2 out of 3 Roads to Quantum Foundations

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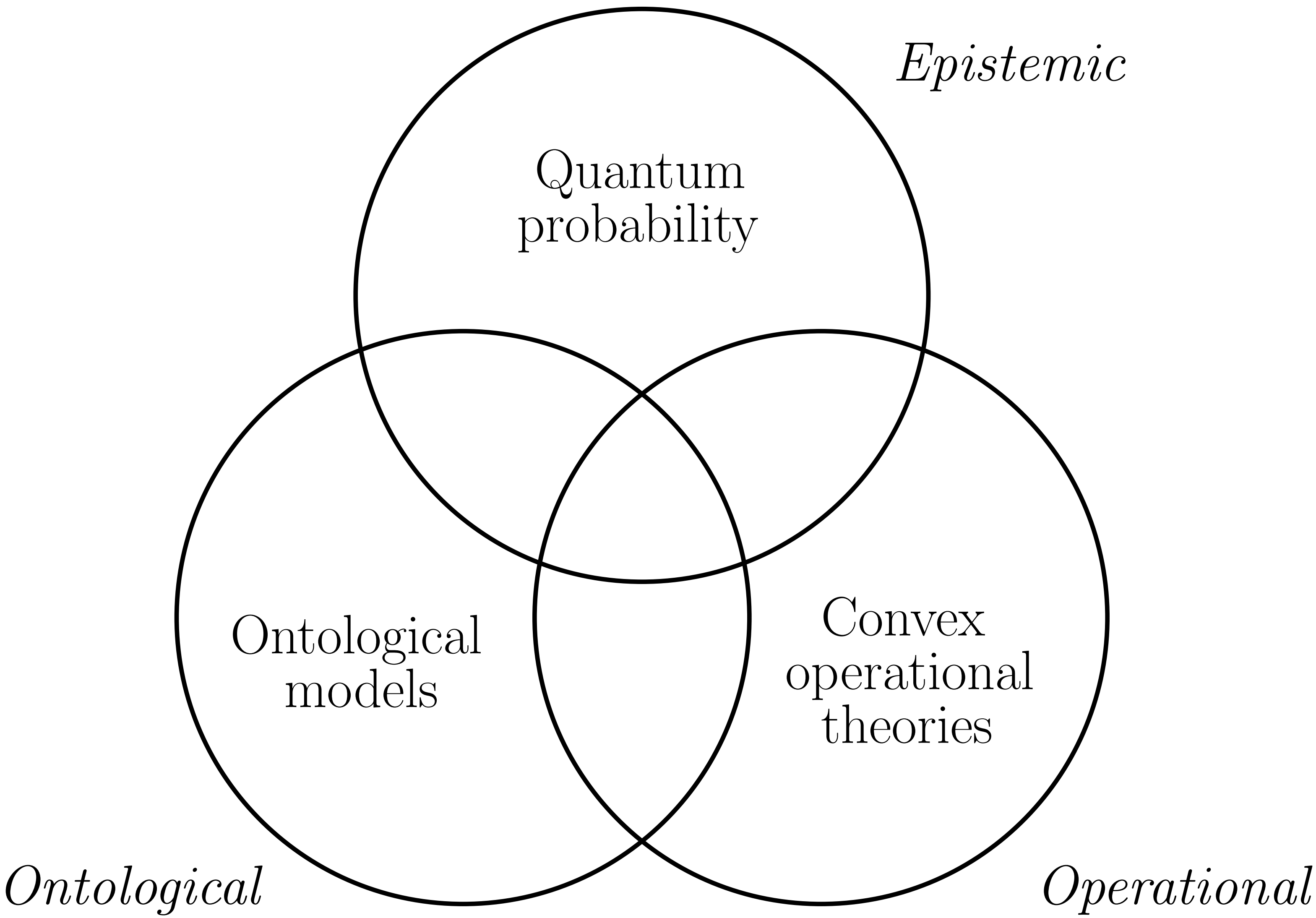
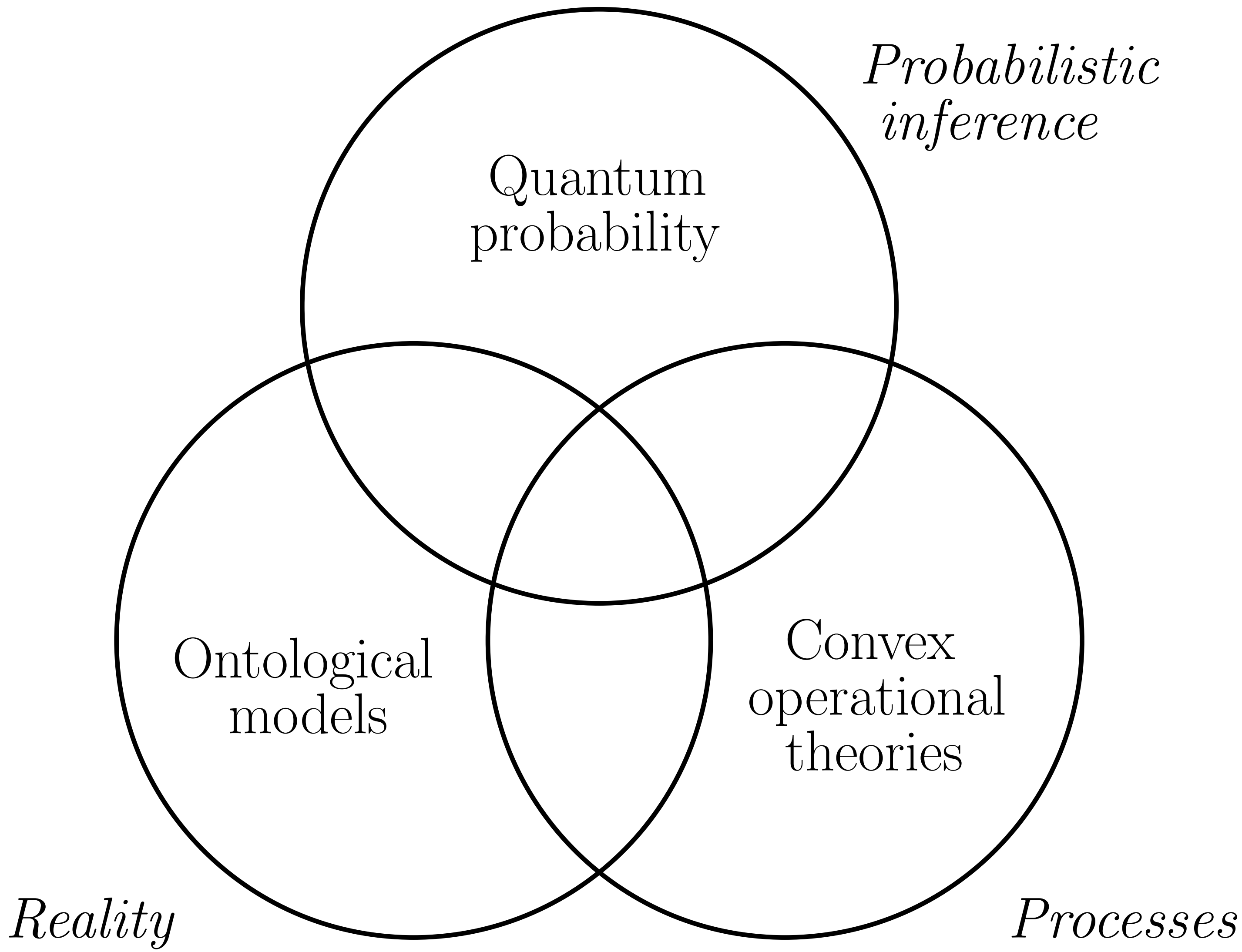
1st half based on:

PRL 112:160404 (2014)
Quanta 3:67–155 (2014).

