Is the wavefunction real?

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Introduction
Epistemic vs. ontic
Classical states
Bohr and Einstein: ψ-epistemicists
Penrose: ψ-ontologist
Interpretations
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

Introduction

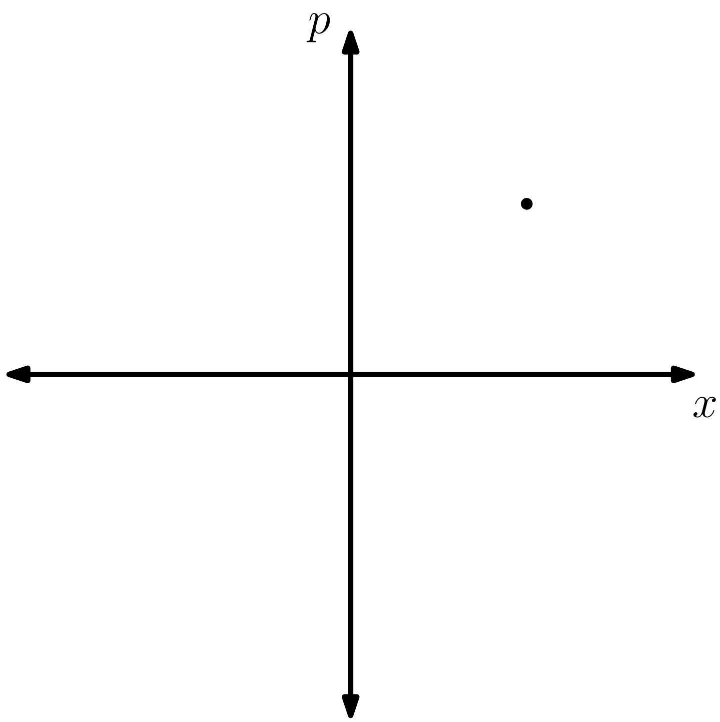
Introduction
Epistemic vs. ontic
Classical states
Bohr and Einstein: ψ-epistemicists
Penrose: ψ-ontologist
Interpretations
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

- *Ontic state*: a state of reality.
  - ψ-*ontic*: the quantum state is ontic.
- *Epistemic state*: a state of knowledge or information.
  - ψ-*epistemic*: the quantum state is epistemic.

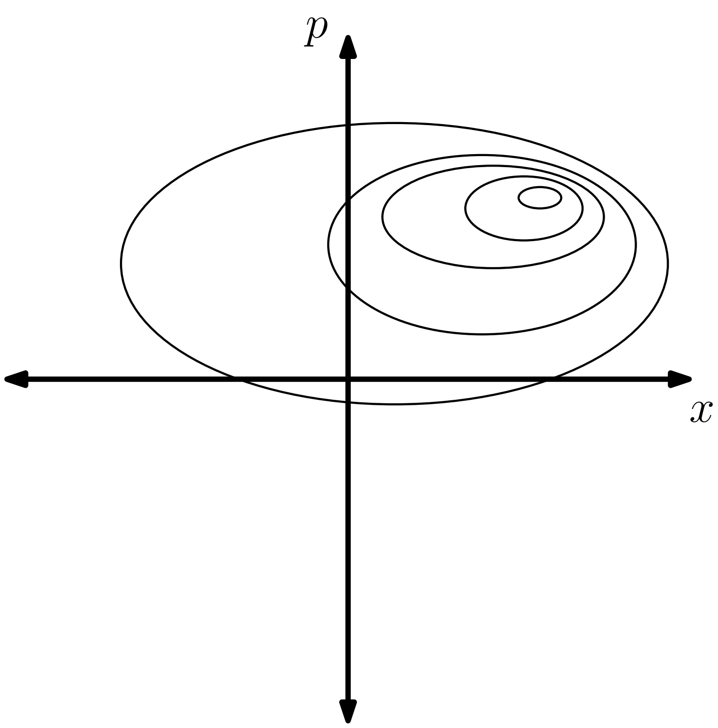
Classical states

Introduction
Epistemic vs. ontic
Classical states
Bohr and Einstein: ψ-epistemicists
Penrose: ψ-ontologist
Interpretations
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

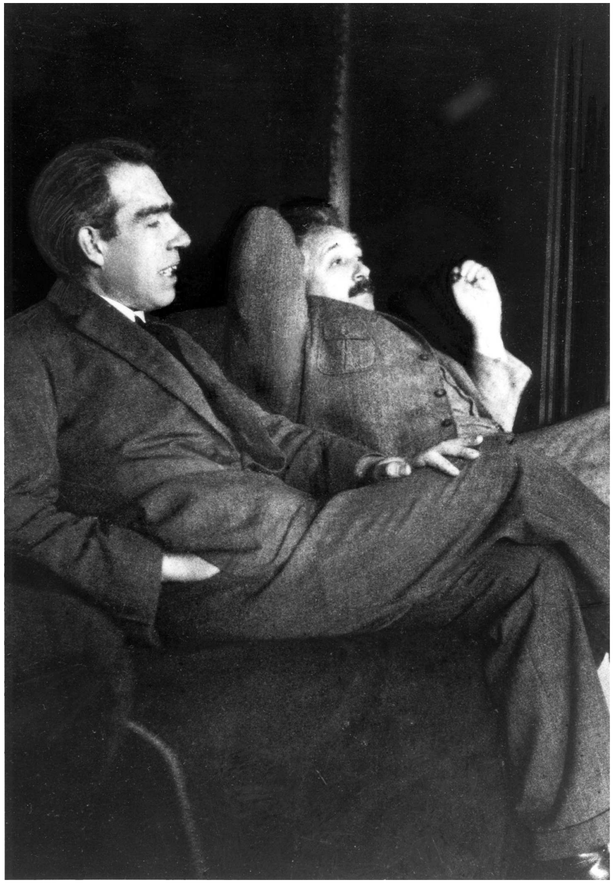
Ontic state



Epistemic state



Bohr and Einstein: ψ-epistemicists



Source: http://en.wikipedia.org/

> There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature. — Niels Bohr[a]

> [t]he ψ-function is to be understood as the description not of a single system but of an ensemble of systems. — Albert Einstein[b]

[a] Quoted in A. Petersen, “The philosophy of Niels Bohr”, *Bulletin of the Atomic Scientists* Vol. 19, No. 7 (1963)

[b] P. A. Schilpp, ed., *Albert Einstein: Philosopher Scientist* (Open Court, 1949)

Penrose: ψ-ontologist

It is often asserted that the state-vector is merely a convenient description of ‘our knowledge’ concerning a physical system—or, perhaps, that the state-vector does not really describe a single system but merely provides probability information about an ‘ensemble’ of a large number of similarly prepared systems. Such sentiments strike me as unreasonably timid concerning what quantum mechanics has to tell us about the *actuality* of the physical world. — Sir Roger Penrose[1]

