Is the wavefunction real?

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Based on:

PRL 112:160404 (2014), PRL 110:120401 (2013)

Review article: to appear in Quanta http://mattleifer.info/publications

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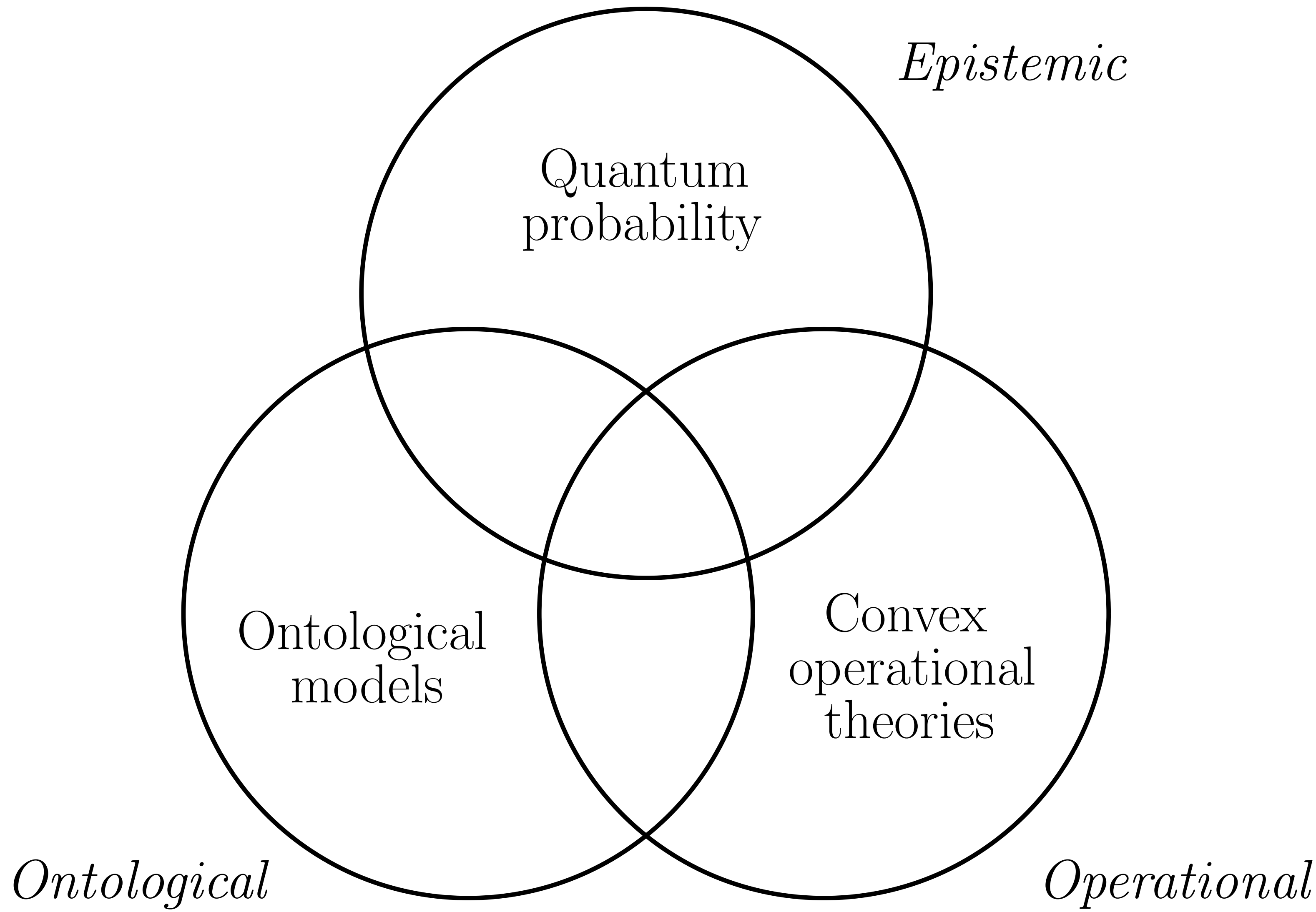
- No-go theorems expose explanatory gaps in our existing realist frameworks for understanding quantum theory.
  - *Bell's theorem*: Nonlocal influences that do not permit superluminal signaling require fine-tuning.
  - *Contextuality*: Distinctions that do not make a difference require fine-tuning.
  - *Reality of the wavefunction*: Many quantum phenomena are most easily explained if the quantum state is not real.
- In my view, none of the existing interpretations close these gaps and there are probably more waiting to be discovered.

Goal

- Goal: Develop a *coherent* and *compelling* explanatory framework for quantum theory.
  - *Coherent*: Free of explanatory gaps.
  - *Compelling*: Essential to the practical applications of the theory.
    - Allows new applications to be developed more easily because we have the right explanations for and intuitions about quantum phenomena, e.g. in Quantum Information.
    - Provides an essential guide to new theory construction, e.g. Quantum Gravity.
- Coherence will likely require exotic ontologies, i.e. a re-evaluation of what is required for a realist account of quantum theory.
- We need to first clearly articulate the nature of the explanatory gaps.