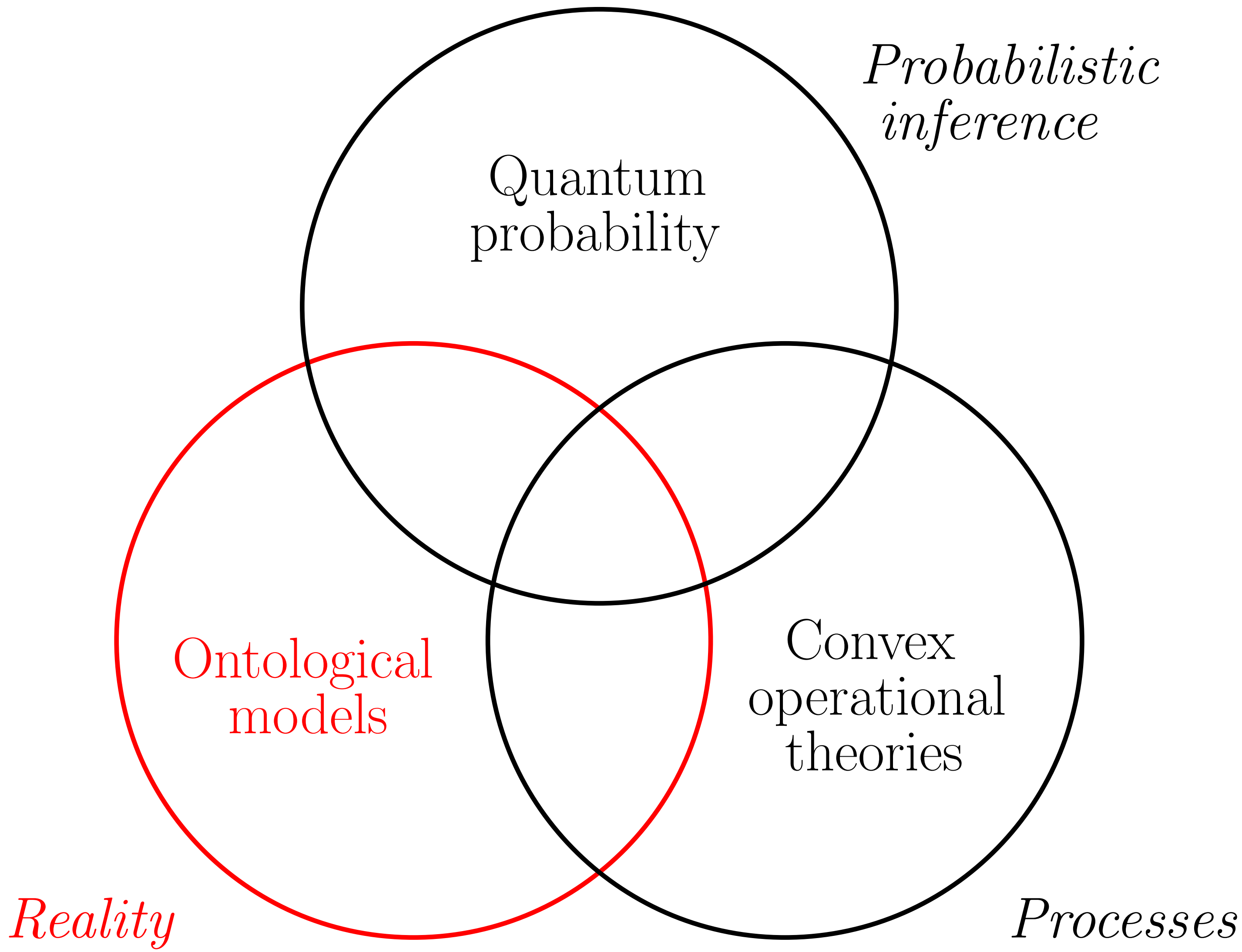
10th December 2014


Epistemic
Quantum probability
Ontological models
Convex operational theories
Ontological
Operational

Overview of my research



Overview of my research

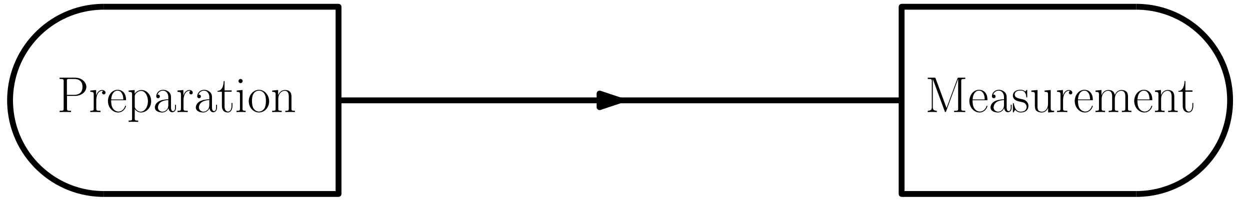


Reality of the Quantum State

Interpretations of quantum theory

	ψ-epistemic	ψ-ontic
Copenhagenish	Copenhagen neo-Copenhagen (e.g. QBism, Peres, Zeilinger, Healey)	
Realist	Einstein Ballentine? Spekkens ?	Dirac-von Neumann Many worlds Bohmian mechanics Spontaneous collapse Modal interpretations

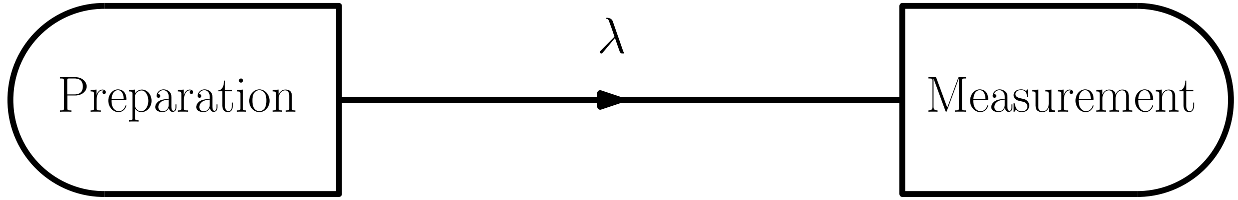
Prepare-and-measure experiments: Quantum description



$|\psi\rangle \in \mathbb{C}^d$ $\qquad$ $M = \{|a\rangle\,, |b\rangle\,, \ldots\}$

