradox

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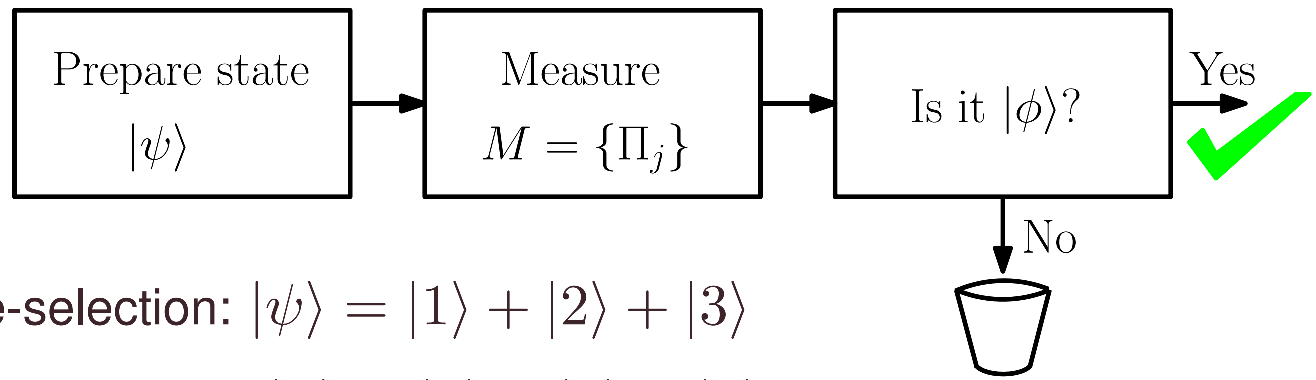

- Pre-selection: $|\psi\rangle = |1\rangle + |2\rangle + |3\rangle$
- Post-selection: $|\phi\rangle = |1\rangle + |2\rangle - |3\rangle$
- Two possible intermediate measurements:
  - M_1: Is ball in box 1? $\Pi_1 = |1\rangle\langle 1|, \quad \Pi_{2\vee 3} = |2\rangle\langle 2| + |3\rangle\langle 3|$

    $$\mathbb{P}(\Pi_1|\psi, M_1, \phi) = 1$$

  - M_2: Is ball in box 2? $\Pi_2 = |2\rangle\langle 2|, \quad \Pi_{1\vee 3} = |1\rangle\langle 1| + |3\rangle\langle 3|$

    $$\mathbb{P}(\Pi_2|\psi, M_2, \phi) = 1$$

Y. Aharonov and L. Vaidman, *J. Phys. A* 24 pp. 2315–2328 (1991).

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- *Outcome determinism*: At any given time, the system has a definite value for every observable.
  - For every projective measurement $\{\Pi_j\}$, precisely one projector is asigned the value 1, the rest 0.
- *Noncontextuality*: The outcome assigned to an observable does not depend on which other (commuting) observables it is measured with.
  - The value assigned to a projector does not depend on which other projectors are measured with it , e.g.

$$|1\rangle\langle 1| \, , |2\rangle\langle 2| \, , |3\rangle\langle 3|$$

$$|1\rangle\langle 1| \, , |2\rangle\langle 2| + |3\rangle\langle 3|$$

$$|2\rangle\langle 2| \, , |1\rangle\langle 1| + |3\rangle\langle 3|$$

S. Kochen and E. Specker, *J. Math. Mech.* 1 pp. 59–87 (1967).

A Kochen-Specker noncontextual model

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- Pre-selection:
  - Place ball in box 1, 2 or 3 at random.
- Intermediate measurement:
  - Open box j.
  - Observe whether ball is present.
  - Leave lid open.
- Post selection:
  - Is there a ball in the box with an open lid?

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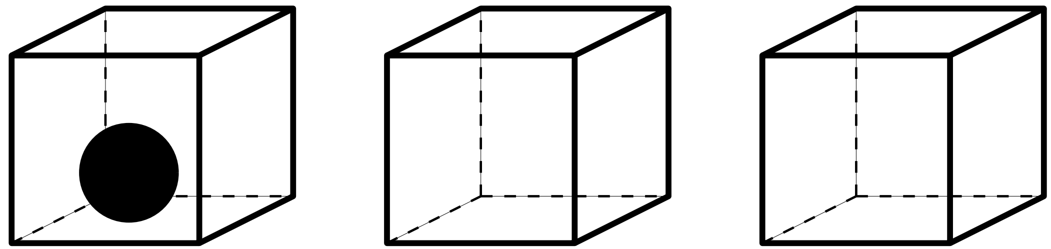