Overview of my research



Overview of this talk

Introduction

Arguments for Epistemic Quantum States

Ontological Models

ψ-ontology theorems

ψ-epistemic models

Overlap measures

Overlap bounds

Conclusions

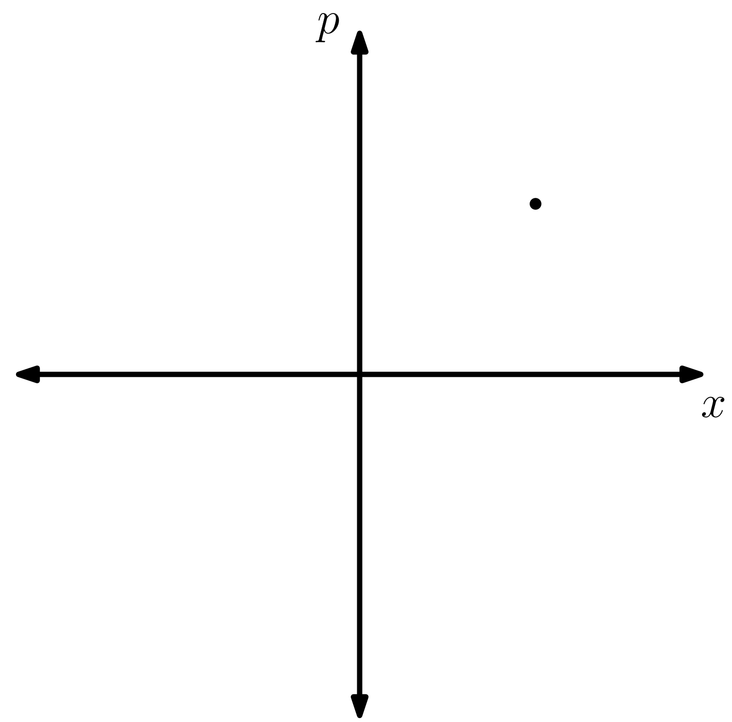
Introduction

ψ-epistemic vs. ψ-ontic

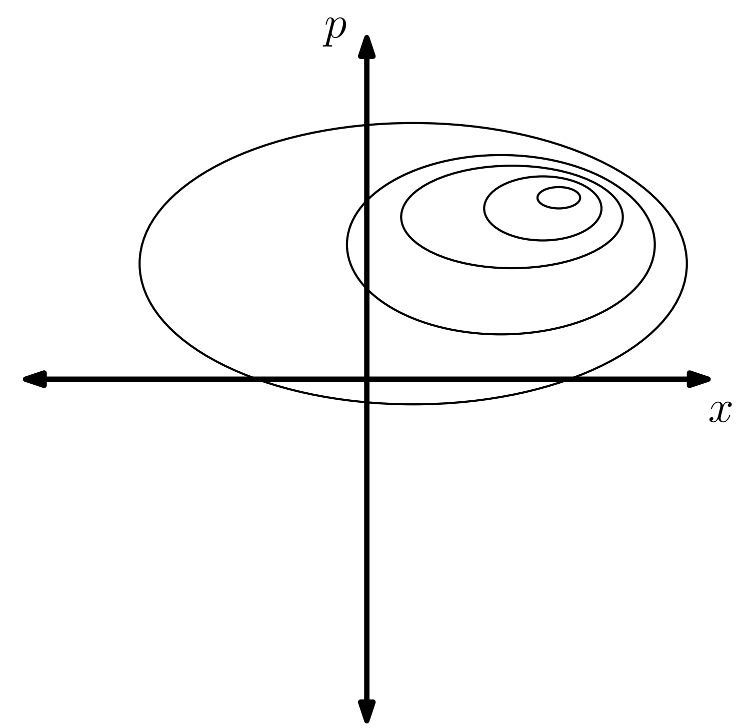
- *Ontic state*: a state of reality.
  - ψ*-ontic*: the quantum state is ontic.
- *Epistemic state*: a state of knowledge or information.
  - ψ*-epistemic*: the quantum state is epistemic.

Classical states

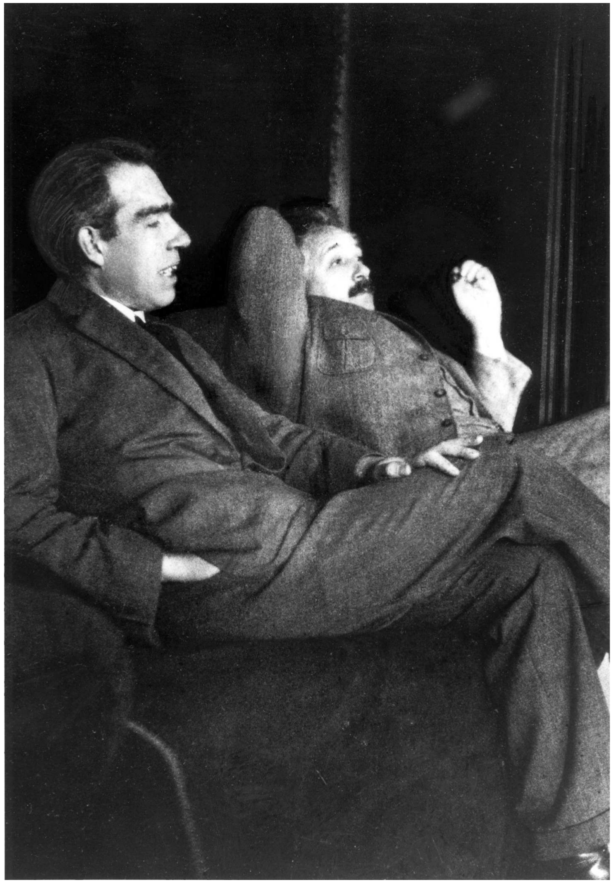
Ontic state



Epistemic state



Bohr and Einstein: ψ-epistemicists



Source: http://en.wikipedia.org/

> There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature. — Niels Bohr[a]

> [t]he ψ-function is to be understood as the description not of a single system but of an ensemble of systems. — Albert Einstein[b]

[a] Quoted in A. Petersen, "The philosophy of Niels Bohr", *Bulletin of the Atomic Scientists* Vol. 19, No. 7 (1963)

[b] P. A. Schilpp, ed., *Albert Einstein: Philosopher Scientist* (Open Court, 1949)

Penrose: ψ-ontologist

It is often asserted that the state-vector is merely a convenient description of ‘our knowledge’ concerning a physical system—or, perhaps, that the state-vector does not really describe a single system but merely provides probability information about an ‘ensemble’ of a large number of similarly prepared systems. Such sentiments strike me as unreasonably timid concerning what quantum mechanics has to tell us about the *actuality* of the physical world. — Sir Roger Penrose[1]

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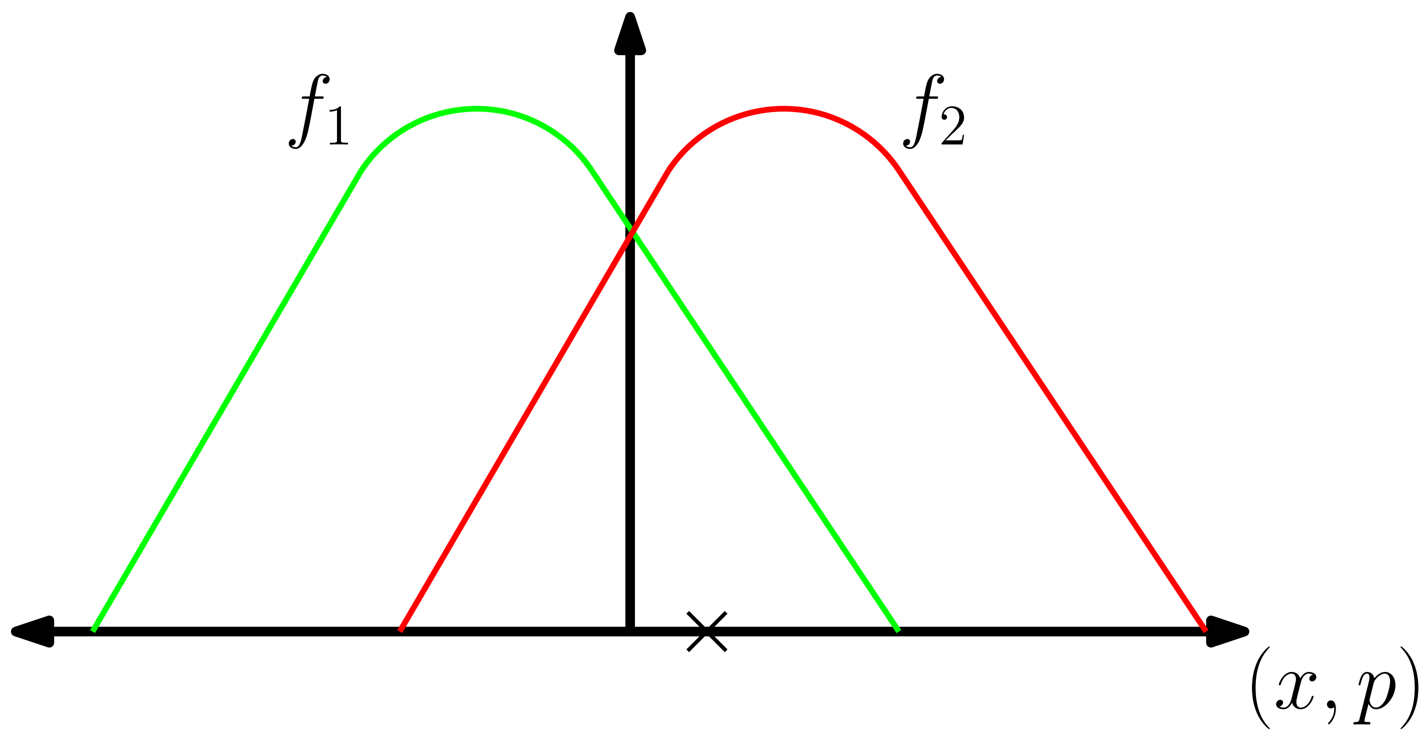
[1] R. Penrose, *The Emperor’s New Mind* pp. 268–269 (Oxford, 1989)