$$\mathrm{Prob}(a|\psi, M) = |\langle a|\psi\rangle|^2$$

Prepare-and-measure experiments: Ontological description



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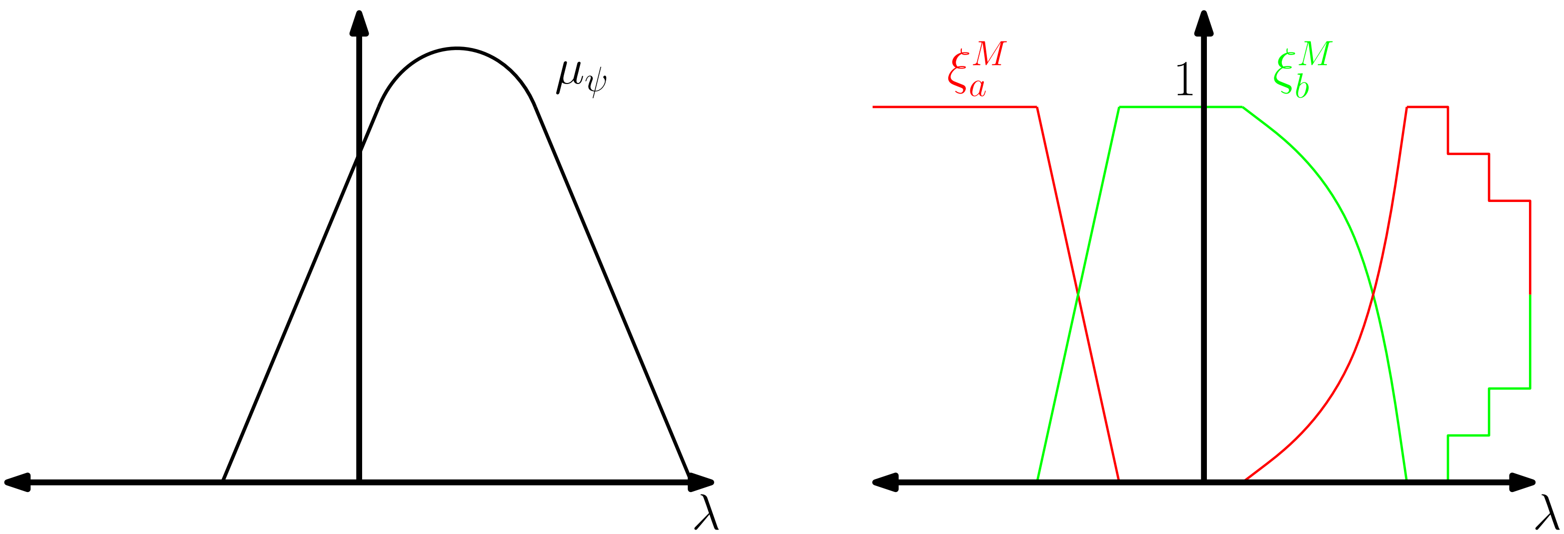
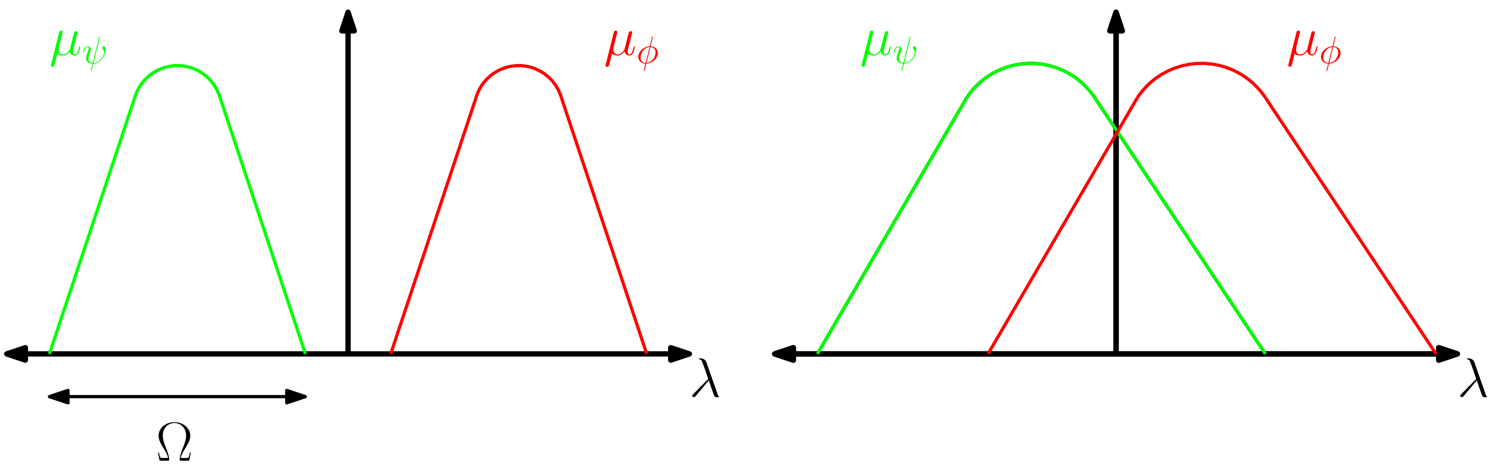


$$\mathrm{Prob}(a|\psi, M) = \int \xi_a^M(\lambda) d\mu_\psi$$

ψ-ontic and ψ-epistemic models

- $|\psi\rangle$ and $|\phi\rangle$ are *ontologically distinct* in an ontological model if there exists $\Omega \in \Sigma$ s.t.

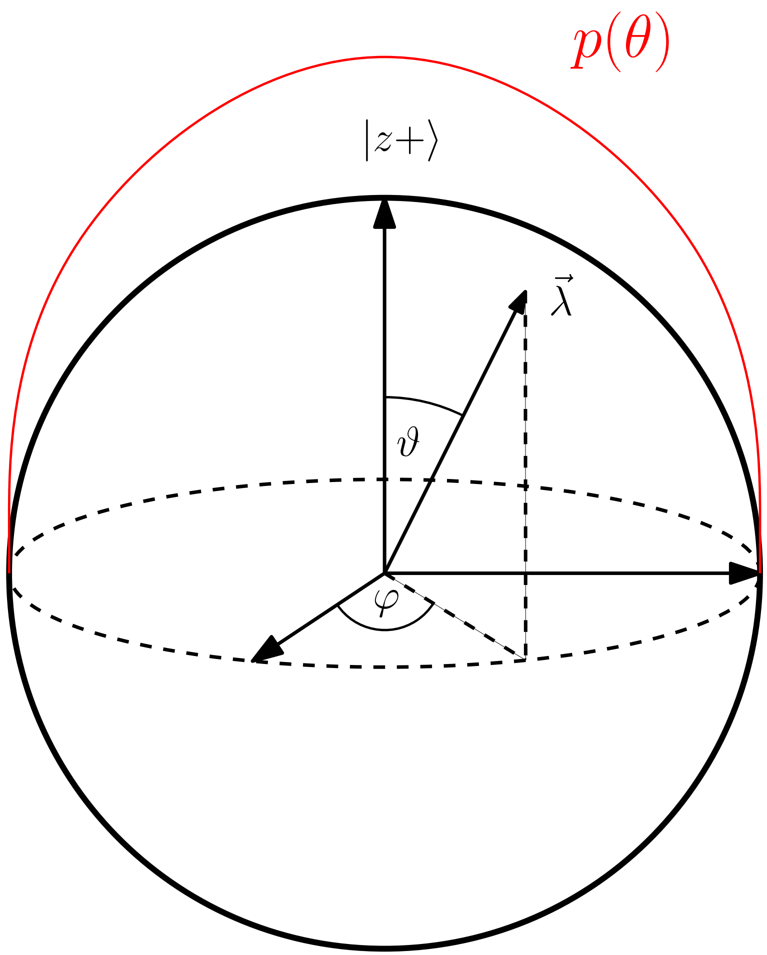
$$\mu_\psi(\Omega) = 1 \qquad \mu_\phi(\Omega) = 0.$$



- An ontological model is *ψ-ontic* if every pair of states is ontologically distinct. Otherwise it is *ψ-epistemic*.

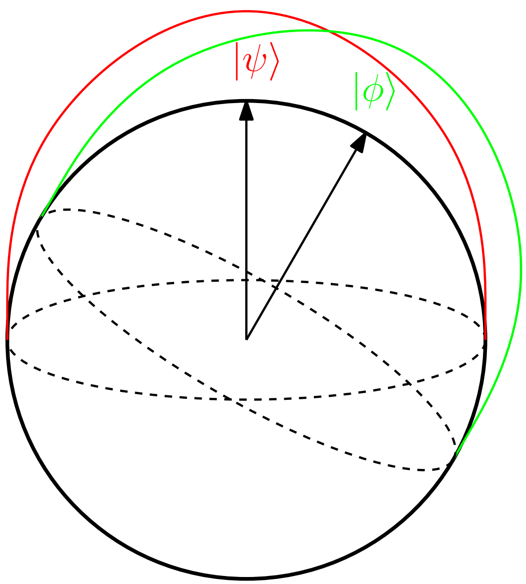
- The Colbeck-Renner theorem: R. Colbeck and R. Renner, arXiv:1312.7353 (2013).
- Hardy's theorem: L. Hardy, *Int. J. Mod. Phys. B*, 27:1345012 (2013) arXiv:1205.1439
- The Pusey-Barrett-Rudolph theorem: M. Pusey et. al., *Nature Physics*, 8:475–478 (2012) arXiv:1111.3328