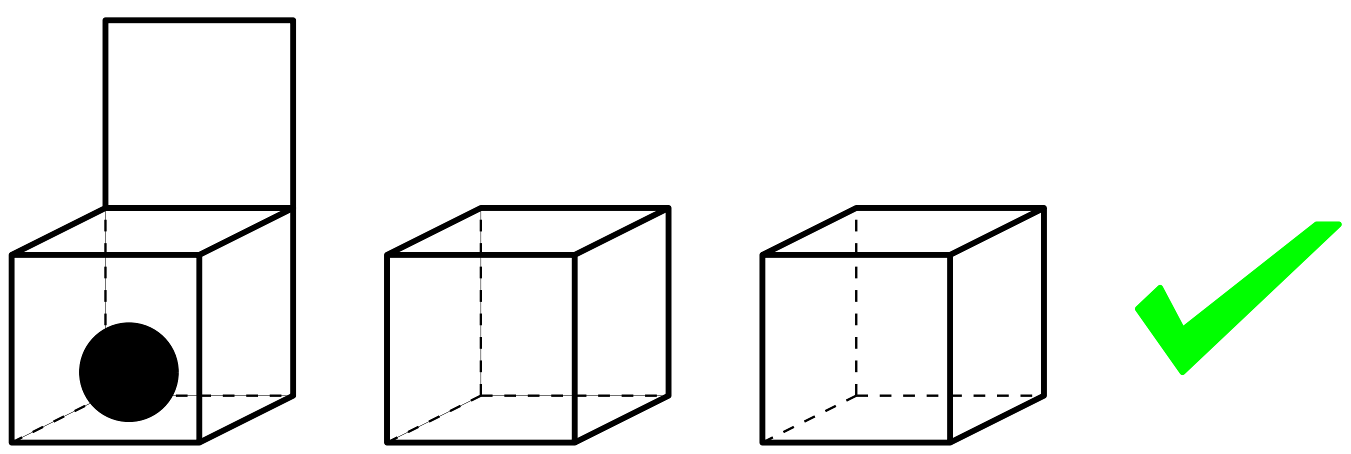

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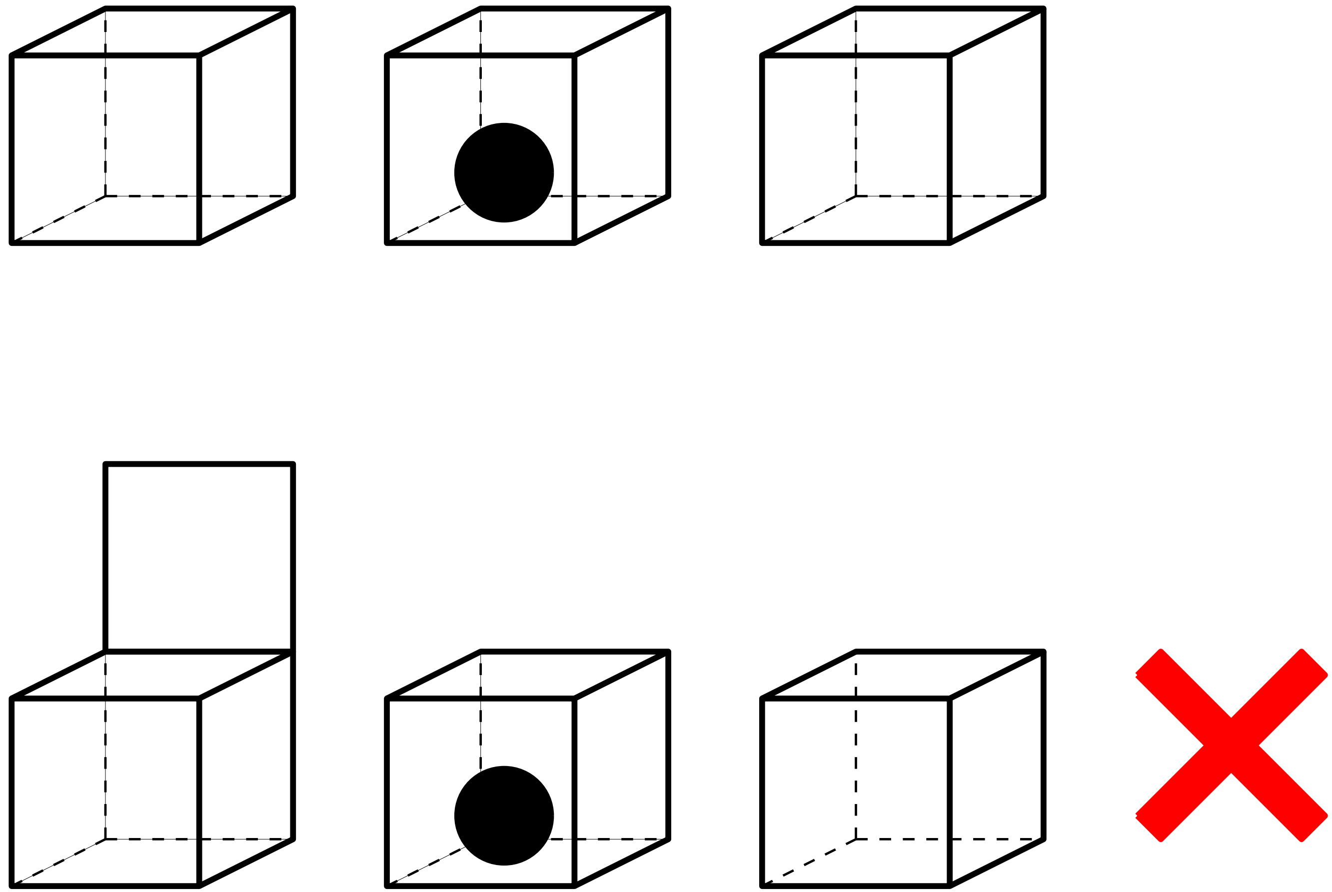

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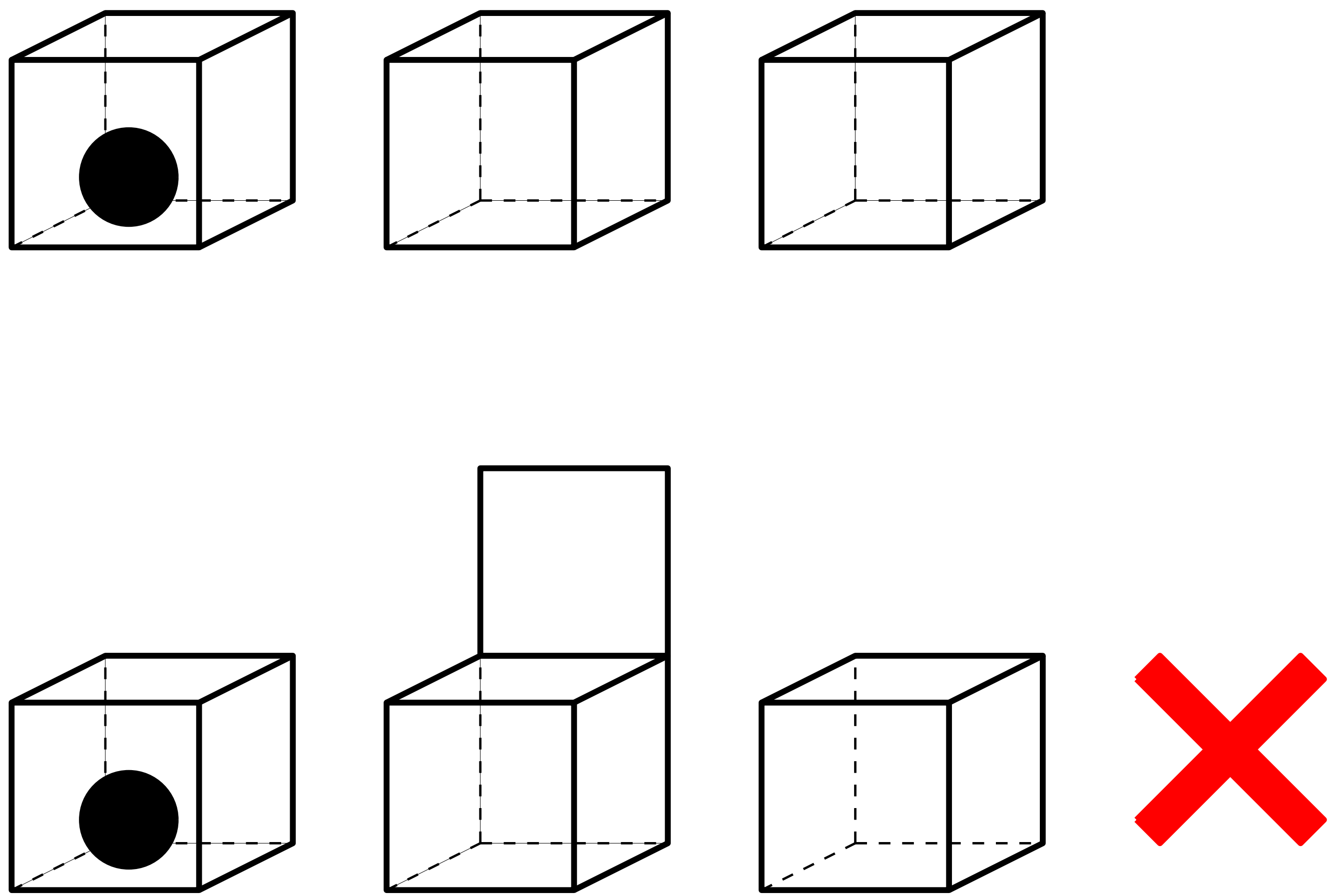