Interpretations of quantum theory

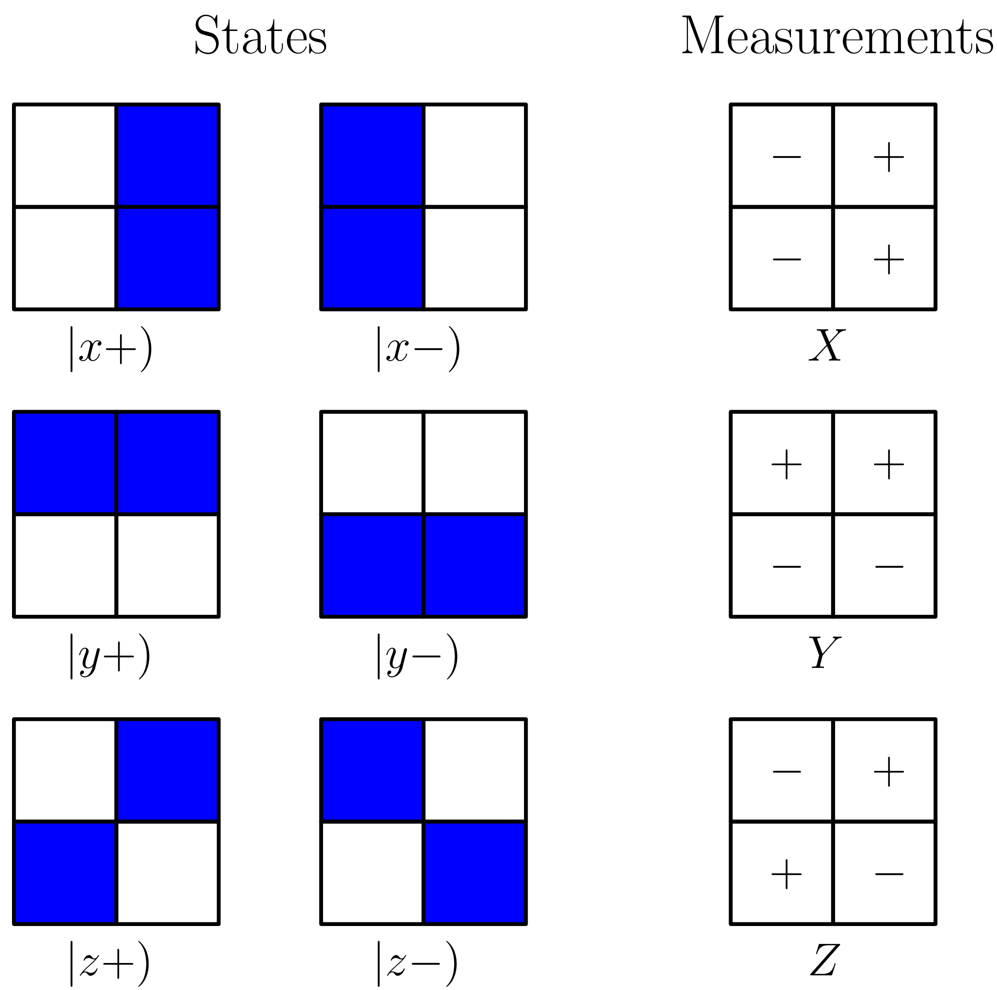
	ψ-epistemic	ψ-ontic
Anti-realist	Copenhagen neo-Copenhagen (e.g. QBism, Peres, Zeilinger, Healey)	
Realist	Einstein Ballentine? Spekkens ?	Dirac-von Neumann Many worlds Bohmian mechanics Spontaneous collapse Modal interpretations

Arguments for Epistemic Quantum States

Epistemic states overlap

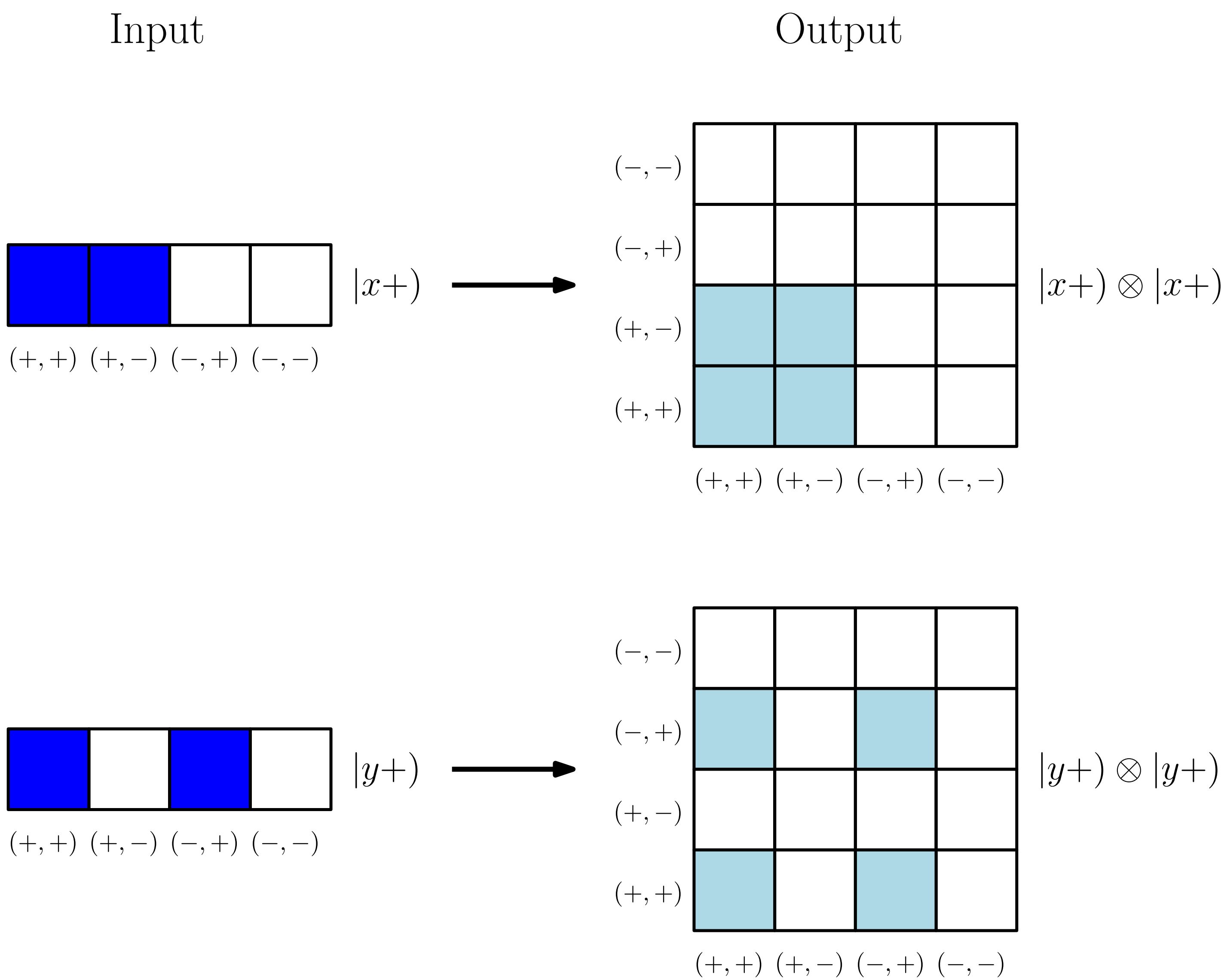


Spekkens' toy theory



R. W. Spekkens, *Phys. Rev. A* 75(3):032110 (2007) arXiv:quant-ph/0401052

No cloning



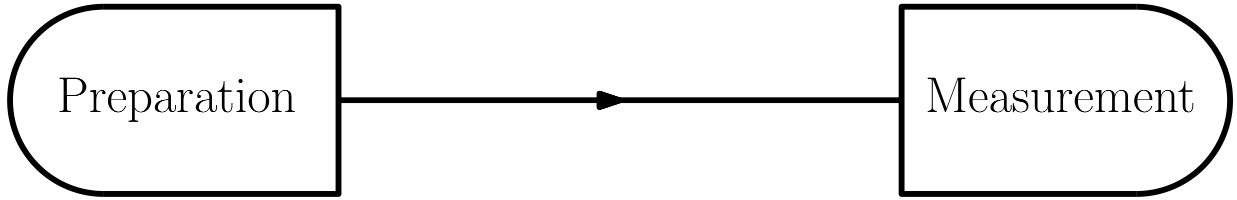
- Collapse of the wavefunction
- Generalized probability theory
- Excess baggage