The Kochen-Specker model for a qubit



$$\mu_{z+}(\Omega) = \int_\Omega p(\vartheta) \sin\vartheta d\vartheta d\varphi$$

$$p(\vartheta) = \begin{cases} \frac{1}{\pi}\cos\vartheta, & 0 \leq \vartheta \leq \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < \vartheta \leq \pi \end{cases}$$



S. Kochen and E. Specker, *J. Math. Mech.*, 17:59–87 (1967)

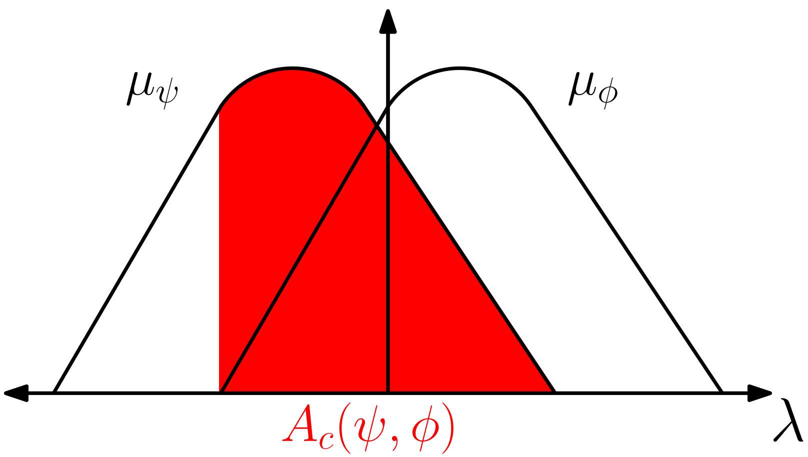
Models for arbitrary finite dimension

- Lewis et. al. provided a ψ-epistemic model for all finite d.
  - P. G. Lewis et. al., *Phys. Rev. Lett.* 109:150404 (2012) arXiv:1201.6554
- Aaronson et. al. provided a similar model in which every pair of nonorthogonal states is ontologically indistinct.
  - S. Aaronson et. al., *Phys. Rev. A* 88:032111 (2013) arXiv:1303.2834
- These models have the feature that, for a fixed inner product, the amount of overlap decreases with d.

Asymmetric overlap

- *Classical asymmetric overlap*:

$$A_c(\psi,\phi) := \inf_{\{\Omega\in\Sigma \,|\, \mu_\phi(\Omega)=1\}} \mu_\psi(\Omega)$$



- An ontological model is *maximally ψ-epistemic* if

$$A_c(\psi,\phi) = |\langle\phi|\psi\rangle|^2$$

Main result

- Let $\mathcal{D} = \{|\phi_j\rangle\}_{j=1}^N$ be a set of quantum states and let $|\psi\rangle$ be any other quantum state. Then, in any ontological model,

$$\sum_{j=1}^{N} A_c(\phi_j, \psi) \leq \mathsf{NC}_{\mathcal{D}},$$

  where $\mathsf{NC}_{\mathcal{D}}$ is the maximum number of states in $\mathcal{D}$ that can be assigned the value 1 in a noncontextual model.

- Define:

$$\bar{k}_{\mathcal{D}}(\psi) = \frac{\sum_{j=1}^N A_c(\psi_j, \phi)}{\sum_{j=1}^N |\langle \phi_j|\psi\rangle|^2}.$$

- Using a KS proof[1], I found a set of states in $\mathbb{C}^d$ for which

$$k_{\mathcal{D}}(\psi) \leq 2de^{-cd}.$$

- All other results of this type found so far can alternatively be derived from the above result.

[1] ML, Phys. Rev. Lett. 112:160404 (2014)

- Ringbauer et. al. obtained

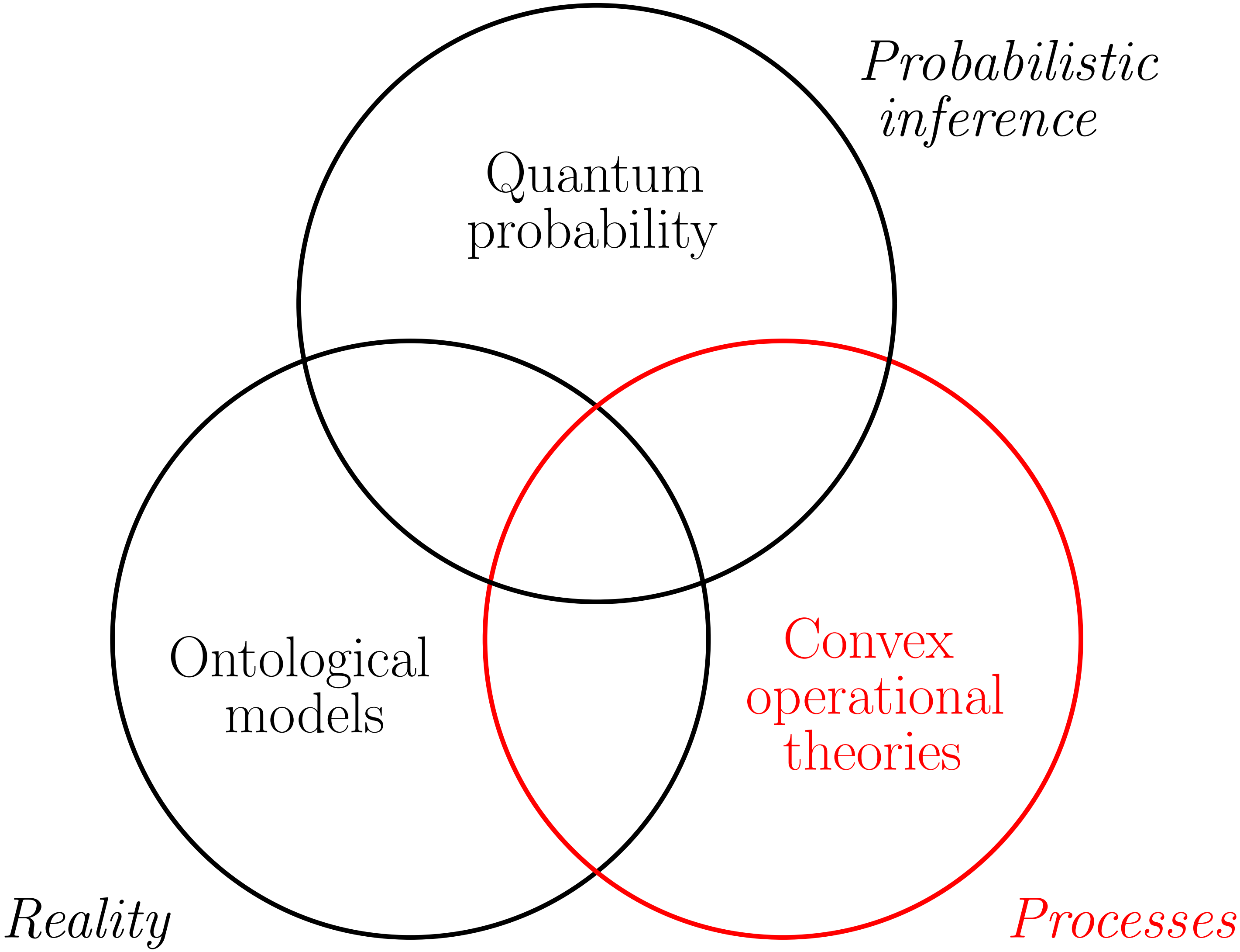
$$k_{\mathcal{D}}(\psi) \leq 0.690 \pm 0.001$$

  in an optical system for $d = 4$.

- Ringbauer et. al. experiments required a fair sampling assumption and estimated $\approx 98\%$ detector efficiency required to do with out.