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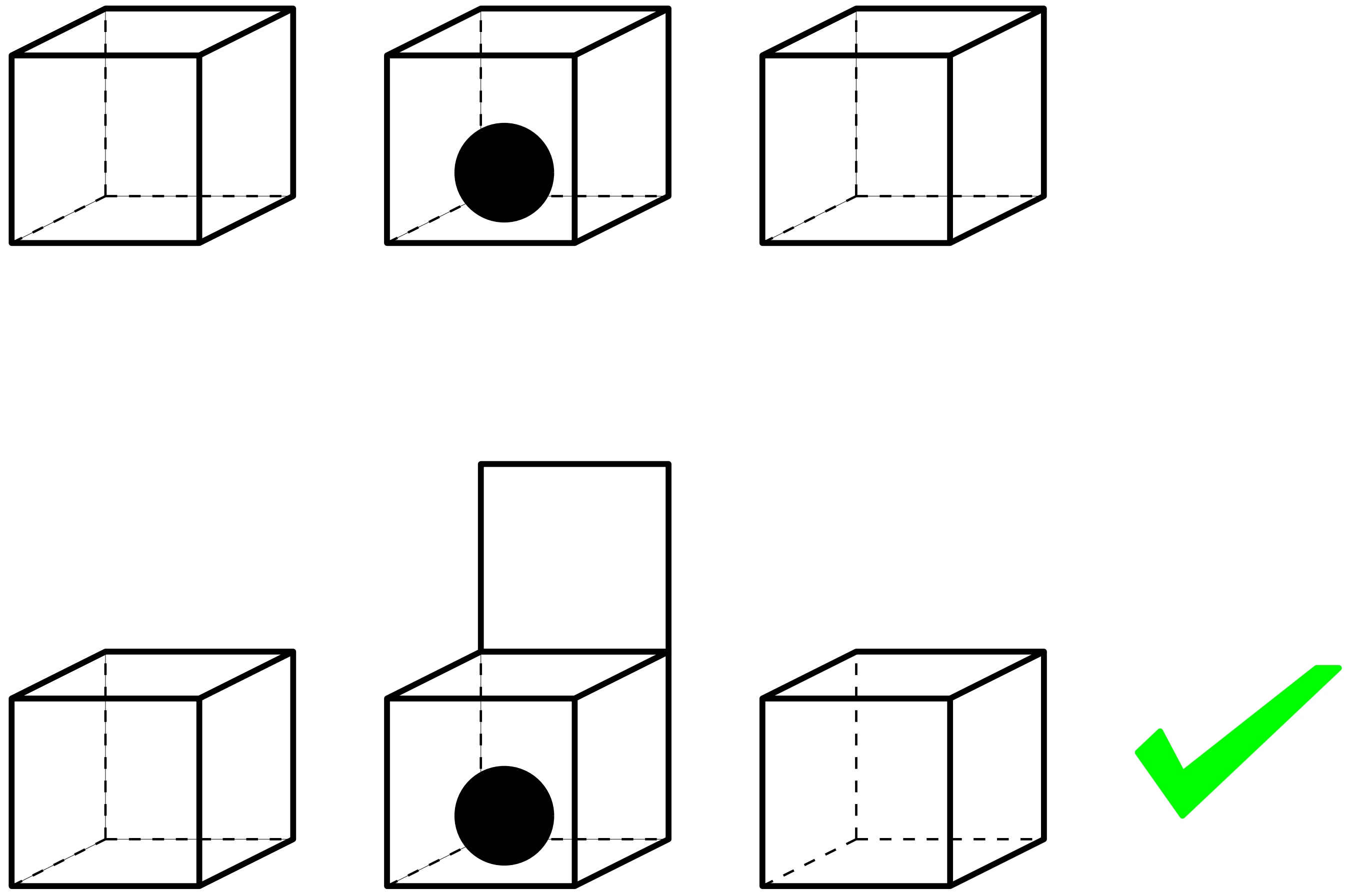

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- See arXiv:1207.3114 for a model that completely reproduces the quantum predictions.