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[1] R. Penrose, *The Emperor’s New Mind* pp. 268–269 (Oxford, 1989)

Interpretations of quantum theory

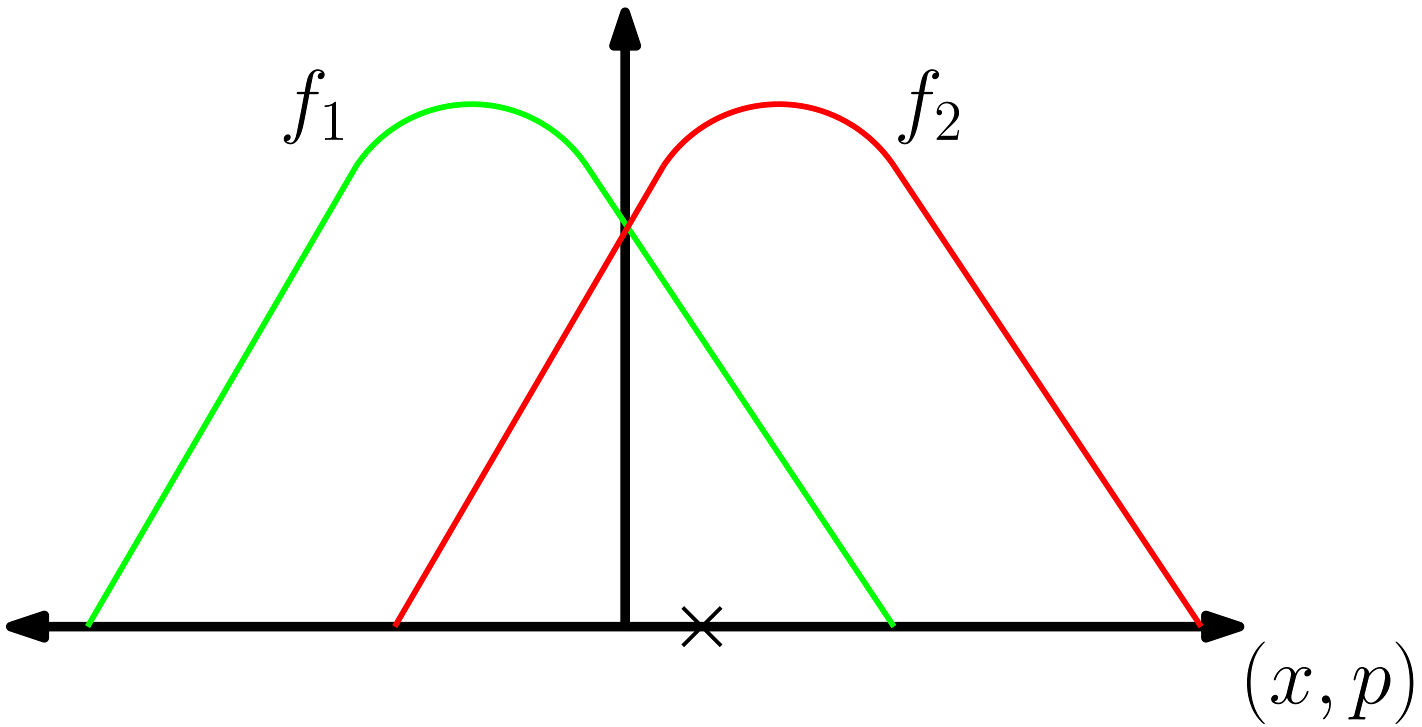
	ψ-epistemic	ψ-ontic
Anti-realist	Copenhagen neo-Copenhagen (e.g. QBism, Peres, Zeilinger, Healey)	
Realist	Einstein Ballentine? Spekkens ?	Dirac-von Neumann Many worlds Bohmian mechanics Spontaneous collapse Modal interpretations

Introduction
Arguments for Epistemic Quantum States
Overlap
Other arguments
Arguments
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

Arguments for Epistemic Quantum States

Introduction
Arguments for Epistemic Quantum States
Overlap
Other arguments
Arguments
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

Epistemic states overlap



Other arguments

Introduction
Arguments for Epistemic Quantum States
Overlap
Other arguments
Arguments
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

- Collapse of the wavefunction
- Generalized probability theory
- Excess baggage

Arguments for ontic quantum states

Introduction

Arguments for Epistemic Quantum States

Overlap

Other arguments

Arguments

Ontological Models

ψ-ontology theorems

ψ-epistemic models

Overlap measures

Overlap bounds

Conclusions

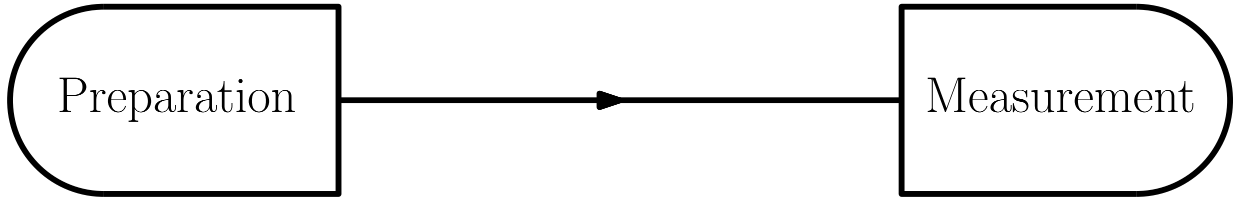
- Interference
- Eigenvalue-eigenstate link
- Lack of imagination
- Quantum computing

Introduction
Arguments for Epistemic Quantum States
Ontological Models
Quantum description
Ontic description
Formal definition
ψ-ontic vs. ψ-epistemic
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

Ontological Models

Prepare-and-measure experiments: Quantum description

Introduction
Arguments for Epistemic Quantum States
Ontological Models
Quantum description
Ontic description
Formal definition
ψ-ontic vs. ψ-epistemic
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

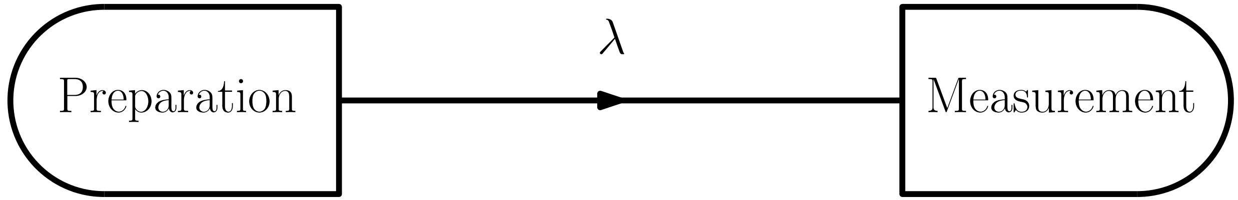


$|\psi\rangle \in \mathbb{C}^d$ $\qquad$ $M = \{|a\rangle, |b\rangle, \ldots\}$

$$\mathrm{Prob}(a|\psi, M) = |\langle a|\psi\rangle|^2$$

Prepare-and-measure experiments: Ontological description

Introduction
Arguments for Epistemic Quantum States
Ontological Models
Quantum description
Ontic description
Formal definition
ψ-ontic vs. ψ-epistemic
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions



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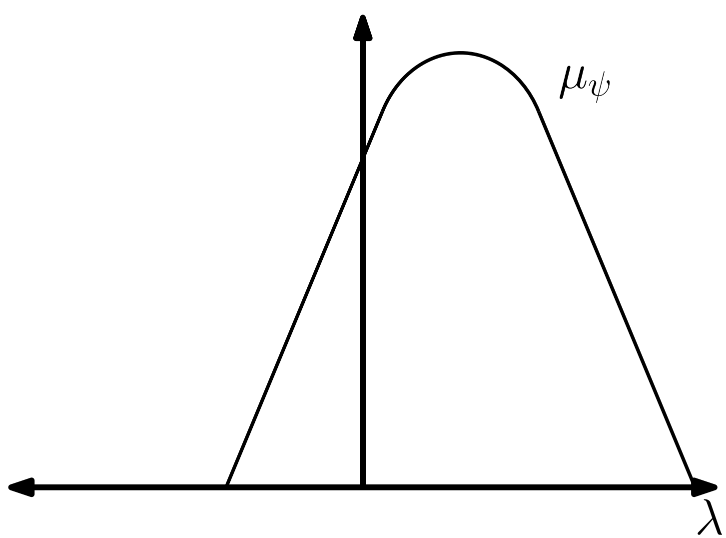

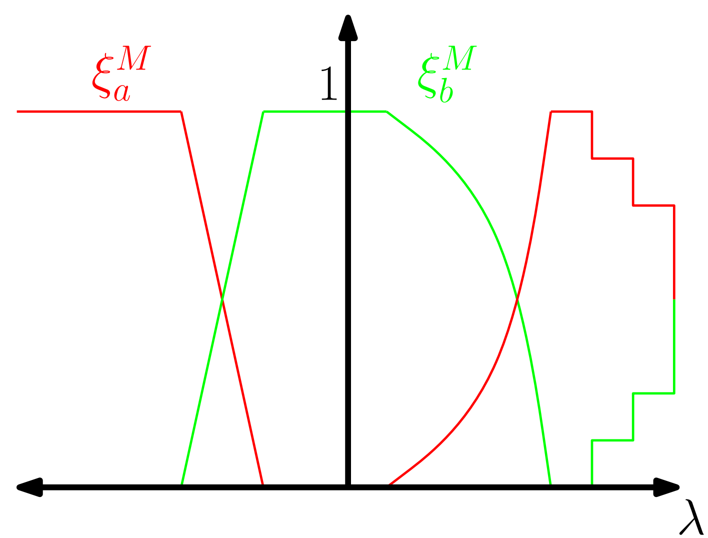

$$\mathrm{Prob}(a|\psi, M) = \int \xi_a^M(\lambda) d\mu_\psi$$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
Quantum description
Ontic description
Formal definition
ψ-ontic vs. ψ-epistemic
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

An ontological model for $\mathbb{C}^d$ consists of:

- A measurable space (Λ, Σ).

Introduction
Arguments for Epistemic Quantum States
Ontological Models
Quantum description
Ontic description
Formal definition
ψ-ontic vs. ψ-epistemic
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

An ontological model for $\mathbb{C}^d$ consists of:

- A measurable space (Λ, Σ).
- For each state $|\psi\rangle \in \mathbb{C}^d$, a probability measure $\mu_\psi : \Sigma \to [0, 1]$.

Introduction
Arguments for Epistemic Quantum States
Ontological Models
Quantum description
Ontic description
Formal definition
ψ-ontic vs. ψ-epistemic
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

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- A measurable space (Λ, Σ).
- For each state $|\psi\rangle \in \mathbb{C}^d$, a probability measure $\mu_\psi : \Sigma \to [0,1]$.
- For each orthonormal basis $M = \{|a\rangle, |b\rangle, \ldots\}$, a set of response functions $\xi_a^M : \Lambda \to [0,1]$ satisfying

$$\forall \lambda, \sum_{|a\rangle \in M} \xi_a^M(\lambda) = 1.$$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
Quantum description
Ontic description
Formal definition
ψ-ontic vs. ψ-epistemic
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

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$$\forall\lambda, \sum_{|a\rangle \in M} \xi_a^M(\lambda) = 1.$$

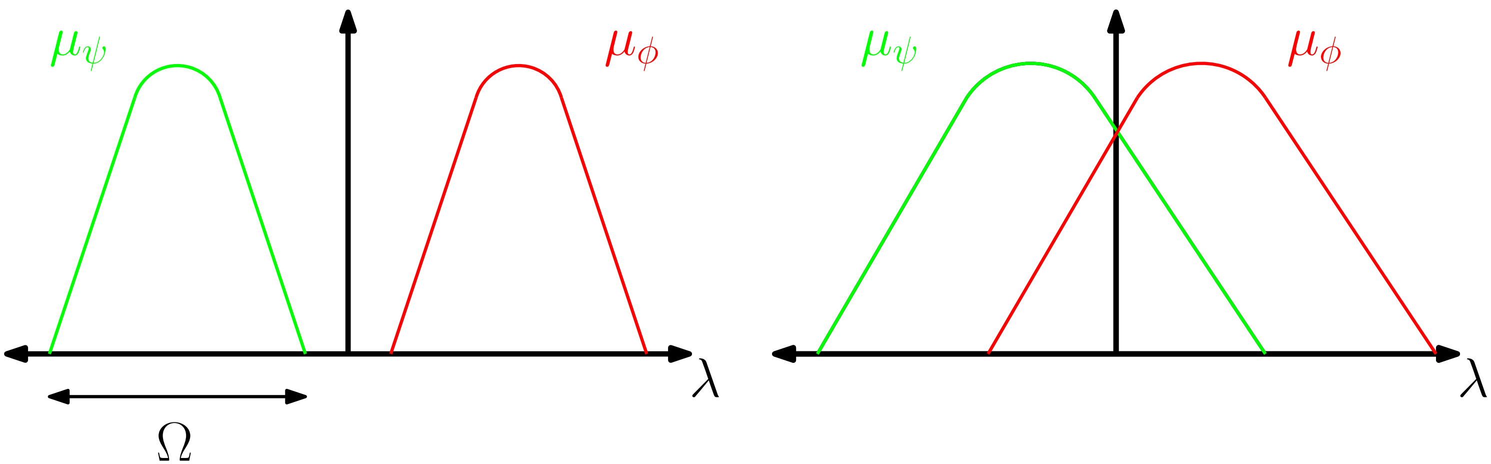
The model is required to reproduce the quantum predictions, i.e.

$$\int_\Lambda \xi_a^M(\lambda) d\mu_\psi = |\langle a|\psi\rangle|^2 .$$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
Quantum description
Ontic description
Formal definition
ψ-ontic vs. ψ-epistemic
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

- $|\psi\rangle$ and $|\phi\rangle$ are *ontologically distinct* in an ontological model if there exists $\Omega \in \Sigma$ s.t.

$$\mu_\psi(\Omega) = 1 \qquad \mu_\phi(\Omega) = 0.$$



- An ontological model is *ψ-ontic* if every pair of states is ontologically distinct. Otherwise it is *ψ-epistemic*.