Arguments for ontic quantum states

- Interference
- Eigenvalue-eigenstate link
- Lack of imagination
- Quantum computing

Ontological Models

Prepare-and-measure experiments: Quantum description

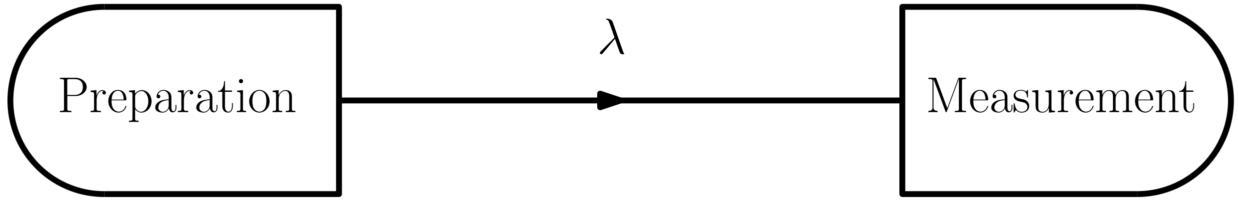


$|\psi\rangle \in \mathbb{C}^d$

$M = \{|a\rangle, |b\rangle, \ldots\}$

$$\mathrm{Prob}(a|\psi, M) = |\langle a|\psi\rangle|^2$$

Prepare-and-measure experiments: Ontological description



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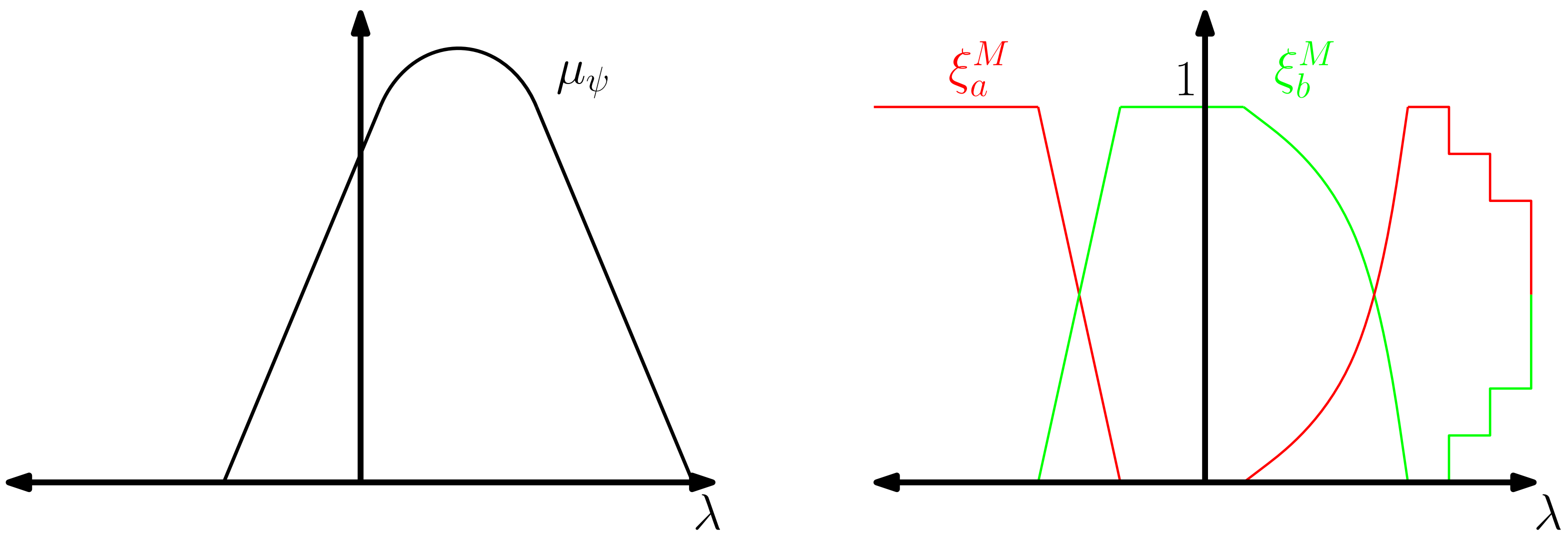

$$\mathrm{Prob}(a|\psi, M) = \int \xi_a^M(\lambda) d\mu_\psi$$

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- A measurable space (Λ, Σ).

Formal definition

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- For each orthonormal basis $M = \{|a\rangle, |b\rangle, \ldots\}$, a set of response functions $\xi_a^M : \Lambda \to [0, 1]$ satisfying

$$\forall\lambda, \sum_{|a\rangle \in M} \xi_a^M(\lambda) = 1.$$

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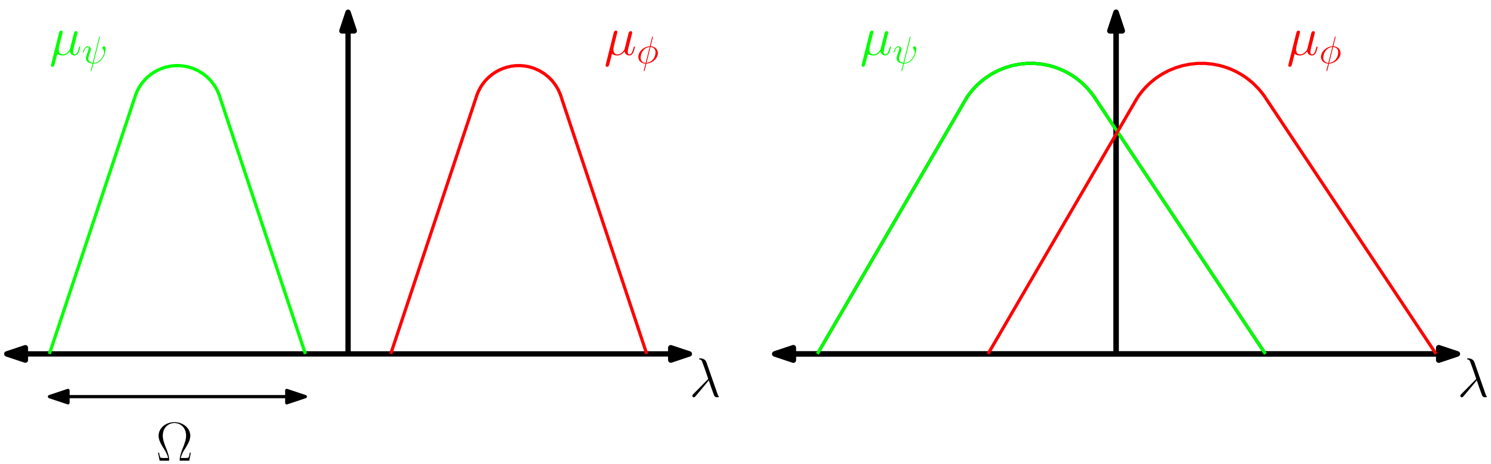
$$\forall \lambda, \quad \sum_{|a\rangle \in M} \xi_a^M(\lambda) = 1.$$

The model is required to reproduce the quantum predictions, i.e.

$$\int_\Lambda \xi_a^M(\lambda) d\mu_\psi = |\langle a|\psi\rangle|^2 .$$

- $|\psi\rangle$ and $|\phi\rangle$ are *ontologically distinct* in an ontological model if there exists $\Omega \in \Sigma$ s.t.

$$\mu_\psi(\Omega) = 1 \qquad \mu_\phi(\Omega) = 0.$$



- An ontological model is *ψ-ontic* if every pair of states is ontologically distinct. Otherwise it is *ψ-epistemic*.