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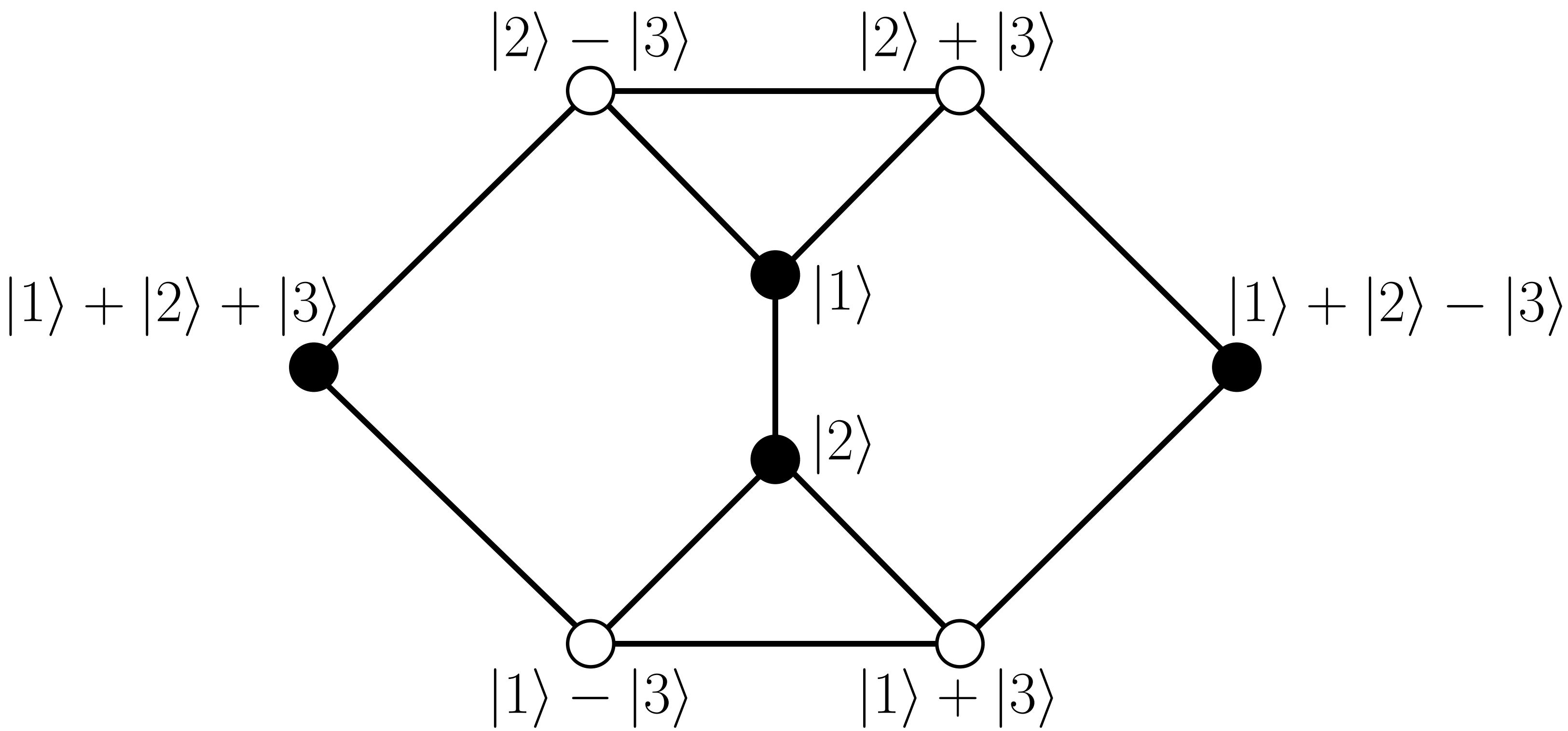

- All logical pre- and post-selection paradoxes are related to a proof of (BS) contextuality in the same way[3].

R. Clifton, *Am. J. Phys.* 61 443 (1993).

[3] M. Leifer and R. Spekkens, *Phys. Rev. Lett.* 95 200405 (2005).

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- Spekkens proposed a more general and operational definition of noncontextuality[4].
  - The reason why projectors receive the same value is because they are always assigned the same probability in quantum theory.
  - General principle: Operationally indistinguishable experimental procedures should be represented the same way in the underlying model.
  - *Transformation noncontextuality*: Two procedures corresponding to the same CPT map must be represented in the same way.

[4] R. Spekkens, *Phys. Rev. A* 71:052108 (2005).

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Theorem. *Let $\{\Pi_j\}$ be a projective measurement and let $\mathcal{E}$ be the nonselective state-update rule*

$$\mathcal{E}(\rho) = \sum_j \Pi_j \rho \Pi_j .$$

Then,

$$\mathcal{E}(\rho) = p\rho + (1-p)\mathcal{C}(\rho),$$

where $\mathcal{C}$ is a completely-positive, trace-preserving map and $0 < p \leq 1$.

- Proof for special case $\{\Pi_1, \Pi_2\}$:

$$U_1 = \Pi_1 + \Pi_2 = I \qquad U_2 = \Pi_1 - \Pi_2$$

$$\mathcal{E}(\rho) = \frac{1}{2}U_1\rho U_1^\dagger + \frac{1}{2}U_2\rho U_2^\dagger = \frac{1}{2}\rho + \frac{1}{2}U_2\rho U_2^\dagger .$$

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Theorem. *The three box paradox is a proof of (Spekkens) contextuality.*

- Assume transformation noncontextuality.
- Since $|\langle\psi|\phi\rangle|^2 > 0$, there must be some hidden states that assign value 1 to both $|\psi\rangle\langle\psi|$ and $|\phi\rangle\langle\phi|$.
- With probability at least $1/2$, the intermediate measurement does not change the hidden state.
- Therefore, these hidden states must assign probability 1 to $|1\rangle\langle 1|$ in M_1 and probability 1 to $|2\rangle\langle 2|$ in M_2, but this is measurement contextual.

Further details

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- Read arXiv:1506.07850 for:
  - Generalization to all logical pre- and post-selection paradoxes.
    - Quantum pigeonhole principle, failure of the product rule, . . .
  - Proof using measurement noncontextuality instead of transformation noncontextuality.
  - Relation to weak measurement paradoxes.
  - Importance of 0/1 probabilities and von-Neumann update rule.

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- Anomalous weak values have classical analogues:
  - C. Ferrie and J. Combes, *Phys. Rev. Lett.* 113 120404 (2014).
- But, if you try to simulate the quantum predictions exctly, the model must be (Spekkens) contextual:
  - M. Pusey, *Phys. Rev. Lett.* 113 200401 (2014).