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

ψ-ontology theorems

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions

- The Colbeck-Renner theorem: R. Colbeck and R. Renner, arXiv:1312.7353 (2013).

- Hardy's theorem: L. Hardy, *Int. J. Mod. Phys. B*, 27:1345012 (2013) arXiv:1205.1439

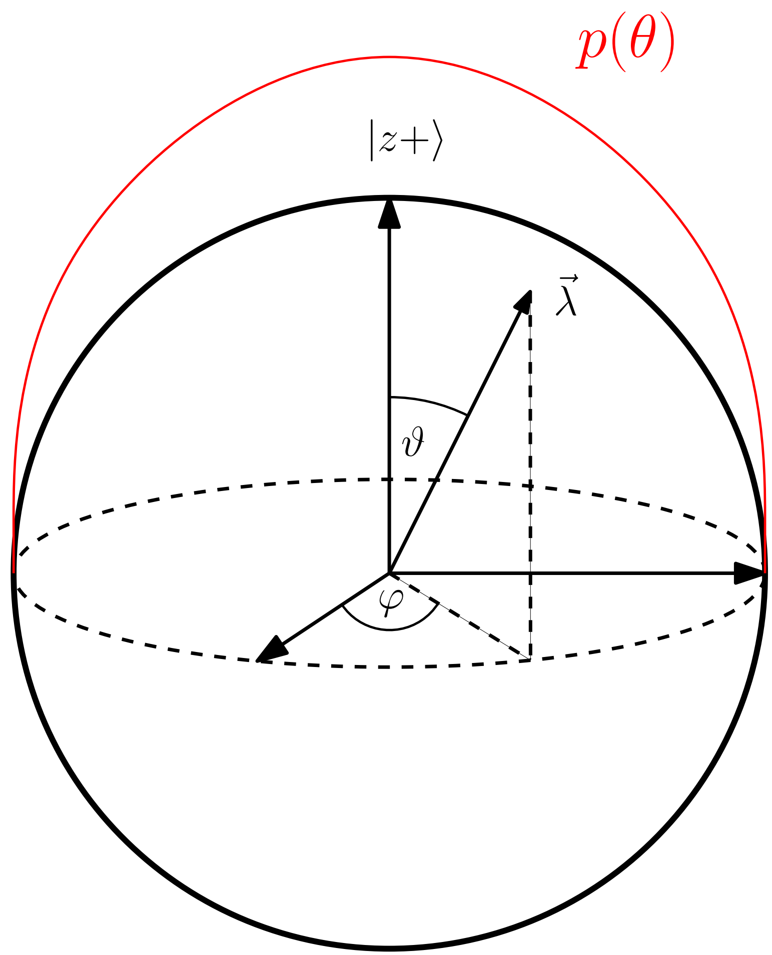
- The Pusey-Barrett-Rudolph theorem: M. Pusey et. al., *Nature Physics*, 8:475–478 (2012) arXiv:1111.3328

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
The Kochen-Specker model
Models for arbitrary finite dimension
Overlap measures
Overlap bounds
Conclusions

ψ-epistemic models

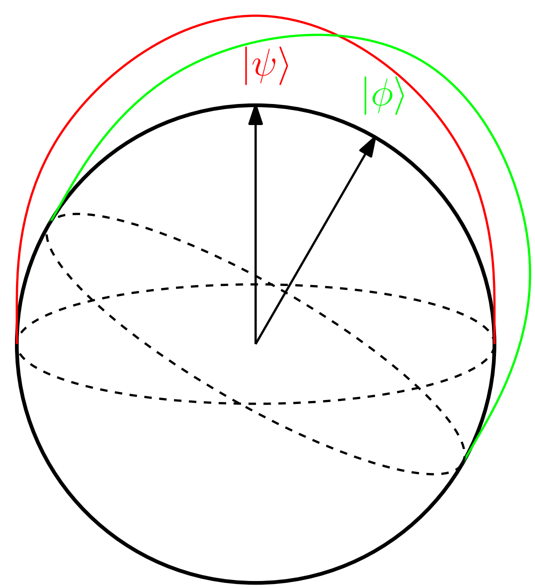
Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
The Kochen-Specker model
Models for arbitrary finite dimension
Overlap measures
Overlap bounds
Conclusions

The Kochen-Specker model for a qubit



$$\mu_{z+}(\Omega) = \int_{\Omega} p(\vartheta) \sin\vartheta d\vartheta d\varphi$$

$$p(\vartheta) = \begin{cases} \frac{1}{\pi}\cos\vartheta, & 0 \leq \vartheta \leq \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < \vartheta \leq \pi \end{cases}$$



S. Kochen and E. Specker, *J. Math. Mech.*, 17:59–87 (1967)

Models for arbitrary finite dimension

Introduction

Arguments for Epistemic Quantum States

Ontological Models

ψ-ontology theorems

ψ-epistemic models

The Kochen-Specker model

Models for arbitrary finite dimension

Overlap measures

Overlap bounds

Conclusions

- Lewis et. al. provided a ψ-epistemic model for all finite d.
  - P. G. Lewis et. al., *Phys. Rev. Lett.* 109:150404 (2012) arXiv:1201.6554
- Aaronson et. al. provided a similar model in which every pair of nonorthogonal states is ontologically indistinct.
  - S. Aaronson et. al., *Phys. Rev. A* 88:032111 (2013) arXiv:1303.2834
- These models have the feature that, for a fixed inner product, the amount of overlap decreases with d.