ψ-ontology theorems

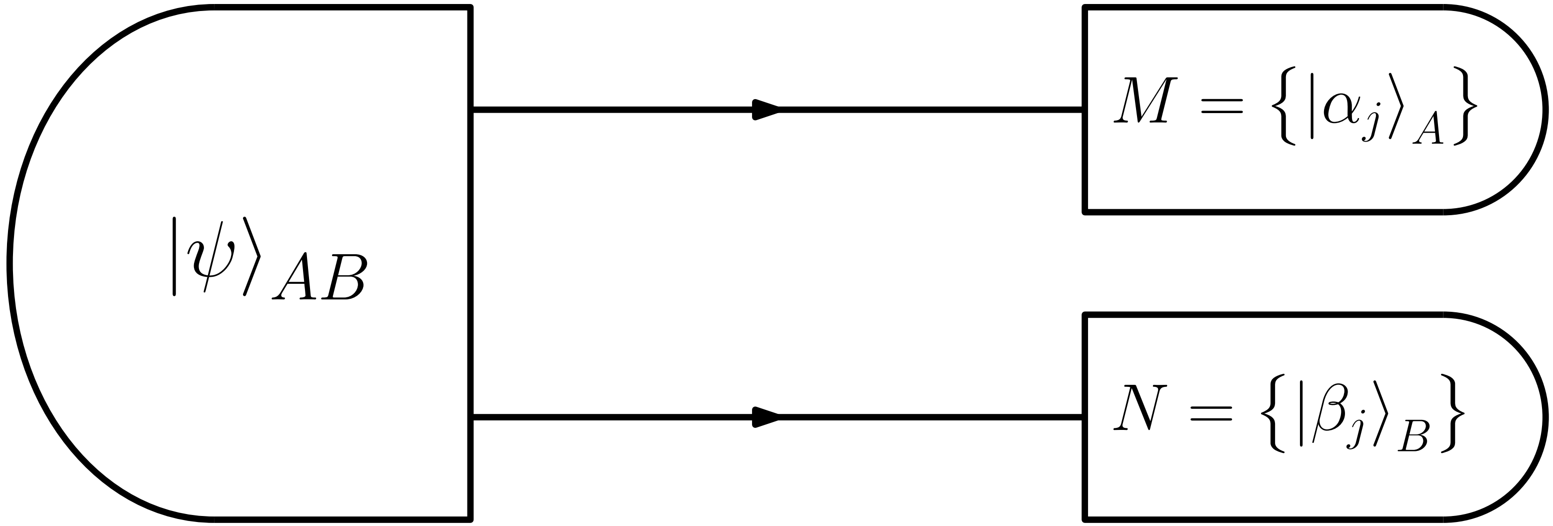
- The Colbeck-Renner theorem: R. Colbeck and R. Renner, arXiv:1312.7353 (2013).
- Hardy's theorem: L. Hardy, *Int. J. Mod. Phys. B*, 27:1345012 (2013) arXiv:1205.1439
- The Pusey-Barrett-Rudolph theorem: M. Pusey et. al., *Nature Physics*, 8:475–478 (2012) arXiv:1111.3328

Criteria for evaluating the assumptions

- If an assumption is violated by viable ψ-ontic models, e.g. Bohmian mechanics, then the theorem does not tell specifically against ψ-epistemic models.

- If an assumption is violated by Spekkens' toy theory then it is too strong, as that is a viable ψ-epistemic model for a subset of quantum theory.

- If an assumption is rendered suspect by an existing no-go theorem, e.g. Bell's theorem, then that is OK, but we would rather do without that assumption.



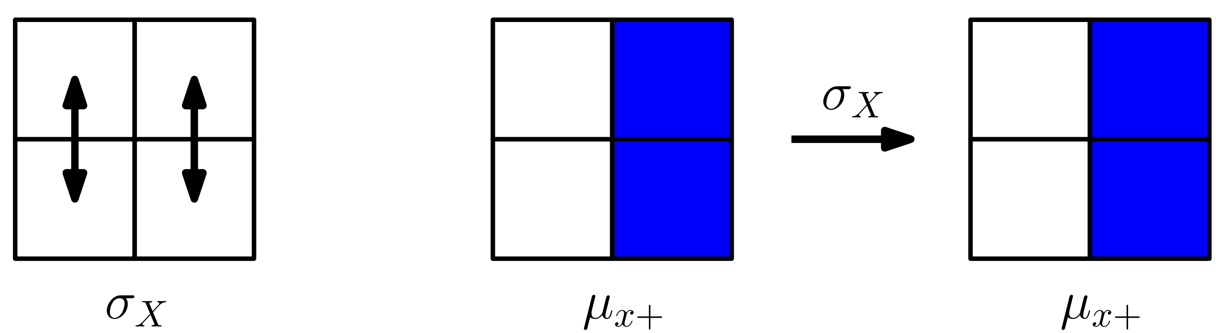
- *Parameter Independence*:
  - $P(a_j|M, N, \lambda) = P(a_j|M, \lambda)$
  - $P(b_k|M, N, \lambda) = P(b_k|N, \lambda)$

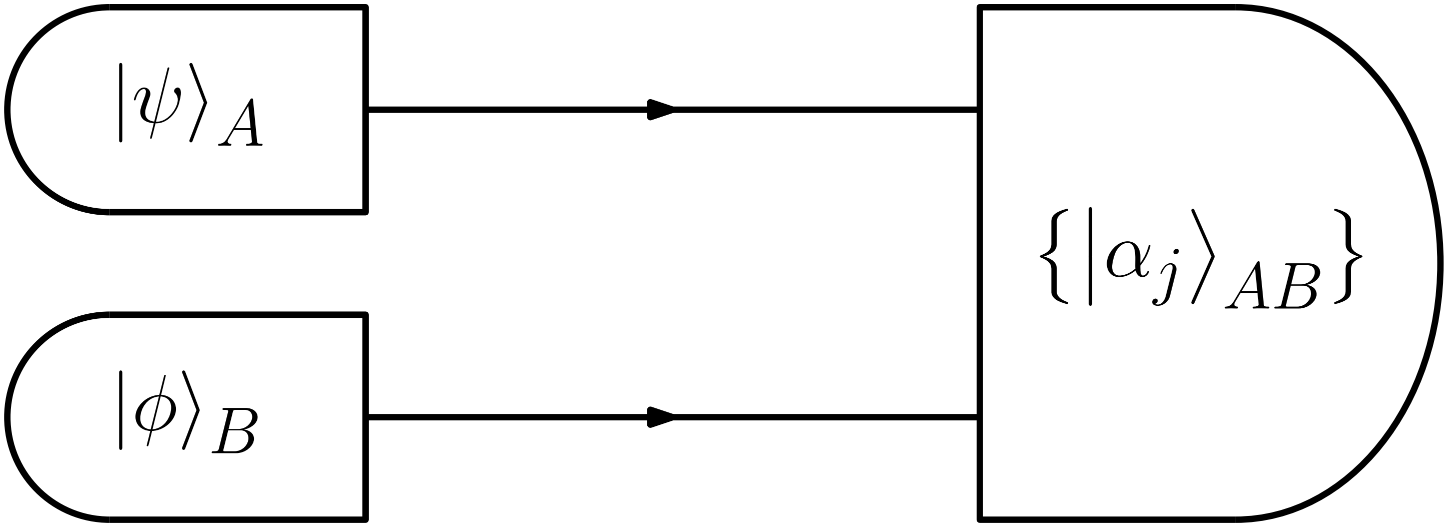
Hardy's Theorem

- *Ontic indifference*: If $U|\psi\rangle = |\psi\rangle$ then all of the ontic states in the support of μ_ψ should be left invariant by U.