- Values close to zero are needed to convincingly rule out ψ-epistemic theories.

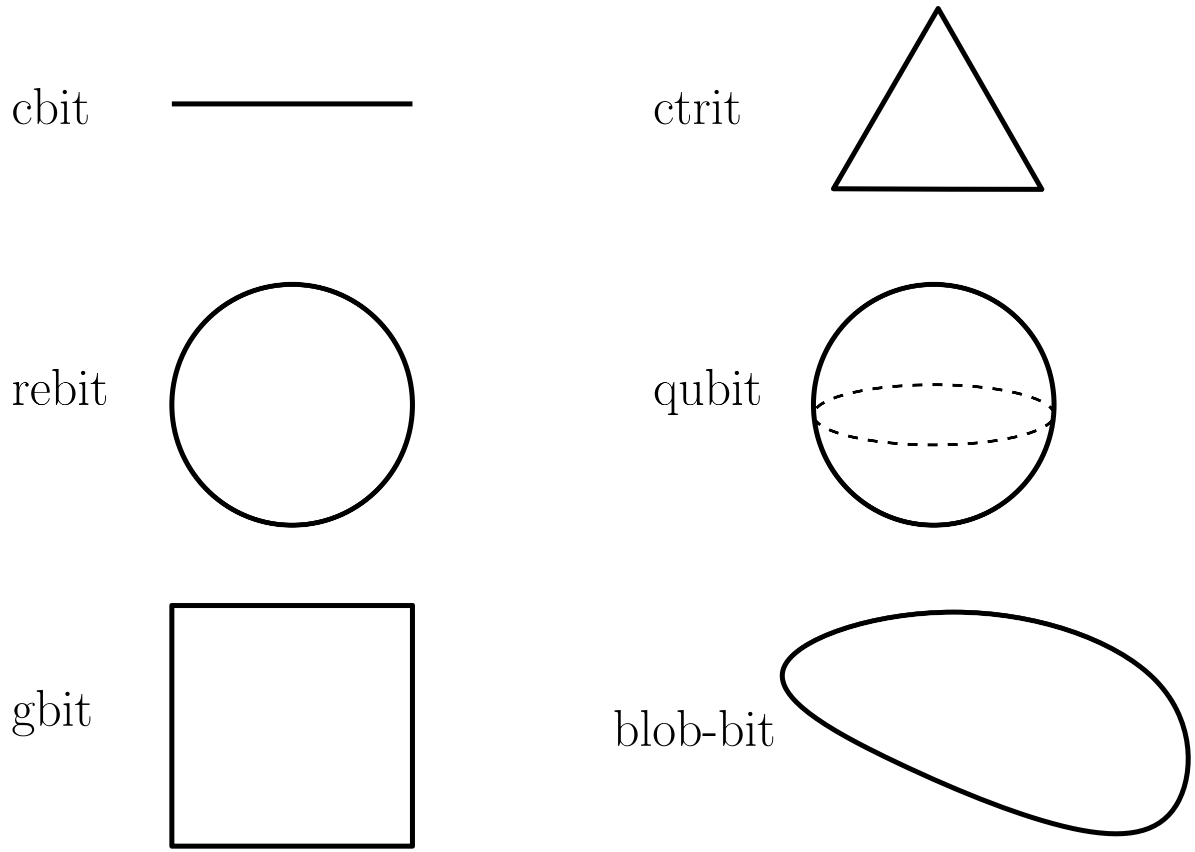
- Since we now know these results can be derived from noncontextuality inequalities, we can now search for optimal experiments.


Probabilistic inference
Quantum probability
Ontological models
Convex operational theories
Reality
Processes

The Second Law of Thermodynamics as an Axiom for Quantum Theory

Convex Operational Theories

- General framework for probabilistic theories that includes classical probability, quantum theory, PR-boxes, . . . as special cases.
- State space of a system is an arbitary compact convex set.



- Identifying the logical structure of information processing
  - Connection between cloning, broadcasting and distinguishability[2].
  - Nonclassicality + No entanglement $\Rightarrow$ Bit commitment[3].
  - de Finetti theorem[4].
  - Requirements for teleportation[5].
- Axiomatic reconstructions of quantum theory
  - L. Hardy, arXiv:quant-ph/0101012, arXiv:1104.2066.
  - B. Dakic, C. Brukner, in H. Halvorson (ed.) *Deep Beauty*, pp. 365–392 (CUP, 2011).
  - L. Masanes, M. Müller, New. J. Phys. 13:063001 (2011).
  - G. Chiribella, G. M. D'Ariano, P. Perinotti, Phys. Rev. A. 84:012311 (2011).

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[2] H. Barnum, J. Barrett, ML, A. Wilce, Phys. Rev. Lett. 99:240501 (2007).

[3] H. Barnum, O. Dahlsten, ML, B. Toner, Proc. IEEE Info. Theory Workshop, 2008, pp. 386–390.

[4] J. Barrett, ML, New J. Phys. 11:033024 (2009).

[5] H. Barnum, J. Barrett, ML, A. Wilce, Proc. Clifford Lectures 2008 (2012).

The Resource Theory of (Classical) Nonuniformity

- Thermodynamics can be formulated as a *resource theory*. If $H =$ const. then this reduces to the theory of *nonuniformity*[6] .
- States: Probability distributions $\boldsymbol{p}$.
- Free operations:
  - Reversible transformations
  - Adding uniform ancillas $(\frac{1}{d}, \frac{1}{d}, \ldots, \frac{1}{d})$.
  - Discarding subsystems.
- Second law: If $\boldsymbol{p} \to \boldsymbol{p}'$ is possible under free operations (with $\boldsymbol{p}$, $\boldsymbol{p}'$ defined on the same space) then

$$S(\boldsymbol{p}') \geq S(\boldsymbol{p}).$$

[6] G. Gour, M. Müller, V. Narasimachar, R. Spekkens, N. Halpern, arXiv:1309.6586.

The Resource Theory of (COT) Nonuniformity

- For an arbitrary COT, this cannot be formulated so easily.
- States: Elements ω of a convex set.
- Free operations:
  - Reversible transformations (automorphism group)
  - *Adding maximally mixed ancillas?*
    - Generally there is no unique notion of a uniform state.
  - Discarding subsystems.
- *Second law?*
  - Although some entropy functions have been proposed[7], it is not clear whether they are relevant to thermodynamics, or indeed if there is a unique thermodynamic entropy at all.

[7] H. Barnum, J. Barrett, L. Clark, ML, R. Spekkens, N. Stepanik, A. Wilce, R. Wilke, New J. Phys. 12:033024 (2010). A. Short, S. Wehner, New J. Phys. 12:033023 (2010).

- We can consider hybrid theories in which we can have both classical and COT systems.
- States: Elements $\boldsymbol{p} \otimes \omega$ of the joint state space.
- Free operations:
  - Reversible transformations (automorphism group)
  - *At least, we should be able to add uniform classical ancillas* $(\frac{1}{d}, \frac{1}{d}, \ldots, \frac{1}{d})$.
  - Discarding subsystems.
- *Second Law: At least we expect that if* $\boldsymbol{p} \otimes \omega \to \boldsymbol{p}' \otimes \omega$ *is possible under free operations (with* $\boldsymbol{p}, \boldsymbol{p}'$ *defined on the same space) then*

$$S(\boldsymbol{p}') \geq S(\boldsymbol{p}).$$

von Neumann's Assumption

- If ω_0 and ω_1 are distinguishable pure states then

$$[p\omega_0 + (1-p)\omega_1] \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \leftrightarrow p\omega_0 \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (1-p)\omega_1 \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

- If the automorphism group of the COT is transitive then von Neumann's assumption implies that mixedness can be SWAPped between classical bits and COT system, i.e. if ω_0 and ω_1 are distinguishable pure states then

$$\begin{pmatrix} p \\ 1-p \end{pmatrix} \otimes \omega_0 \leftrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes [p\omega_0 + (1-p)\omega_1].$$