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Protective measurement & The reality of the quantum state

Collaborators

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- In 1993, Aharonov, Anandan and Vaidman introduced a method of determining the quantum state of a single copy of a quantum system, provided the system is *protected* during the course of measurement[5].
- Protection is a procedure for preventing the quantum state from changing during the course of a measurement. Two types:
  - Protection via the quantum Zeno effect.
  - Hamiltonian protection.

[5] Y. Aharonov, J. Anandan and L. Vaidman, *Phys. Rev. A* 47:6 4616–4626 (1993).

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- Protection is a procedure for preventing the quantum state from changing during the course of a measurement. Two types:

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  - Hamiltonian protection.

- Does this imply the reality of the quantum state?

[7] Y. Aharonov, J. Anandan and L. Vaidman, *Phys. Rev. A* 47:6 4616–4626 (1993).

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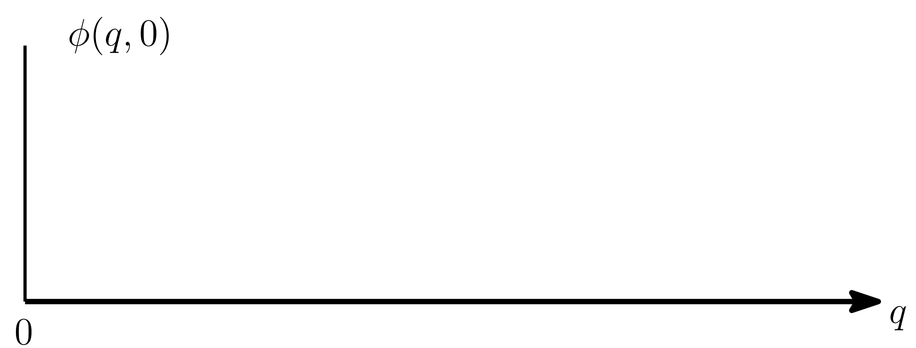
Zeno protected measurement

Alice Person trying to determine the quantum state
Bob Person who protects the quantum system.

- Bob sends Alice a quantum system prepared in a state $|\psi\rangle$.
- The protection: Every Δt Bob performs a measurement in a basis $\{|\psi_j\rangle\}$ that includes $|\psi\rangle$ as an eigenstate.
- To measure an observable, Alice couples it to a pointer system with wavefunction $\phi(q,t)$ and initial state $\phi(q,0) = \delta(q)$.



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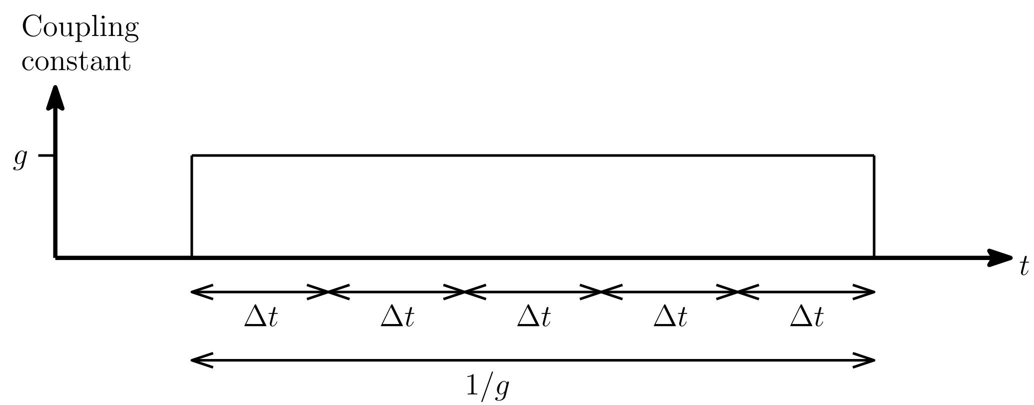
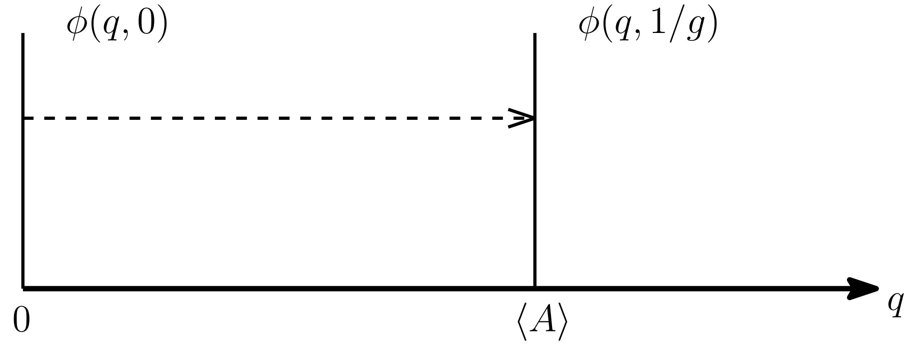
- To measure A, Alice couples the pointer to the system via a Hamiltonian $H = gAp$ for time $1/g$ s.t. $\Delta t \ll 1/g$.



- When $\Delta t \to 0$, the pointer ends up pointing to $\langle A \rangle = \langle \psi | A | \psi \rangle$ and the system remains in state $|\psi\rangle$.



Measuring the quantum state

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- Since the state of the system is unchanged, Alice can perform as many protective measurements of different observables as she likes.
- If she measures a tomographically complete set, she can determine the quantum state.
- So does this imply the reality of the quantum state?
- We give 3 arguments against:
  - First two suggest that almost all the information about the state is coming from the protection operation rather than the system.
  - Last is to construct a ψ-epistemic toy model. Suggests protective measurement is more like tomography on an infinite ensemble than a measurement of a single copy.

Heuristic resource counting argument

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- In protective measurement, Alice effectively has access to the following resources:
  - A single copy of the system prepared in the state $|\psi\rangle$.
  - The ability to perform a measurment in the basis $\{|\psi_j\rangle\}$ an unlimited number of times, i.e. the channel

$$\mathcal{E}(\rho) = \sum_j |\psi_j\rangle\langle\psi_j|\rho|\psi_j\rangle\langle\psi_j|.$$

- A simpler way to determine $|\psi\rangle$ is to:
  - Perform process tomography on $\mathcal{E}$.
  - Calculate the basis $\{|\psi_j\rangle\}$ from its fixed point set.
  - Measure $|\psi\rangle$ in that basis.