- Example: For a spin-$1/2$ particle, $\sigma_X|x+\rangle = |x+\rangle$.

- But in Spekkens' toy theory:

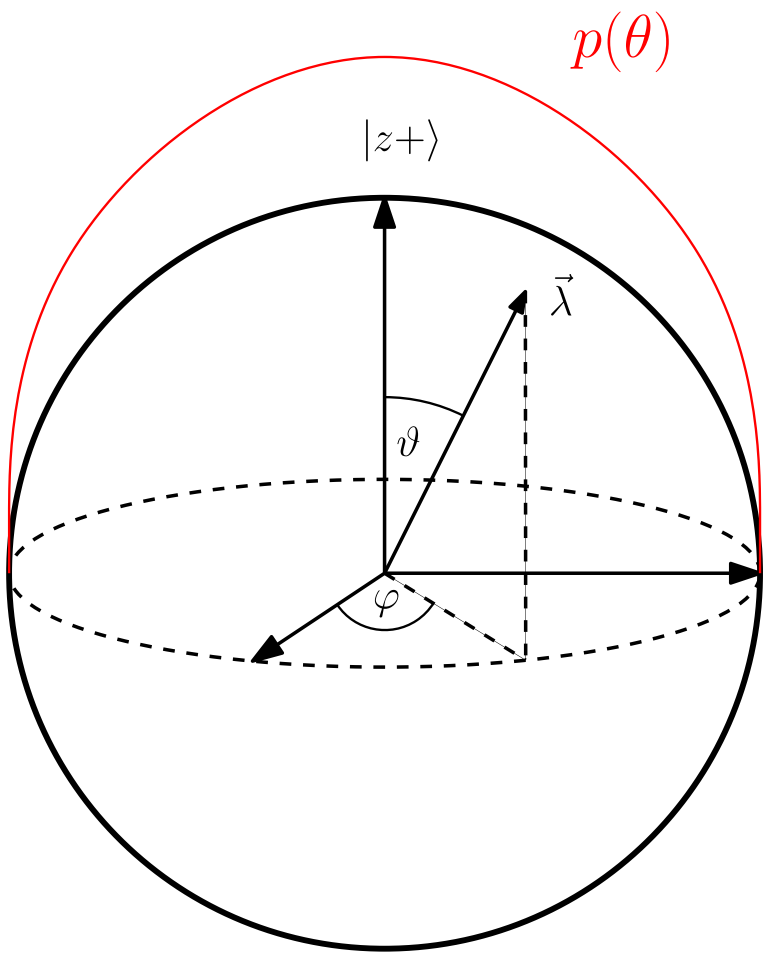




- The *Preparation Independence Postulate*:
  - $(\Lambda_{AB}, \Sigma_{AB}) = (\Lambda_A \times \Lambda_B, \Sigma_A \otimes \Sigma_B)$
  - $\mu_{AB} = \mu_A \times \mu_B$

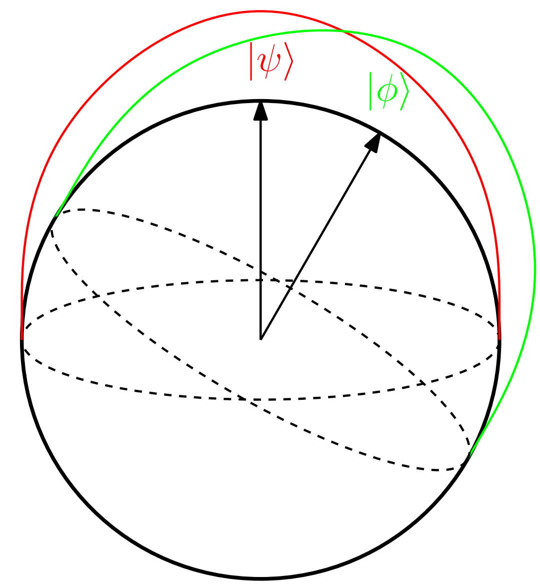
ψ-epistemic models

The Kochen-Specker model for a qubit



$$\mu_{z+}(\Omega) = \int_{\Omega} p(\vartheta) \sin\vartheta d\vartheta d\varphi$$

$$p(\vartheta) = \begin{cases} \frac{1}{\pi}\cos\vartheta, & 0 \leq \vartheta \leq \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < \vartheta \leq \pi \end{cases}$$



S. Kochen and E. Specker, *J. Math. Mech.*, 17:59–87 (1967)

Models for arbitrary finite dimension

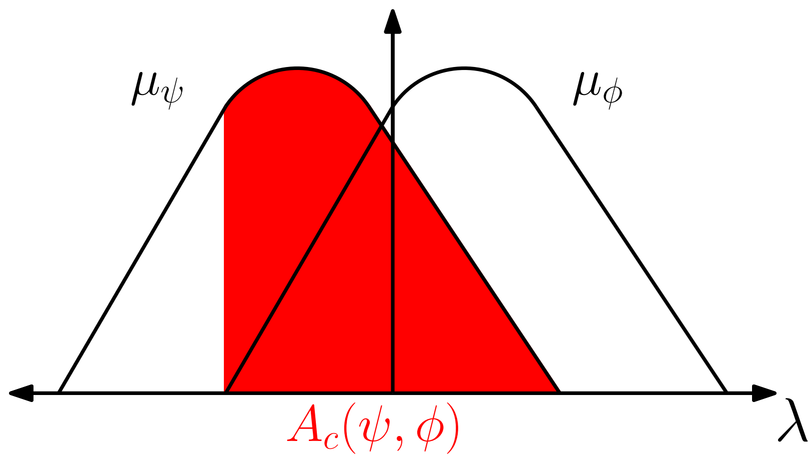
- Lewis et. al. provided a ψ-epistemic model for all finite d.
  - P. G. Lewis et. al., *Phys. Rev. Lett.* 109:150404 (2012) arXiv:1201.6554
- Aaronson et. al. provided a similar model in which every pair of nonorthogonal states is ontologically indistinct.
  - S. Aaronson et. al., *Phys. Rev. A* 88:032111 (2013) arXiv:1303.2834
- These models have the feature that, for a fixed inner product, the amount of overlap decreases with d.