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Asymmetric overlap
Classical Symmetric overlap
Quantum Symmetric overlap
Relationships between overlap measures
Overlap bounds
Conclusions

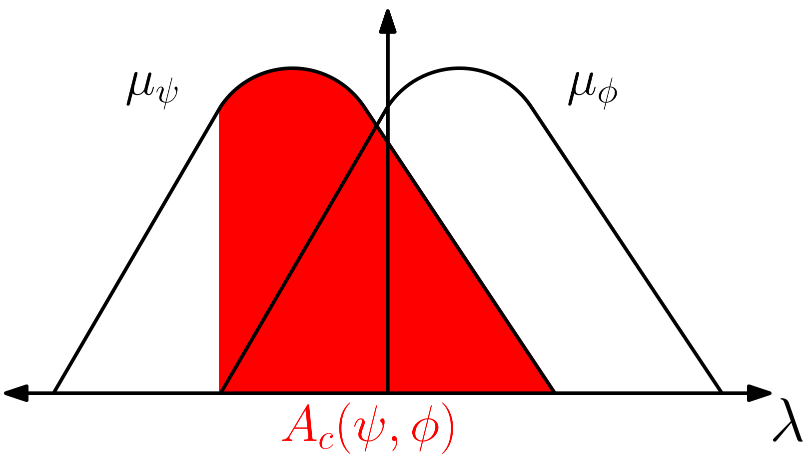
Overlap measures

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Asymmetric overlap
Classical Symmetric overlap
Quantum Symmetric overlap
Relationships between overlap measures
Overlap bounds
Conclusions

Asymmetric overlap

- *Classical asymmetric overlap*:

$$A_c(\psi, \phi) := \inf_{\left\{\Omega \in \Sigma \middle| \mu_\phi(\Omega) = 1\right\}} \mu_\psi(\Omega)$$



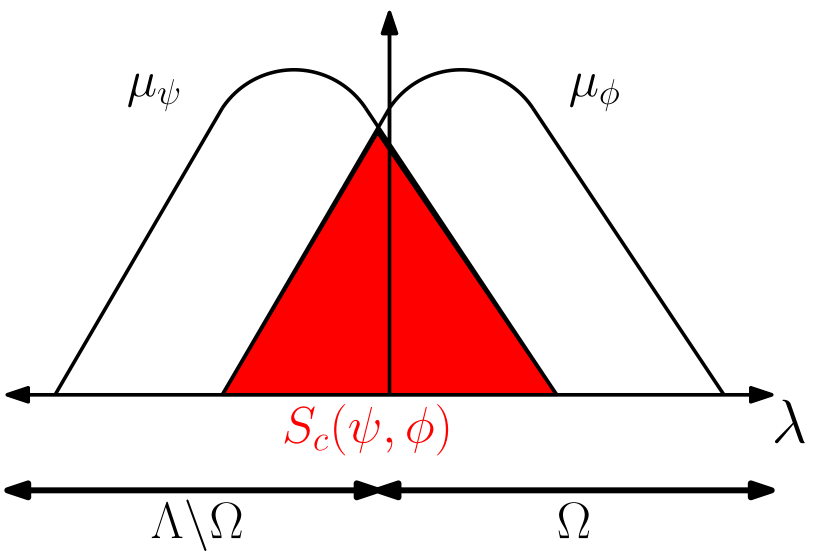
- An ontological model is *maximally ψ-epistemic* if

$$A_c(\psi, \phi) = |\langle\phi|\psi\rangle|^2$$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Asymmetric overlap
Classical Symmetric overlap
Quantum Symmetric overlap
Relationships between overlap measures
Overlap bounds
Conclusions

- *Classical symmetric overlap*:

$$S_c(\psi, \phi) := \inf_{\Omega \in \Sigma} \left[\mu_\psi(\Omega) + \mu_\phi(\Lambda \backslash \Omega) \right]$$



- Optimal success probability of distinguishing $|\psi\rangle$ and $|\phi\rangle$ if you know λ:

$$p_c(\psi, \phi) = \frac{1}{2} \left(2 - S_c(\psi, \phi)\right)$$

Quantum Symmetric overlap

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Asymmetric overlap
Classical Symmetric overlap
Quantum Symmetric overlap
Relationships between overlap measures
Overlap bounds
Conclusions

- *Classical symmetric overlap*:

$$S_c(\psi, \phi) := \inf_{\Omega \in \Sigma} \left[\mu_\psi(\Omega) + \mu_\phi(\Lambda \backslash \Omega) \right]$$

- *Quantum symmetric overlap*:

$$S_q(\psi, \phi) := \inf_{0 \leq E \leq I} \left[\langle \psi | E | \psi \rangle + \langle \phi | (I - E) | \phi \rangle \right]$$

- Optimal success probability of distinguishing $|\psi\rangle$ and $|\phi\rangle$ based on a quantum measurement:

$$p_q(\psi, \phi) = \frac{1}{2} \left(2 - S_q(\psi, \phi) \right)$$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Asymmetric overlap
Classical Symmetric overlap
Quantum Symmetric overlap
Relationships between overlap measures
Overlap bounds
Conclusions

- Classical overlap measures:

$$S_c(\psi, \phi) \leq A_c(\psi, \phi)$$

- Quantum overlap measures:
  - $S_q(\psi, \phi) = 1 - \sqrt{1 - |\langle \phi | \psi \rangle|^2}$
  - $S_q(\psi, \phi) \geq \frac{1}{2} |\langle \phi | \psi \rangle|^2$

- Hence:

$$\frac{S_c(\psi, \phi)}{S_q(\psi, \phi)} \leq 2 \frac{A_c(\psi, \phi)}{|\langle \phi | \psi \rangle|^2}.$$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

Overlap bounds

Previous results

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

- Define:

$$k(\psi,\phi) = \frac{A_c(\psi,\phi)}{|\langle\phi|\psi\rangle|^2}.$$

- Maroney showed $k(\psi,\phi) < 1$ for some states. ML and Maroney showed this follows from KS theorem.

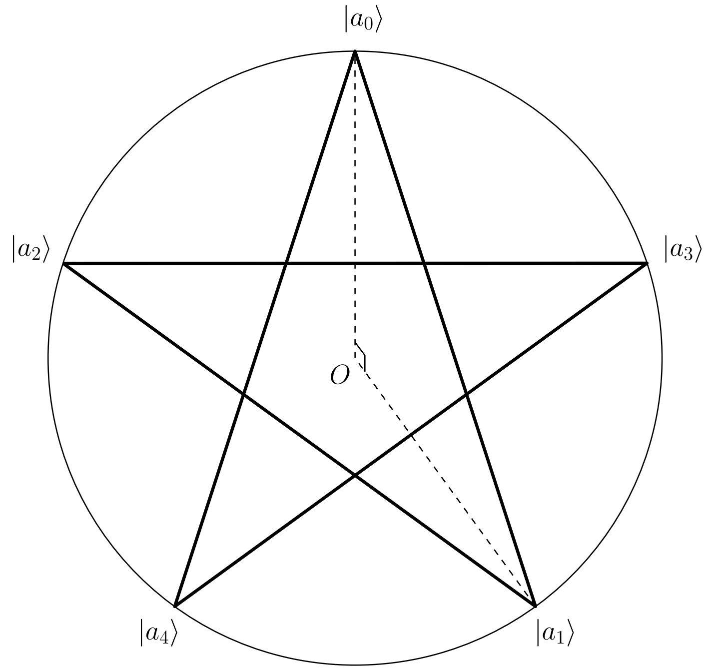
- Barrett et. al. exhibited a family of states in $\mathbb{C}^d$ such that, for $d \geq 4$:

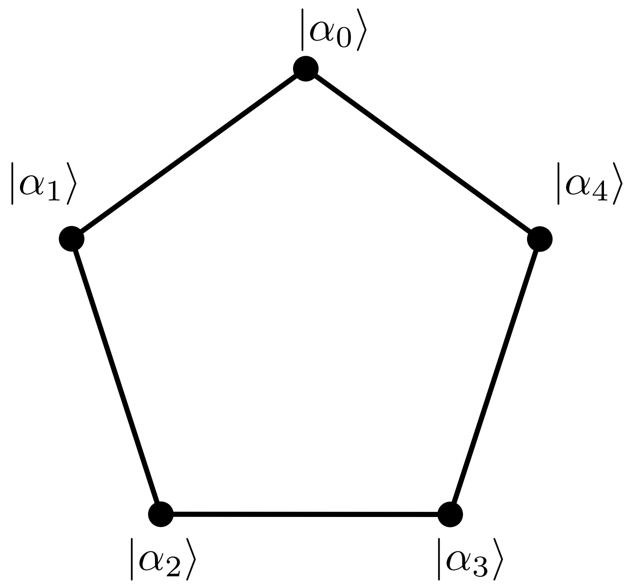
$$k(\psi,\phi) \leq \frac{4}{d-1}.$$

- Today: $k(\psi,\phi) \leq de^{-cd}$ for d divisible by 4.

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

- Example: Klyachko states
  - $|a_j\rangle = \sin\vartheta \cos\varphi_j |0\rangle + \sin\vartheta \sin\varphi_j |1\rangle + \cos\vartheta |2\rangle$
  - $\varphi_j = \frac{4\pi j}{5}$ and $\cos\vartheta = \frac{1}{\sqrt[4]{5}}$

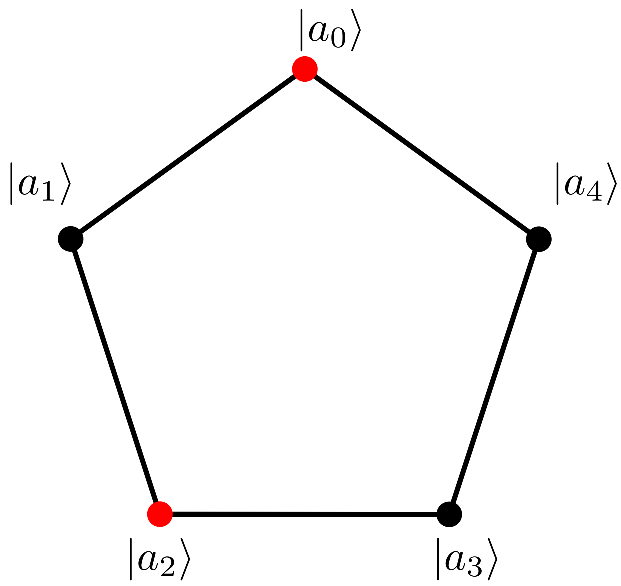




Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

- The *independence number* $\alpha(G)$ of a graph G is the cardinality of the largest subset of vertices such that no two vertices are connected by an edge.

- Example: $\alpha(G) = 2$



Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

Theorem: Let V be a finite set of states in $\mathbb{C}^d$ an let $G = (V, E)$ be its orthogonality graph. For $|\psi\rangle \in \mathbb{C}^d$ define

$$\bar{k}(\psi) = \frac{1}{|V|} \sum_{|a\rangle \in V} k(\psi, a).$$

Then, in any ontological model

$$\bar{k}(\psi) \leq \frac{\alpha(G)}{|V| \min_{|a\rangle \in V} |\langle a|\psi\rangle|^2}.$$

Bound from Klyatchko states

Introduction

Arguments for Epistemic Quantum States

Ontological Models

ψ-ontology theorems

ψ-epistemic models

Overlap measures

Overlap bounds

Previous results

Orthogonality graphs

Independence number

Main result

Klyatchko bound

Exponential bound

Main result

Proof of main result:1

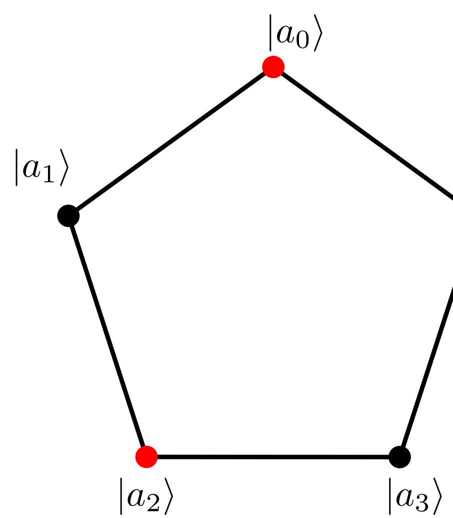
Proof of main result:2

Proof of main result:3

Contextuality

Conclusions

- $|a_j\rangle = \sin\vartheta\cos\varphi_j\,|0\rangle + \sin\vartheta\sin\varphi_j\,|1\rangle + \cos\vartheta\,|2\rangle$
- $\varphi_j = \frac{4\pi j}{5}$ and $\cos\vartheta = \frac{1}{\sqrt[4]{5}}$
- $|\psi\rangle = |2\rangle$



$$\bar{k}(\psi) \leq \frac{\alpha(G)}{5\min_j |\langle a_j|\psi\rangle|^2} = \frac{2}{5\times\frac{1}{\sqrt{5}}} \sim 0.8944$$

Exponential bound: Hadamard states

Introduction

Arguments for Epistemic Quantum States

Ontological Models

ψ-ontology theorems

ψ-epistemic models

Overlap measures

Overlap bounds

- Previous results
- Orthogonality graphs
- Independence number
- Main result
- Klyatchko bound
- Exponential bound
- Main result
- Proof of main result:1
- Proof of main result:2
- Proof of main result:3
- Contextuality

Conclusions

- For $\boldsymbol{x} = (x_0, x_1, \ldots, x_{d-1}) \in \{0,1\}^d$, let

$$|a_{\boldsymbol{x}}\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} (-1)^{x_j} |j\rangle .$$

- Let $|\psi\rangle = |0\rangle$.
- By Frankl-Rödl theorem[2], for d divisible by 4, there exists an $\epsilon > 0$ such that $\alpha(G) \leq (2-\epsilon)^d$.

$$\bar{k}(\psi) \leq \frac{\alpha(G)}{2^d \min_{\boldsymbol{x}\in\{0,1\}^d} |\langle a_{\boldsymbol{x}}|\psi\rangle|^2} = \frac{(2-\epsilon)^d}{2^d \times \frac{1}{d}} = de^{-cd}$$

$$c = \ln 2 - \ln(2-\epsilon)$$

[2] P. Frankl and V. Rödl, *Trans. Amer. Math. Soc.* 300:259 (1987)

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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Then, in any ontological model

$$\bar{k}(\psi) \leq \frac{\alpha(G)}{|V| \min_{|a\rangle \in V} |\langle a|\psi\rangle|^2}.$$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

- Let $\mathcal{M}$ be a covering set of bases for V.