Exact analysis of protective measurement

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- Any sequence of operations on a system that results in a classical outcome can be written in terms of a POVM:

$$\mathbb{P}(q) = \text{Tr}\left(E_q \rho\right)$$

- In a protective measurement E_q is correlated with $|\psi\rangle$ via the protection operation, but it depends only on this and not on the initial state of the system.
- For a protective measurement of a two-outcome observable, we have shown that

$$E_q = \sum_j |\psi_j\rangle\langle\psi_j| \delta(q - \langle\psi_j| A |\psi_j\rangle)$$

- Thus, most of the information comes from the protection operation.

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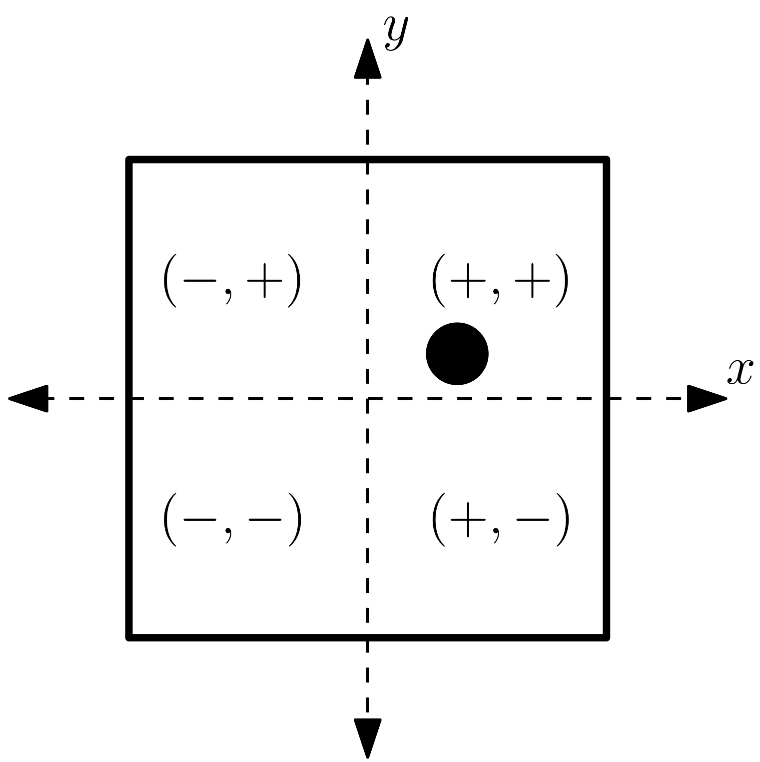
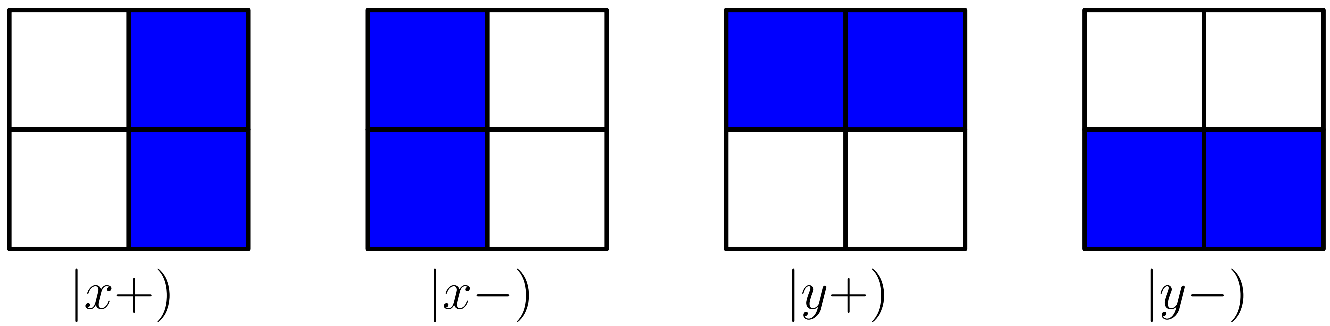
- System described by two classical random variables, X and Y, that take values ± 1 (or $\pm$ for short).
  - (x, y) denotes state in which $X = x$ and $Y = y$.
- Example: Ball in a box:



Toy model: Bob's States

- Assume Bob can prepare the system in four different probability distributions:



Distribution	$\langle X \rangle$	$\langle Y \rangle$
$\|x+)$	+1	0
$\|x-)$	-1	0
$\|y+)$	0	+1
$\|y-)$	0	-1

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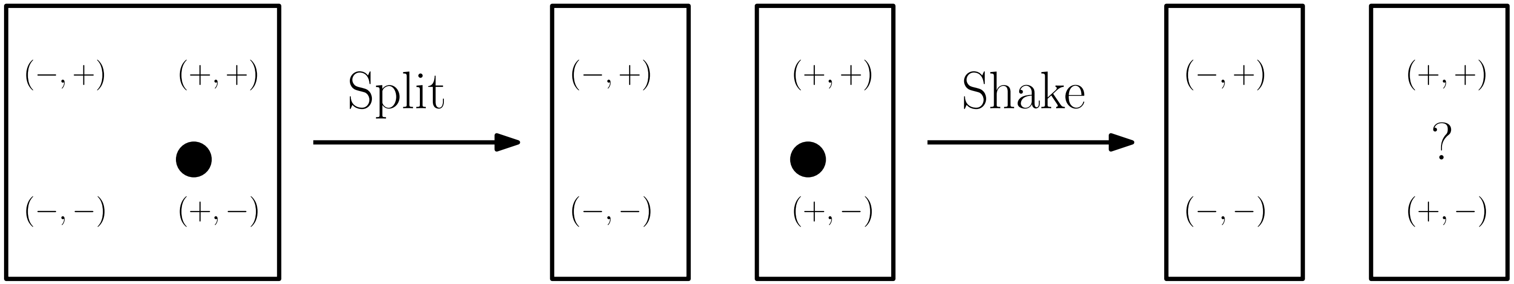
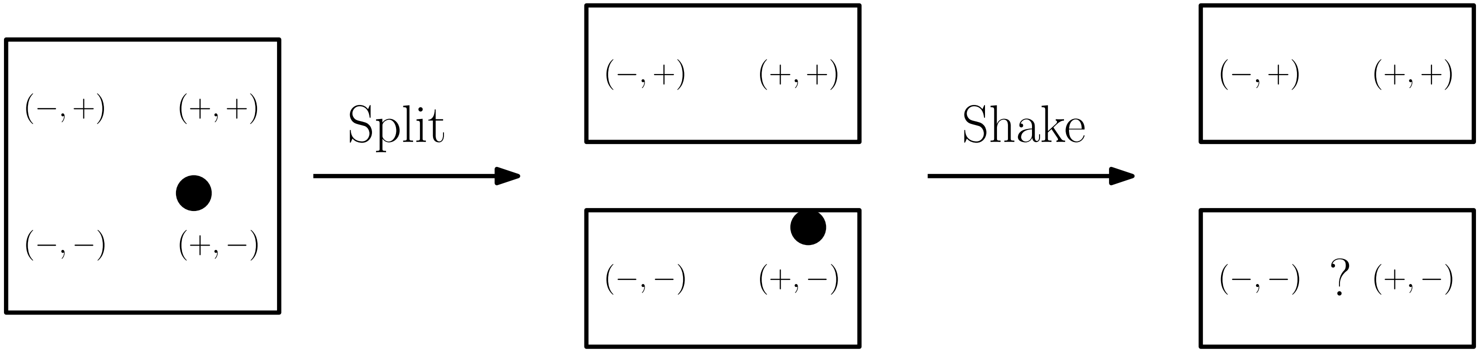
- X-measurement:



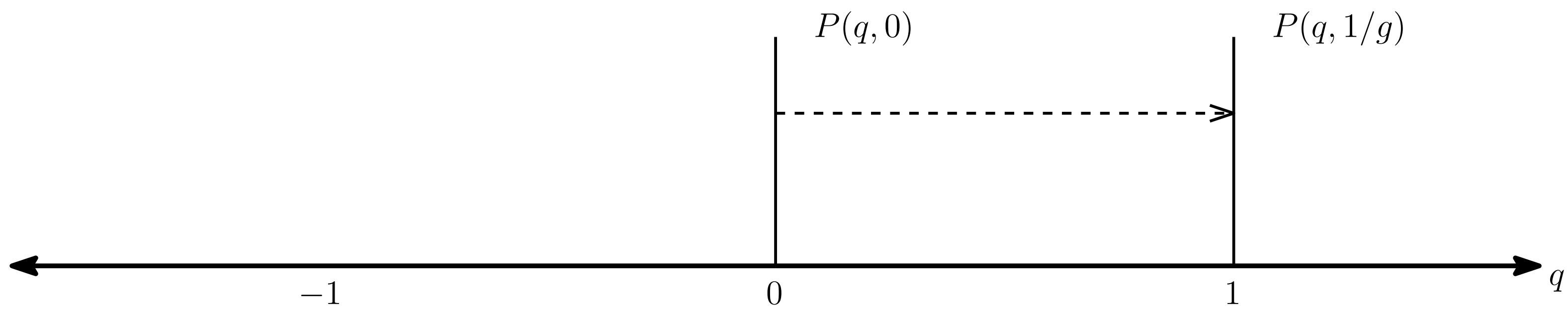
- Y-measurement:



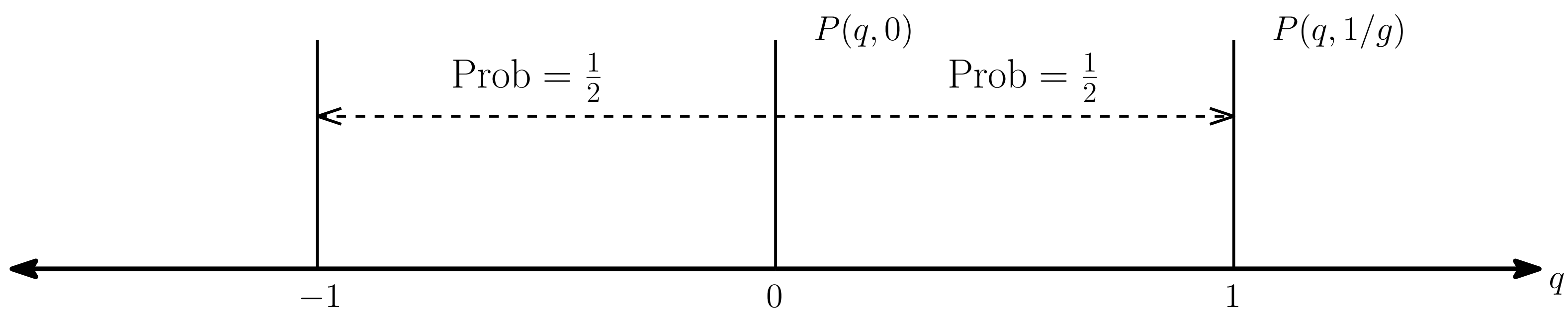
Toy model: Alice's measurements

- System is coupled to a classical pointer prepared in state $q = p = 0$ with Hamiltonian $H = gXp$ or $H = gYp$ for a time $1/g$.

- Without protection, for system prepared in $|x+)$, with $H = gXp$:



- and with $H = gYp$:



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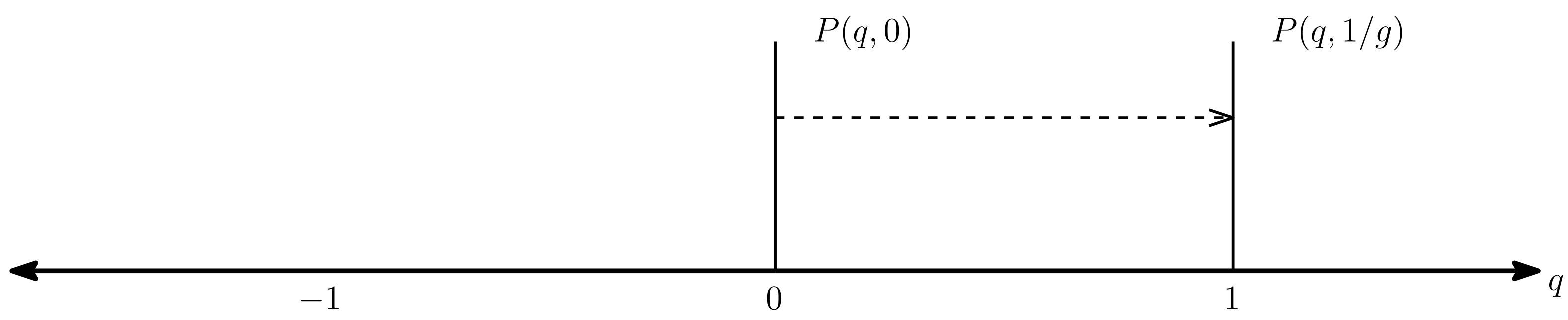
Toy model: Zeno protected measurement

- Now do the same thing whilst at the same time Bob is measuring X every $\Delta t = 1/gN$.