Overlap measures

Asymmetric overlap

- *Classical asymmetric overlap*:

$$A_c(\psi,\phi) := \inf_{\{\Omega\in\Sigma \,|\, \mu_\phi(\Omega)=1\}} \mu_\psi(\Omega)$$

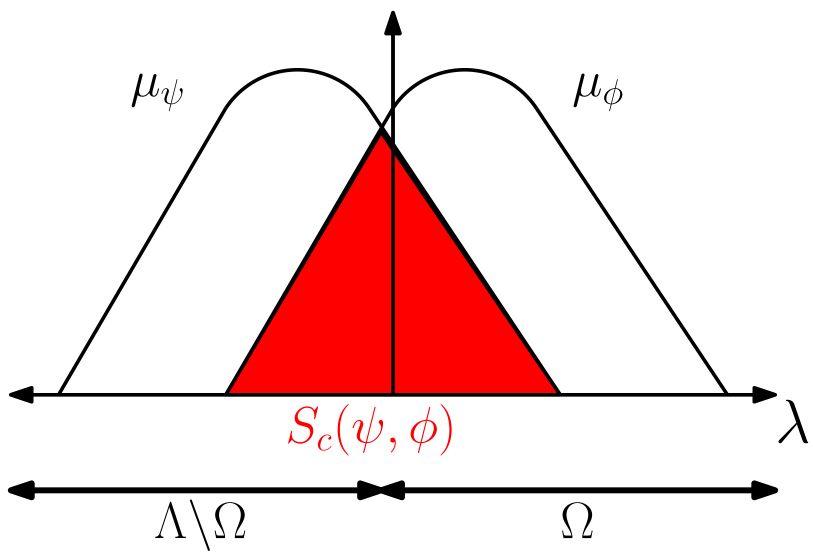


- An ontological model is *maximally ψ-epistemic* if

$$A_c(\psi,\phi) = |\langle\phi|\psi\rangle|^2$$

- *Classical symmetric overlap*:

$$S_c(\psi, \phi) := \inf_{\Omega \in \Sigma} \left[\mu_\psi(\Omega) + \mu_\phi(\Lambda \backslash \Omega)\right]$$



- Optimal success probability of distinguishing $|\psi\rangle$ and $|\phi\rangle$ if you know λ:

$$p_c(\psi, \phi) = \frac{1}{2}\left(2 - S_c(\psi, \phi)\right)$$

- *Classical symmetric overlap*:

$$S_c(\psi, \phi) := \inf_{\Omega \in \Sigma} \left[\mu_\psi(\Omega) + \mu_\phi(\Lambda \backslash \Omega)\right]$$

- *Quantum symmetric overlap*:

$$S_q(\psi, \phi) := \inf_{0 \leq E \leq I} \left[\langle\psi| E |\psi\rangle + \langle\phi| (I - E) |\phi\rangle\right]$$

- Optimal success probability of distinguishing $|\psi\rangle$ and $|\phi\rangle$ based on a quantum measurement:

$$p_q(\psi, \phi) = \frac{1}{2}\left(2 - S_q(\psi, \phi)\right)$$

Relationships between overlap measures

- Classical overlap measures:

$$S_c(\psi,\phi) \leq A_c(\psi,\phi)$$

- Quantum overlap measures:
  - $S_q(\psi,\phi) = 1 - \sqrt{1 - |\langle\phi|\psi\rangle|^2}$
  - $S_q(\psi,\phi) \geq \frac{1}{2}\,|\langle\phi|\psi\rangle|^2$

- Hence:

$$\frac{S_c(\psi,\phi)}{S_q(\psi,\phi)} \leq 2\frac{A_c(\psi,\phi)}{|\langle\phi|\psi\rangle|^2}.$$

Overlap bounds

- Define:

$$k(\psi, \phi) = \frac{A_c(\psi, \phi)}{|\langle \phi | \psi \rangle|^2}.$$

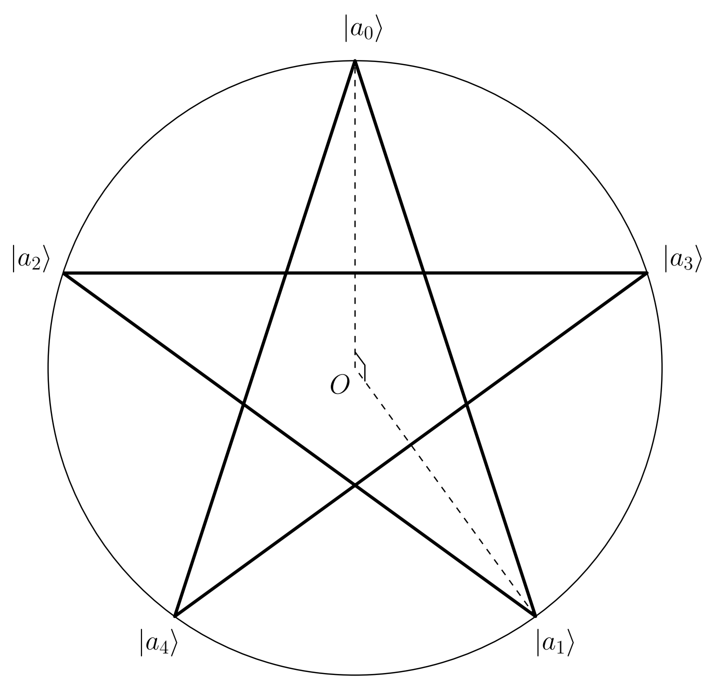
- Maroney showed $k(\psi, \phi) < 1$ for some states. ML and Maroney showed this follows from KS theorem.

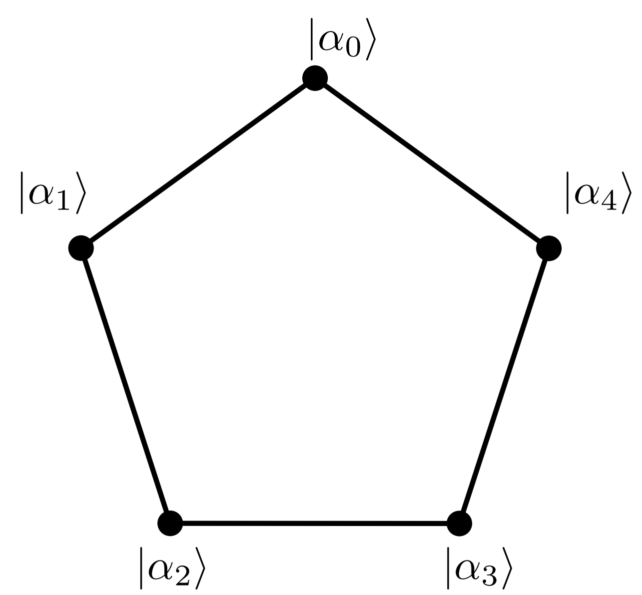
- Barrett et. al. exhibited a family of states in $\mathbb{C}^d$ such that, for $d \geq 4$:

$$k(\psi, \phi) \leq \frac{4}{d-1}.$$

- Today: $k(\psi, \phi) \leq de^{-cd}$ for d divisible by 4.

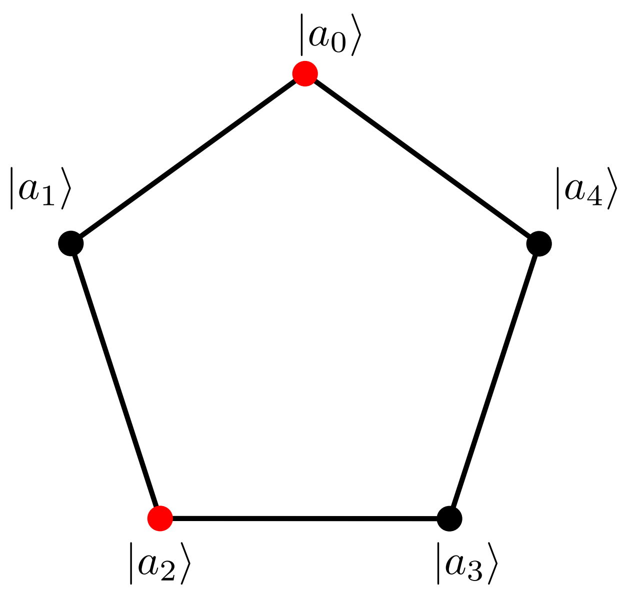
- Example: Klyachko states
  - $|a_j\rangle = \sin\vartheta\cos\varphi_j\,|0\rangle + \sin\vartheta\sin\varphi_j\,|1\rangle + \cos\vartheta\,|2\rangle$
  - $\varphi_j = \frac{4\pi j}{5}$ and $\cos\vartheta = \frac{1}{\sqrt[4]{5}}$





- The *independence number* $\alpha(G)$ of a graph G is the cardinality of the largest subset of vertices such that no two vertices are connected by an edge.