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

Proof of main result:1

- Let $\mathcal{M}$ be a covering set of bases for V.
- For $M \in \mathcal{M}$, let

$$\Gamma_a^M = \{\lambda | \xi_a^M(\lambda) = 1\}$$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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- For $M \in \mathcal{M}$, let

$$\Gamma_a^M = \{\lambda | \xi_a^M(\lambda) = 1\}$$

- $\mu_a(\Gamma_a^M) = 1$ because $\int_\Lambda \xi_a^M(\lambda) d\mu_a = |\langle a|a\rangle|^2 = 1$.

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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- Let
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  - $\mu_a(\Gamma_a^{\mathcal{M}}) = 1$ also.
- Hence, $A_c(\psi, a) = \inf_{\{\Omega \in \Sigma | \mu_a(\Omega) = 1\}} \mu_\psi(\Omega) \leq \mu_\psi(\Gamma_a^{\mathcal{M}})$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

Proof of main result:2

- $$A_c(\psi, a) \leq \mu_\psi(\Gamma_a^{\mathcal{M}})$$
  $$\sum_{|a\rangle \in V} A_c(\psi, a) \leq \sum_{a \in V} \mu_\psi(\Gamma_a^{\mathcal{M}})$$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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- Let
$$\chi_a(\lambda) = \begin{cases} 1, & \lambda \in \Gamma_a^{\mathcal{M}} \\ 0, & \lambda \notin \Gamma_a^{\mathcal{M}} \end{cases}$$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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- Let
$$\chi_a(\lambda) = \begin{cases} 1, & \lambda \in \Gamma_a^{\mathcal{M}} \\ 0, & \lambda \notin \Gamma_a^{\mathcal{M}} \end{cases}$$

- Then,
$$\sum_{a \in V} \mu_\psi(\Gamma_a^{\mathcal{M}}) = \int_\Lambda \left[\sum_{a \in V} \chi_a(\lambda) \right] d\mu_\psi \leq \sup_{\lambda \in \Lambda} \left[\sum_{a \in V} \chi_a(\lambda) \right].$$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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- Hence, if $\lambda \in \Gamma_a^{\mathcal{M}}$ then $\lambda \notin \Gamma_b^{\mathcal{M}}$ for any $|b\rangle \in V$ such that $(|a\rangle, |b\rangle) \in E$.

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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- Hence, if $\lambda \in \Gamma_a^{\mathcal{M}}$ then $\lambda \notin \Gamma_b^{\mathcal{M}}$ for any $|b\rangle \in V$ such that $(|a\rangle\,, |b\rangle) \in E$.
- Hence, $\sup_{\lambda\in\Lambda}\left[\sum_{a\in V}\chi_a(\lambda)\right] \leq \alpha(G)$.

The connection to contextuality

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

- An ontological model for a set of bases $\mathcal{M}$ is *Kochen-Specker noncontextual* if it is:
  - *Outcome deterministic*: $\xi_a^M(\lambda) \in \{0, 1\}$.
  - *Measurement noncontextual*: $\xi_a^M = \xi_a^N$.

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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- In any ontological model $A_c(\psi, \phi) \leq \max \text{Prob}_{\text{N.C.}}(\phi|\psi, M)$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Previous results
Orthogonality graphs
Independence number
Main result
Klyatchko bound
Exponential bound
Main result
Proof of main result:1
Proof of main result:2
Proof of main result:3
Contextuality
Conclusions

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- In any ontological model $A_c(\psi, \phi) \leq \max \text{Prob}_{\text{N.C.}}(\phi|\psi, M)$
- Therefore, any KS contextuality inequality gives an overlap bound.

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions
Summary and Open questions
What now for ψ-epistemicists?
References

Conclusions

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions
Summary and Open questions
What now for ψ-epistemicists?
References

Summary and Open questions

- Summary
  - There exist pairs of states such that $k(\psi, \phi) \leq de^{-cd}$. The ψ-epistemic explanations of indistinguishability, no-cloning, etc. get implausible for these states very radpidly for large d.
  - Any contextuality inequality can be used to derive an overlap bound.
- Open questions
  - Error analysis.
  - Best bounds in small dimensions.
  - Bounds with a fixed inner product.
  - Connection to communication complexity.

What now for ψ-epistemicists?

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions
Summary and Open questions
What now for ψ-epistemicists?
References

- Become neo-Copenhagen.
- Adopt a more exotic ontology:
  - Nonstandard logics and probability theories.
  - Ironic many-worlds.
  - Retrocausality.
  - Relationalism.

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions
Summary and Open questions
What now for ψ-epistemicists?
References

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- Become neo-Copenhagen.
- Adopt a more exotic ontology:
  - Nonstandard logics and probability theories.
  - Ironic many-worlds.
  - Retrocausality.
  - Relationalism.
- Explanatory conservatism: If there is a natural explanation for a quantum phenomenon then we should adopt an interpretation that incorporates it.
  - Suggests exploring exotic ontologies.

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Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions
Extra Slides
The Colbeck-Renner Theorem
Hardy's Theorem
The PBR Theorem

