Does protective measurement imply the reality of the quantum state?

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In 1993, Aharonov, Anandan and Vaidman introduced a method of determining the quantum state of a single copy of a quantum system, provided the system is protected during the course of measurement\(^1\).

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- Protection via the quantum Zeno effect.
- Hamiltonian protection.

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In 1993, Aharonov, Anandan and Vaidman introduced a method of determining the quantum state of a single copy of a quantum system, provided the system is *protected* during the course of measurement\(^3\).

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Does this imply the reality of the quantum state?

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Alice  Person trying to determine the quantum state
Bob   Person who protects the quantum system.

- Bob sends Alice a quantum system prepared in a state $|\psi\rangle$.
- The protection: Every $\Delta t$ Bob performs a measurement in a basis $\{|\psi_j\rangle\}$ that includes $|\psi\rangle$ as an eigenstate.
- To measure an observable, Alice couples it to a pointer system with wavefunction $\phi(q, t)$ and initial state $\phi(q, 0) = \delta(q)$. 

![Graph](image)
To measure $A$, Alice couples the pointer to the system via a Hamiltonian $H = gAp$ for time $1/g$ s.t. $\Delta t \ll 1/g$.

When $\Delta t \to 0$, the pointer ends up pointing to $\langle A \rangle = \langle \psi | A | \psi \rangle$ and the system remains in state $|\psi\rangle$. 
Since the state of the system is unchanged, Alice can perform as many protective measurements of different observables as she likes.

If she measures a tomographically complete set, she can determine the quantum state.

So does this imply the reality of the quantum state?

If we can do the same thing with classical probability distributions then the answer is no.
System described by two classical random variables, $X$ and $Y$, that take values $\pm 1$ (or $\pm$ for short).

$(x, y)$ denotes state in which $X = x$ and $Y = y$.

Example: Ball in a box:
Assume Bob can prepare the system in four different probability distributions:

\[
\begin{array}{c|cc}
\text{Distribution} & \langle X \rangle & \langle Y \rangle \\
\hline
|x+\rangle & +1 & 0 \\
|x-\rangle & -1 & 0 \\
|y+\rangle & 0 & +1 \\
|y-\rangle & 0 & -1 \\
\end{array}
\]
**Toy model: Bob’s Measurements**

- **X-measurement:**
  - Protective measurement
  - Zeno protected measurement
  - Measuring the quantum state
  - Toy model
  - Toy model: Bob’s States
  - Toy model: Alice’s measurements
  - Toy model: Zeno protected measurement

- **Y-measurement:**
  - Protective measurement
  - Zeno protected measurement
  - Measuring the quantum state
  - Toy model
  - Toy model: Bob’s Measurements
  - Toy model: Alice’s measurements
  - Toy model: Zeno protected measurement

**Conclusions**

**Other stuff in the paper**
System is coupled to a classical pointer prepared in state $q = p = 0$ with Hamiltonian $H = gXp$ or $H = gYp$ for a time $1/g$.

Without protection, for system prepared in $|x+\rangle$, with $H = gXp$:

$$P(q, 0) - P(q, 1/g)$$

and with $H = gYp$:

$$Prob = \frac{1}{2}$$
Now do the same thing whilst at the same time Bob is measuring $X$ every $\Delta t = 1/gN$.

For $H = gXp$, the pointer moves as before. The pointer is coupled to $X$, but Bob’s measurement only affects $Y$.

For $H = gYp$, every $\Delta t$ the $y$-coordinate is randomized, so the pointer will keep going in the same direction or switch direction with probability $1/2$ each.

☐ Pointer executes an $N$-step random walk with step size $1/N$. 
For large $N$, distribution of final pointer position is $\approx \mathcal{N}(0, 1/N)$.

Tends to $\delta(q)$ as $N \to \infty$. 

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Implicit assumption that if a measurement does not change a quantum state then the measurement does nothing to the system when it is prepared in that state:

- Not true in our model: Measuring $X$ randomizes the $y$-coordinate even though distribution $|x+\rangle$ is unchanged.

Protective measurement is more like measuring $N$ independently prepared systems than measuring just a single copy.
- Adding back-action to the Zeno toy model.

- Toy model for Hamiltonian protective measurements.

- Operational arguments for why protective measurement does not imply the reality of the quantum state:
  - Resource counting.
  - Protective measurement just implements an ordinary projective measurement in a basis for which the prepared state is an eigenstate.