- Example: $\alpha(G) = 2$



Theorem: Let V be a finite set of states in $\mathbb{C}^d$ an let $G = (V, E)$ be its orthogonality graph. For $|\psi\rangle \in \mathbb{C}^d$ define

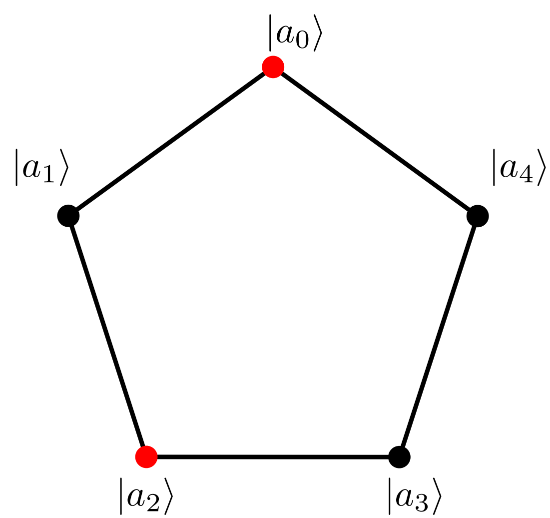
$$\bar{k}(\psi) = \frac{1}{|V|} \sum_{|a\rangle \in V} k(\psi, a).$$

Then, in any ontological model

$$\bar{k}(\psi) \leq \frac{\alpha(G)}{|V| \min_{|a\rangle \in V} |\langle a|\psi\rangle|^2}.$$

Bound from Klyatchko states

- $|a_j\rangle = \sin\vartheta\cos\varphi_j\,|0\rangle + \sin\vartheta\sin\varphi_j\,|1\rangle + \cos\vartheta\,|2\rangle$
- $\varphi_j = \frac{4\pi j}{5}$ and $\cos\vartheta = \frac{1}{\sqrt[4]{5}}$
- $|\psi\rangle = |2\rangle$



$$\bar{k}(\psi) \leq \frac{\alpha(G)}{5\min_j |\langle a_j|\psi\rangle|^2} = \frac{2}{5 \times \frac{1}{\sqrt{5}}} \sim 0.8944$$

- For $\boldsymbol{x} = (x_0, x_1, \ldots, x_{d-1}) \in \{0,1\}^d$, let

$$|a_{\boldsymbol{x}}\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} (-1)^{x_j} |j\rangle .$$

- Let $|\psi\rangle = |0\rangle$.
- By Frankl-Rödl theorem[2], for d divisible by 4, there exists an $\epsilon > 0$ such that $\alpha(G) \leq (2-\epsilon)^d$.

$$\bar{k}(\psi) \leq \frac{\alpha(G)}{2^d \min_{\boldsymbol{x}\in\{0,1\}^d} |\langle a_{\boldsymbol{x}}|\psi\rangle|^2} = \frac{(2-\epsilon)^d}{2^d \times \frac{1}{d}} = de^{-cd}$$

$$c = \ln 2 - \ln(2-\epsilon)$$

[2] P. Frankl and V. Rödl, *Trans. Amer. Math. Soc.* 300:259 (1987)

- An ontological model for a set of bases $\mathcal{M}$ is *Kochen-Specker noncontextual* if it is:
  - *Outcome deterministic*: $\xi_a^M(\lambda) \in \{0, 1\}$.
  - *Measurement noncontextual*: $\xi_a^M = \xi_a^N$.

The connection to contextuality

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- In any ontological model $A_c(\psi, \phi) \leq \max \text{Prob}_{\text{N.C.}}(\phi|\psi, M)$
- Therefore, any KS contextuality inequality gives an overlap bound.

Conclusions

Summary and Open questions

- Summary
  - There exist pairs of states such that $k(\psi, \phi) \leq de^{-cd}$. The ψ-epistemic explanations of indistinguishability, no-cloning, etc. get implausible for these states very radpidly for large d.
  - Any contextuality inequality can be used to derive an overlap bound.
- Open questions
  - Error analysis.
  - Best bounds in small dimensions.
  - Bounds with a fixed inner product.
  - Connection to communication complexity.

What now for ψ-epistemicists?

- Become neo-Copenhagen.
- Adopt a more exotic ontology:
  - Nonstandard logics and probability theories.
  - Ironic many-worlds.
  - Retrocausality.
  - Relationalism.

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- Adopt a more exotic ontology:
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  - Retrocausality.
  - Relationalism.
- Explanatory conservatism: If there is a natural explanation for a quantum phenomenon then we should adopt an interpretation that incorporates it.
  - Suggests exploring exotic ontologies.

References

- Review article:
  - ML, “Is the wavefunction real? A review of ψ-ontology theorems”, to appear in Quanta, http://mattleifer.info/publications
- Connection to contextuality:
  - ML and O. Maroney, *Phys. Rev. Lett.* 110:120401 (2013) arXiv:1208.5132
- Exponential overlap bound:
  - ML, *Phys. Rev. Lett.* 112:160404 (2014) arXiv:1401.7996

Extra Slides

Theorem: Let V be a finite set of states in $\mathbb{C}^d$ an let $G = (V, E)$ be its orthogonality graph. For $|\psi\rangle \in \mathbb{C}^d$ define

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Then, in any ontological model

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  - $\mu_a(\Gamma_a^{\mathcal{M}}) = 1$ also.
- Hence, $A_c(\psi, a) = \inf_{\{\Omega \in \Sigma | \mu_a(\Omega) = 1\}} \mu_\psi(\Omega) \leq \mu_\psi(\Gamma_a^{\mathcal{M}})$

-

$$A_c(\psi, a) \leq \mu_\psi(\Gamma_a^{\mathcal{M}})$$
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- Let
  $$\chi_a(\lambda) = \begin{cases} 1, & \lambda \in \Gamma_a^{\mathcal{M}} \\ 0, & \lambda \notin \Gamma_a^{\mathcal{M}} \end{cases}$$

- $$A_c(\psi, a) \leq \mu_\psi(\Gamma_a^{\mathcal{M}})$$
  $$\sum_{|a\rangle \in V} A_c(\psi, a) \leq \sum_{a \in V} \mu_\psi(\Gamma_a^{\mathcal{M}})$$
- Let
  $$\chi_a(\lambda) = \begin{cases} 1, & \lambda \in \Gamma_a^{\mathcal{M}} \\ 0, & \lambda \notin \Gamma_a^{\mathcal{M}} \end{cases}$$
- Then,
  $$\sum_{a \in V} \mu_\psi(\Gamma_a^{\mathcal{M}}) = \int_\Lambda \left[\sum_{a \in V} \chi_a(\lambda)\right] d\mu_\psi \leq \sup_{\lambda \in \Lambda} \left[\sum_{a \in V} \chi_a(\lambda)\right].$$

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- Hence, $\sup_{\lambda\in\Lambda}\left[\sum_{a\in V}\chi_a(\lambda)\right] \leq \alpha(G)$.