Extra Slides

The Colbeck-Renner Theorem

Introduction

Arguments for Epistemic Quantum States

Ontological Models

ψ-ontology theorems

ψ-epistemic models

Overlap measures

Overlap bounds

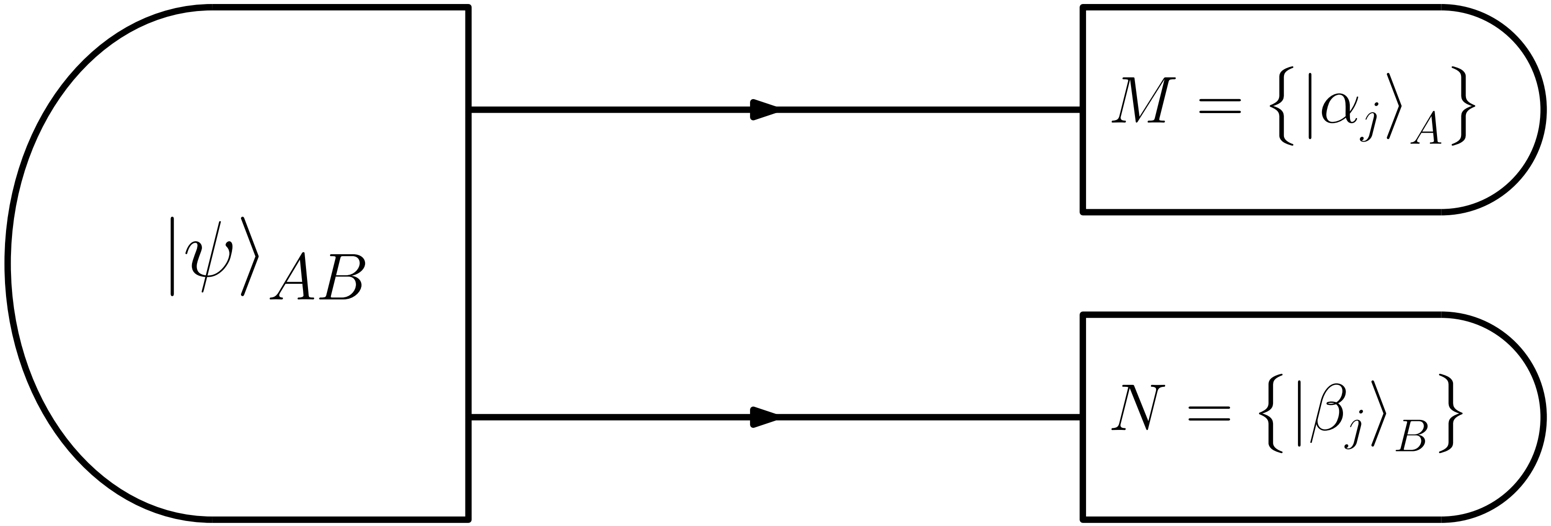
Conclusions

Extra Slides

The Colbeck-Renner Theorem

Hardy's Theorem

The PBR Theorem



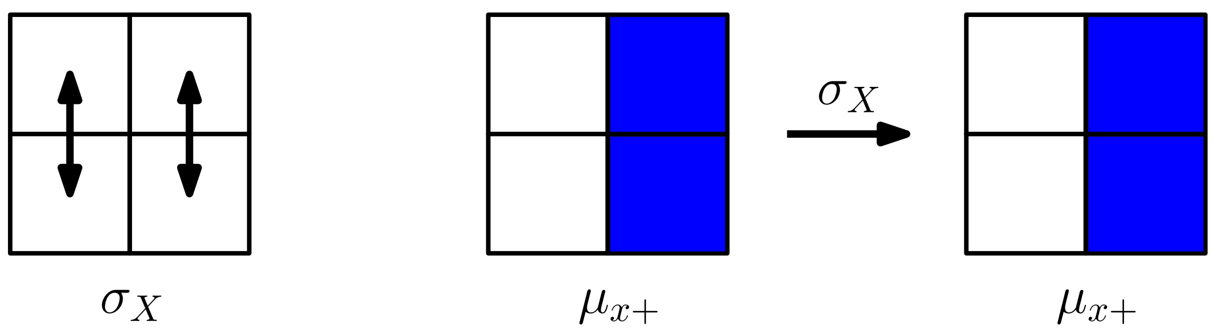
- *Parameter Independence*:
  - $P(a_j|M, N, \lambda) = P(a_j|M, \lambda)$
  - $P(b_k|M, N, \lambda) = P(b_k|N, \lambda)$

Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions
Extra Slides
The Colbeck-Renner Theorem
Hardy's Theorem
The PBR Theorem

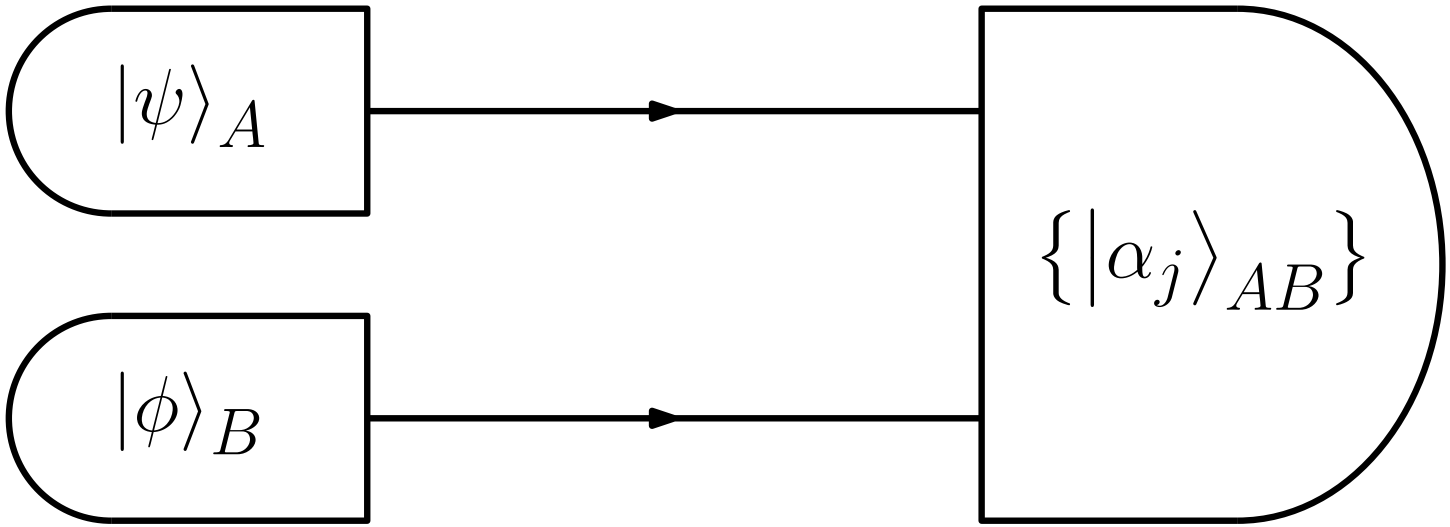
- *Ontic indifference*: If $U|\psi\rangle = |\psi\rangle$ then all of the ontic states in the support of μ_ψ should be left invariant by U.

- Example: For a spin-$1/2$ particle, $\sigma_X|x+\rangle = |x+\rangle$.

- But in Spekkens' toy theory:



Introduction
Arguments for Epistemic Quantum States
Ontological Models
ψ-ontology theorems
ψ-epistemic models
Overlap measures
Overlap bounds
Conclusions
Extra Slides
The Colbeck-Renner Theorem
Hardy's Theorem
The PBR Theorem



- The *Preparation Independence Postulate*:
  - $(\Lambda_{AB}, \Sigma_{AB}) = (\Lambda_A \times \Lambda_B, \Sigma_A \otimes \Sigma_B)$
  - $\mu_{AB} = \mu_A \times \mu_B$