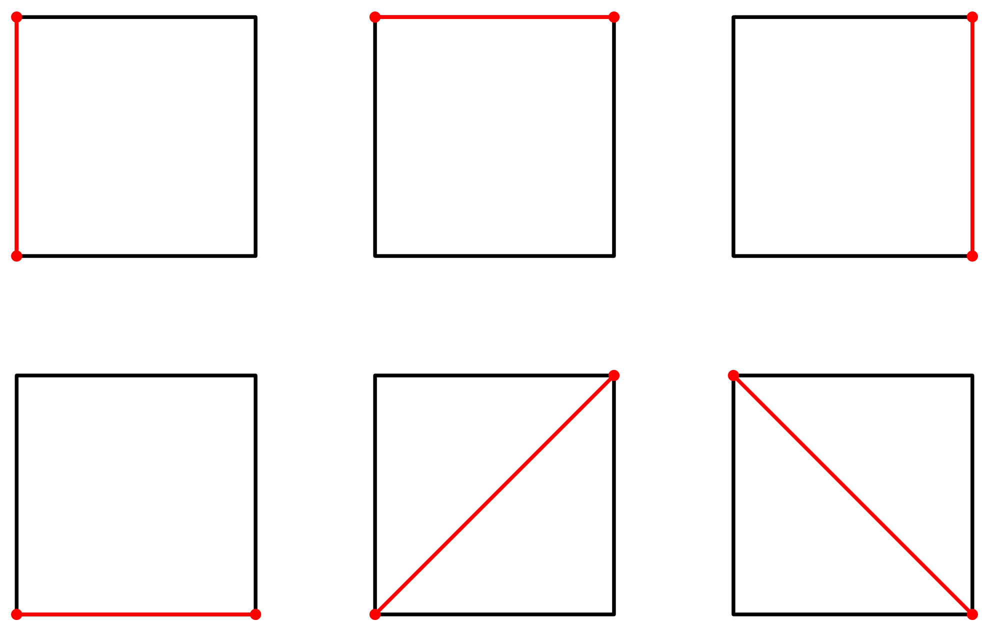
- This is because automorphisms on the COT system, controlled by the value of the classical bit, are always automorphisms of the hybrid system.

Important facts

- Uniform MIXing is always possible, i.e. if $\omega \to \omega_0$ and $\omega \to \omega_1$ then

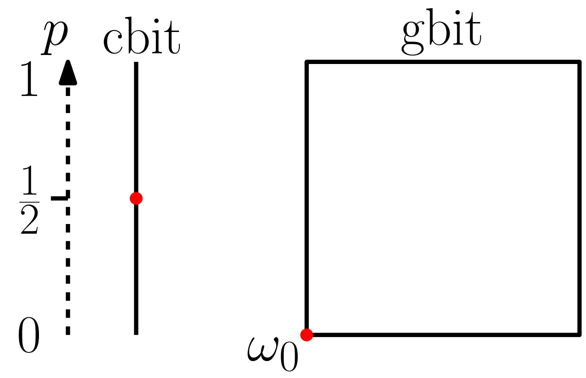
$$\omega \to \frac{1}{2}\omega_0 + \frac{1}{2}\omega_1.$$

- Important fact about gbits: Every pair of pure states is distinguishable (but no triple is) and the automorphism group is transitive.



Gbits Violate the Second Law

- Starting state $\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \otimes \omega_0$ has $S(\boldsymbol{p}) = 1$ bit.



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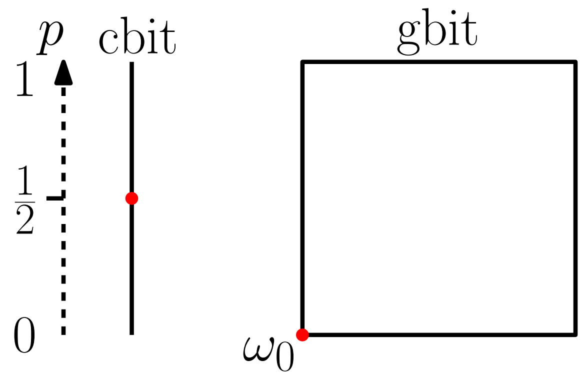

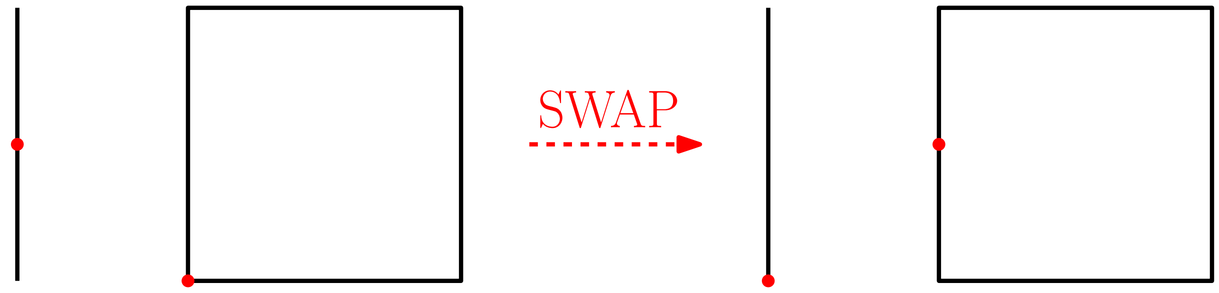

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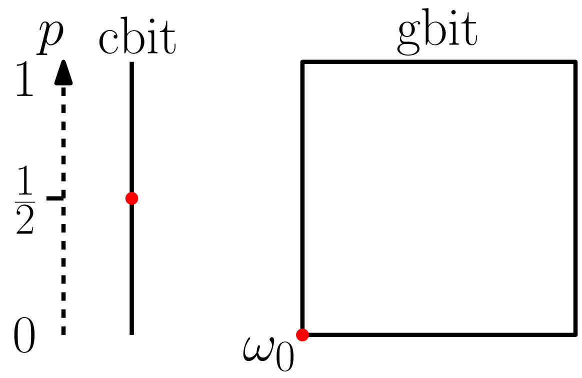

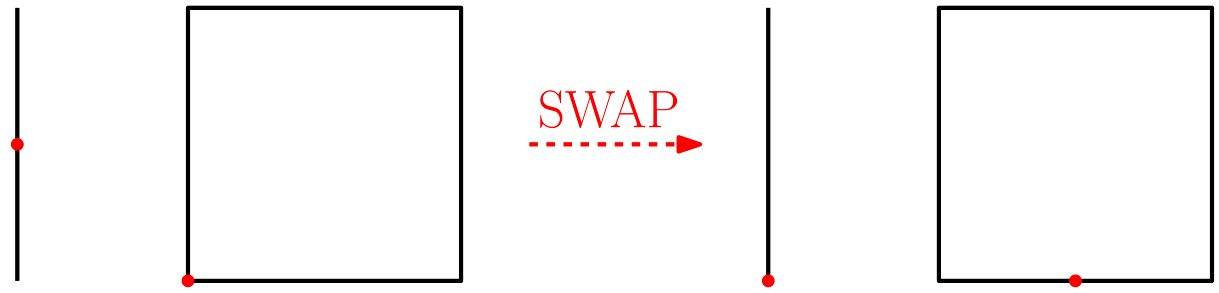

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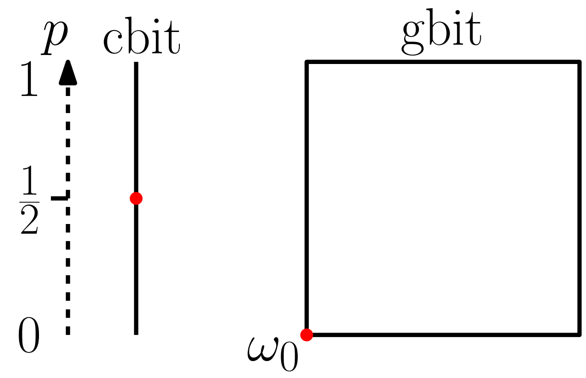