- For $H = gXp$, the pointer moves as before. The pointer is coupled to X, but Bob's measurement only affects Y.



- For $H = gYp$, every Δt the y-coordinate is randomized, so the pointer will keep going in the same direction or switch direction with probability $1/2$ each.
  - Pointer executes an N-step random walk with step size $1/N$.

Toy model: Zeno protected measurement

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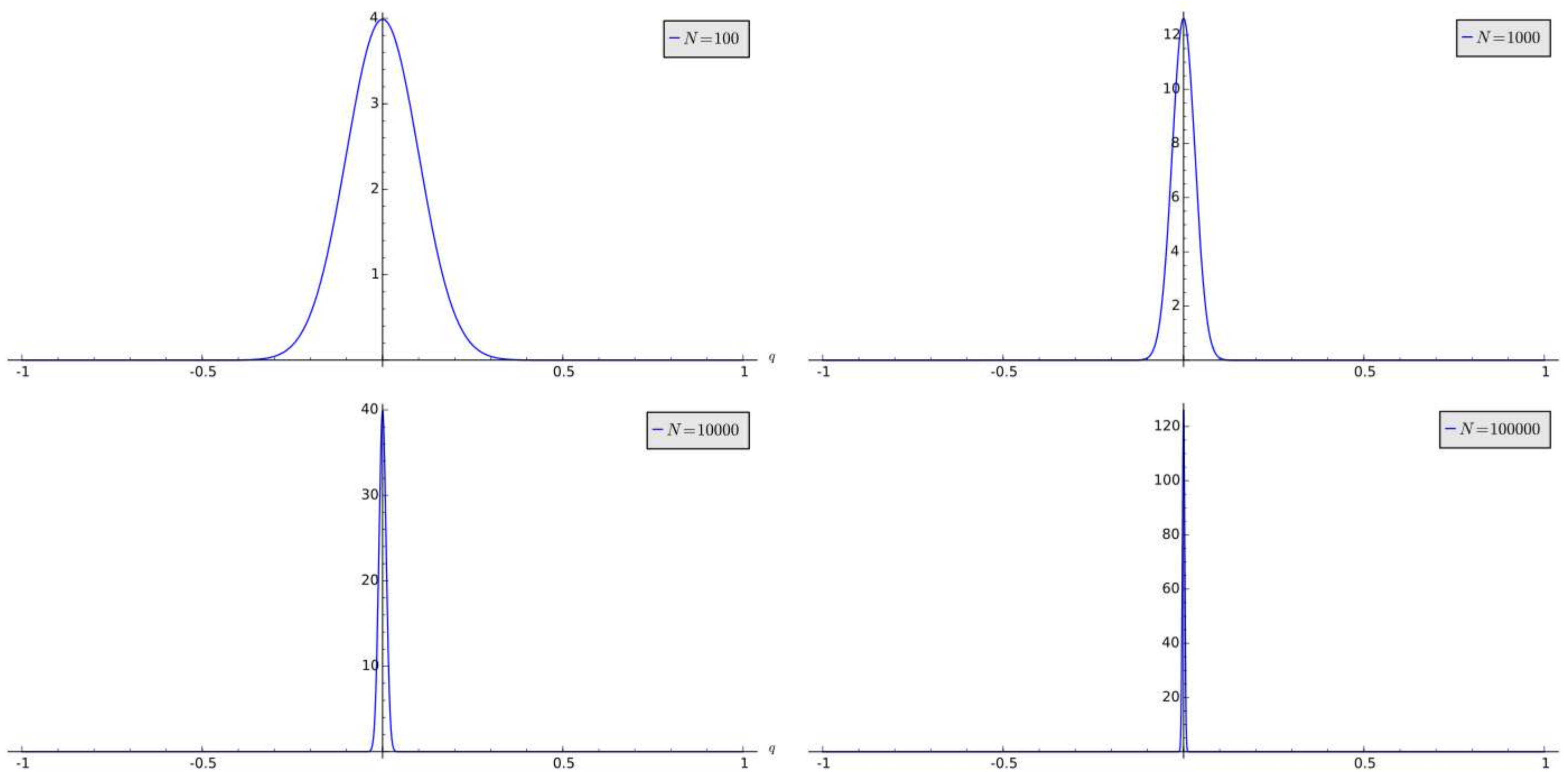
- For large N, distribution of final pointer position is $\approx \mathcal{N}(0, 1/N)$.
- Tends to $\delta(q)$ as $N \to \infty$.



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- Implicit assumption that if a measurement does not change a quantum state then the measurement does nothing to the system when it is prepared in that state:
  - Not true in our model: Measuring X randomizes the y-coordinate even though distribution $|x+)$ is unchanged.
- Protective measurement is more like measuring N independently prepared systems than measuring just a single copy.
- One might worry that there are aspects of protective measurement not captured by the toy model.

Further results

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- Adding back-action to the Zeno toy model.
- Toy model and exact analysis for Hamiltonian protective measurements.

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Conclusions

- There is no such thing as a “classical” or “genuinely quantum” phenomenon without
  - Specifying assumptions for “classical” models.
  - Specifying which aspects of the phenomenon you want to reproduce.
- A well-motivated set of assumptions is:
  - Understandable in a Spekkens noncontextual classical probabilistic theory with restriction on knowledge = “classical”.
  - Spekens Contextual = “quantum”.
- On this classification LPPS paradoxes are “quantum” and protective measurement is “classical”.