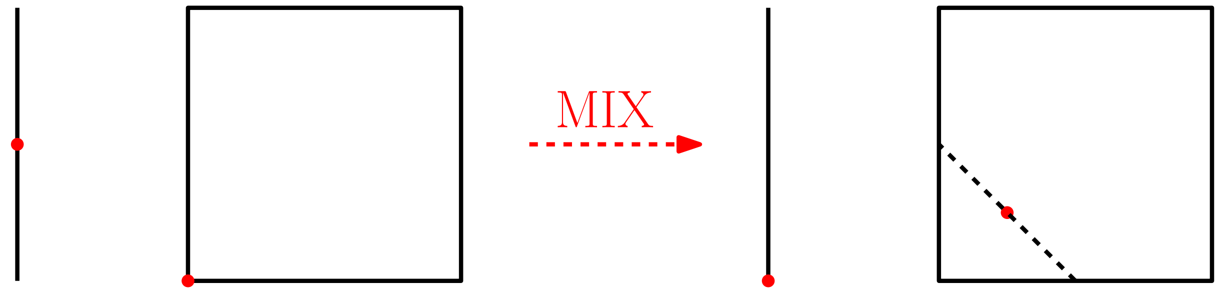

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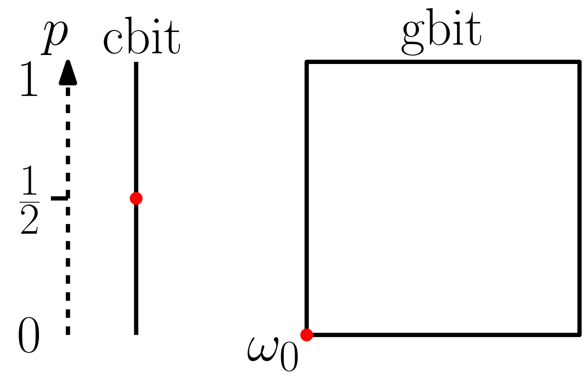

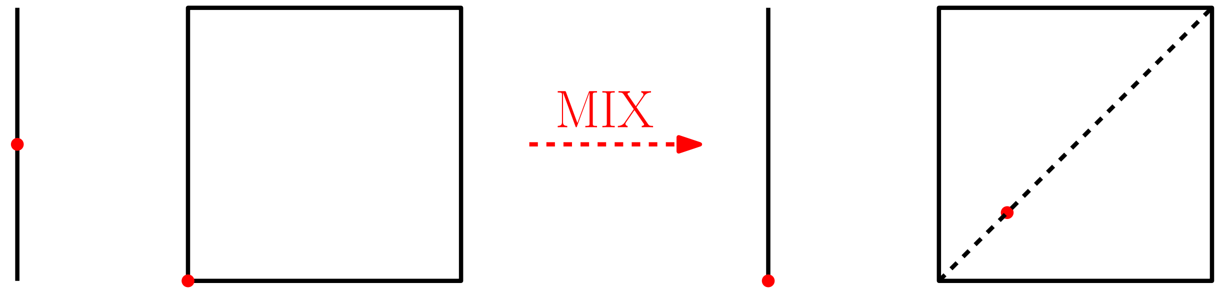

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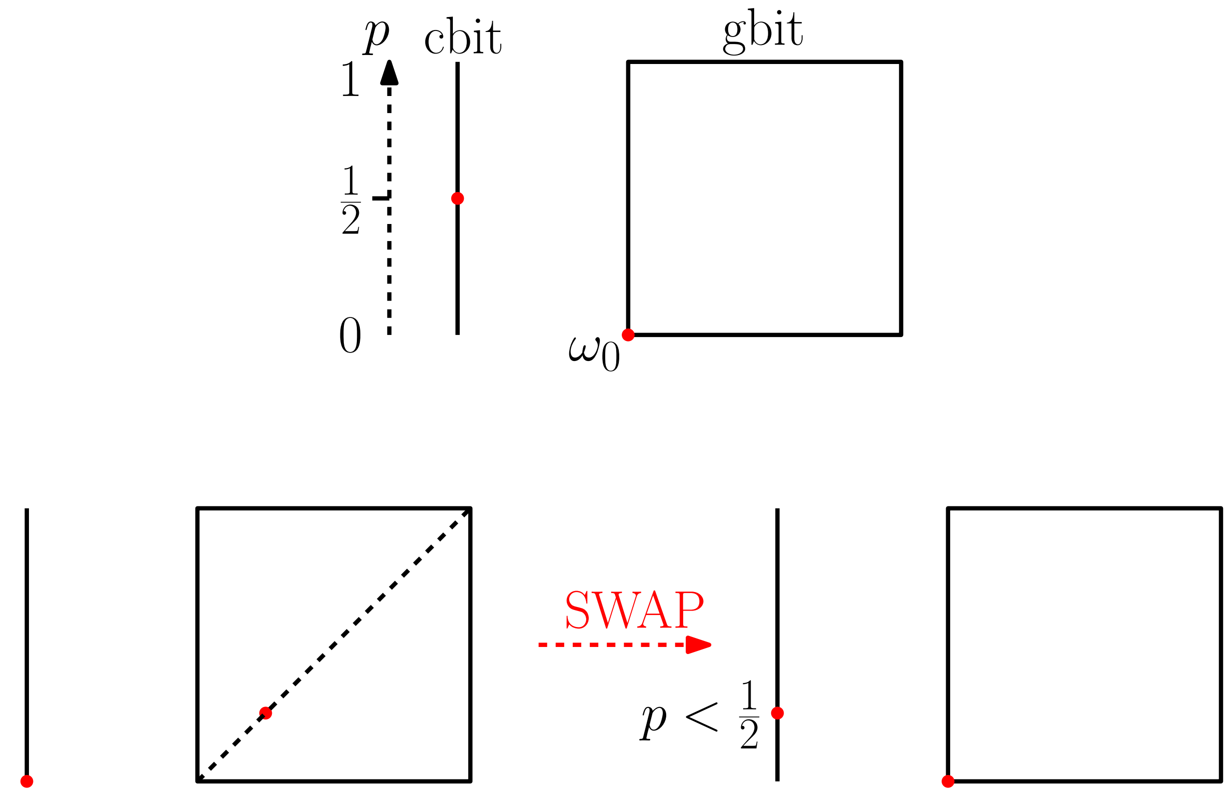


Proposed Axioms for Quantum Theory

1. Automorphism group is transitive.
2. von Neumann's assumption.
3. Second Law for classical systems.

- What I know so far:
  - Rules out polygons with even number of sides in 2D.
  - There are non-classical and non-quantum theories that satisfy the axioms, e.g. hyperspheres.
- Conjecture: Axioms single out state spaces of Jordan algebras.

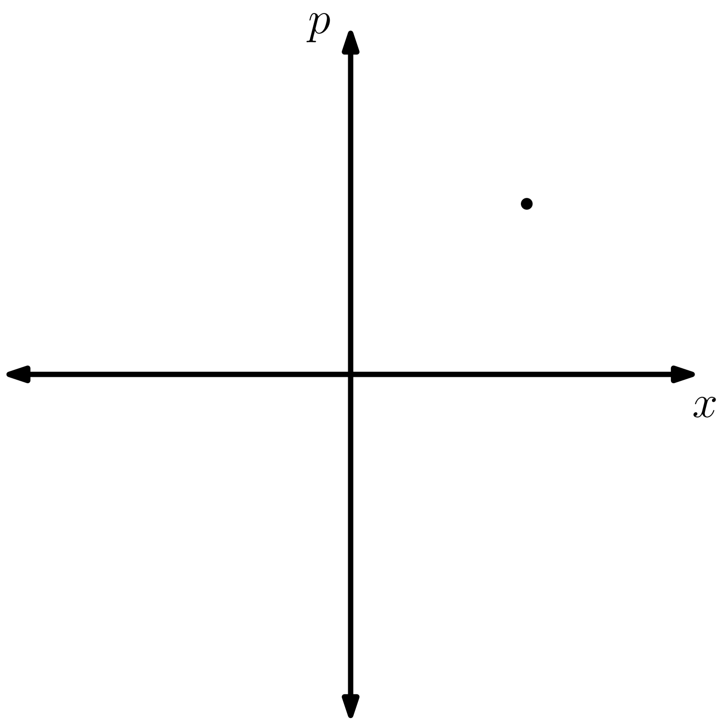
Future Directions

- Ontological models:
  - Find experimentally testable overlap bounds with low $k_{\mathcal{D}}(\psi)$.
  - Develop qinfo. applications, e.g. to communication complexity.
  - Investigate exotic ontologies that may close the explanatory gaps demonstrated by no-go theorems, e.g. retrocausality.
- Convex operational theories:
  - What can be derived from other physical postulates, e.g. existence and conservation of an energy observable, Lorentz symmetry, etc.?
- Quantum probability:
  - Develop a quantum theory of Bayesian inference without a priori causal structure.
  - Develop quantum generalizations of probabilistic machine learning structures and algorithms.
  - Investigate monogamy of conditional states and applications, e.g. to simulation of many-body systems.

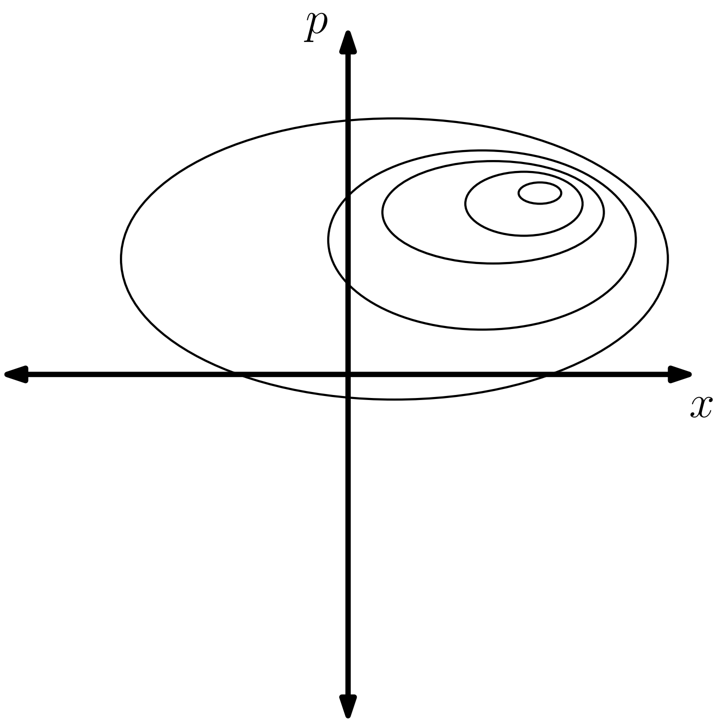
Additional slides

Classical states

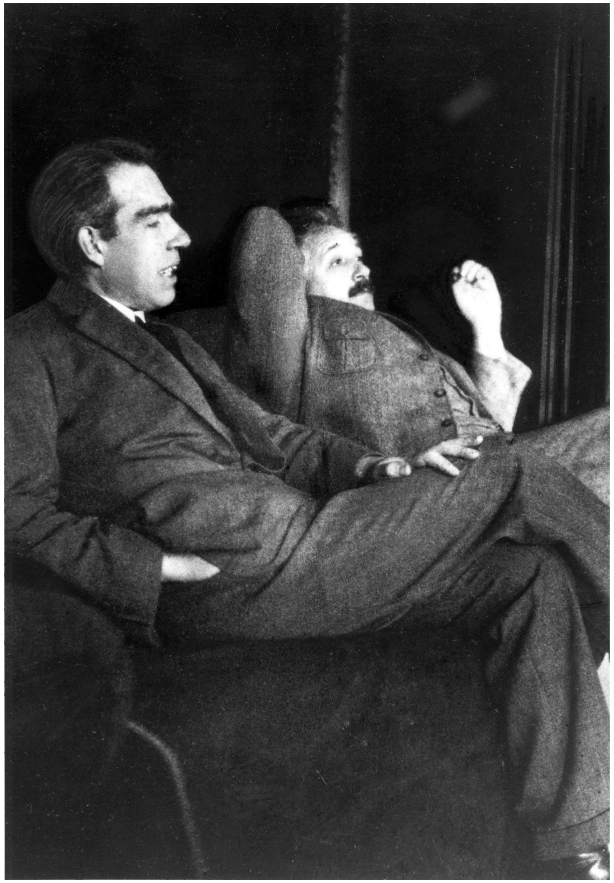
Ontic state



Epistemic state



Bohr and Einstein: ψ-epistemicists



Source: http://en.wikipedia.org/

There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature. — Niels Bohr[a]

[t]he ψ-function is to be understood as the description not of a single system but of an ensemble of systems. — Albert Einstein[b]

[a] Quoted in A. Petersen, "The philosophy of Niels Bohr", *Bulletin of the Atomic Scientists* Vol. 19, No. 7 (1963)

[b] P. A. Schilpp, ed., *Albert Einstein: Philosopher Scientist* (Open Court, 1949)

Penrose: ψ-ontologist

It is often asserted that the state-vector is merely a convenient description of 'our knowledge' concerning a physical system—or, perhaps, that the state-vector does not really describe a single system but merely provides probability information about an 'ensemble' of a large number of similarly prepared systems. Such sentiments strike me as unreasonably timid concerning what quantum mechanics has to tell us about the *actuality* of the physical world. — Sir Roger Penrose[8]

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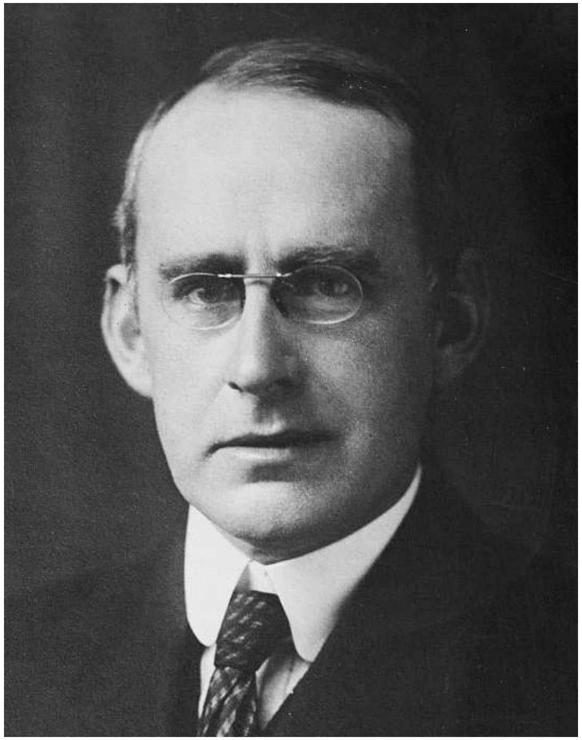
[8] R. Penrose, *The Emperor's New Mind* pp. 268–269 (Oxford, 1989)

Supremacy of the Second Law



George Grantham Bain Collection (Library of Congress)

The law that entropy always increases, holds, I think, the supreme position among the laws of Nature. If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations then so much the worse for Maxwell's equations. If it is found to be contradicted by observation well, these experimentalists do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation. — Sir Arthur Eddington[a]

[a] *The Nature of the Physical World* (Cambridge University Press, 1929) p. 74.