Is the wavefunction real?

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Introduction

ψ-epistemic vs. ψ-ontic

- *Ontic state*: a state of reality.
  - *ψ-ontic*: the quantum state is ontic.
- *Epistemic state*: a state of knowledge or information.
  - *ψ-epistemic*: the quantum state is epistemic.

Eddington: ψ-epistemicist

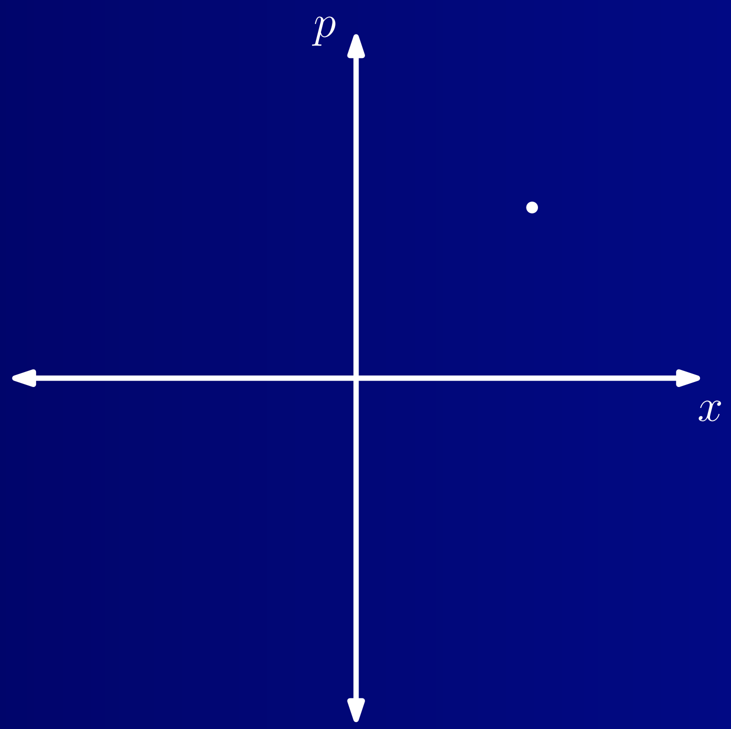
George Grantham Bain Collection (Library of Congress)

The statement often made, that in modern theory the electron is not a particle but a wave, is misleading. The "wave" represents our knowledge of the electron.
— Sir Arthur Eddington[a]

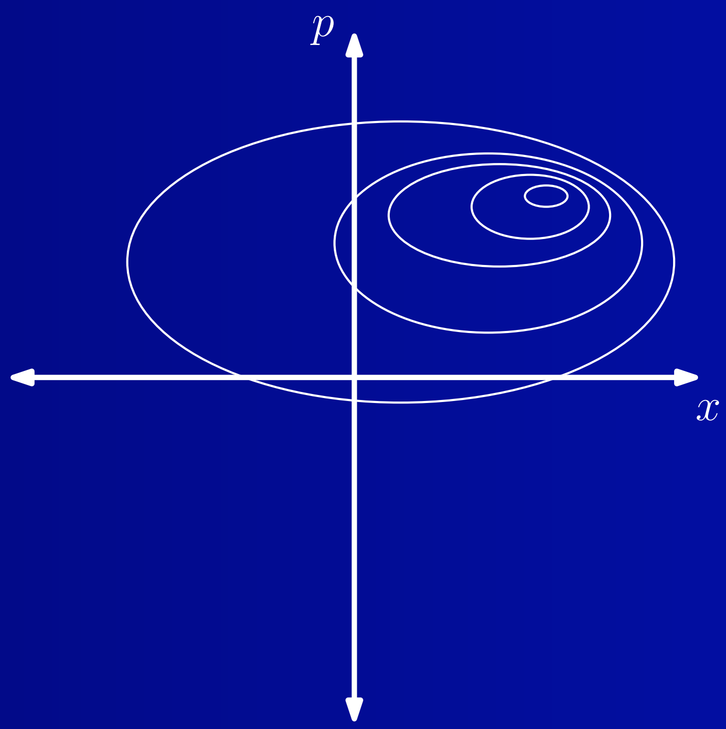
[a] *The Philosophy of Physical Science* (Cambridge University Press, 1939) p. 51.

Classical states

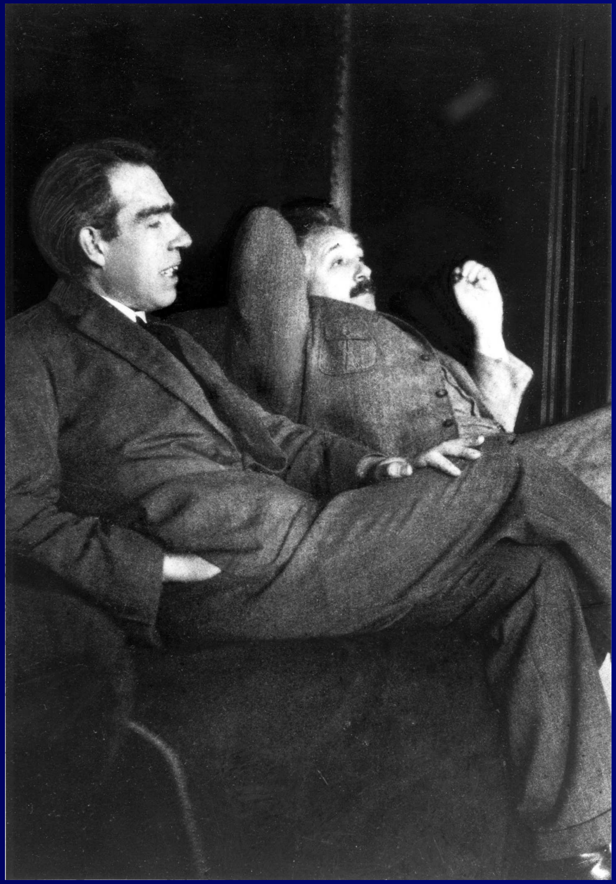
Ontic state



Epistemic state



Bohr and Einstein: More ψ-epistemicists



Source: http://en.wikipedia.org/

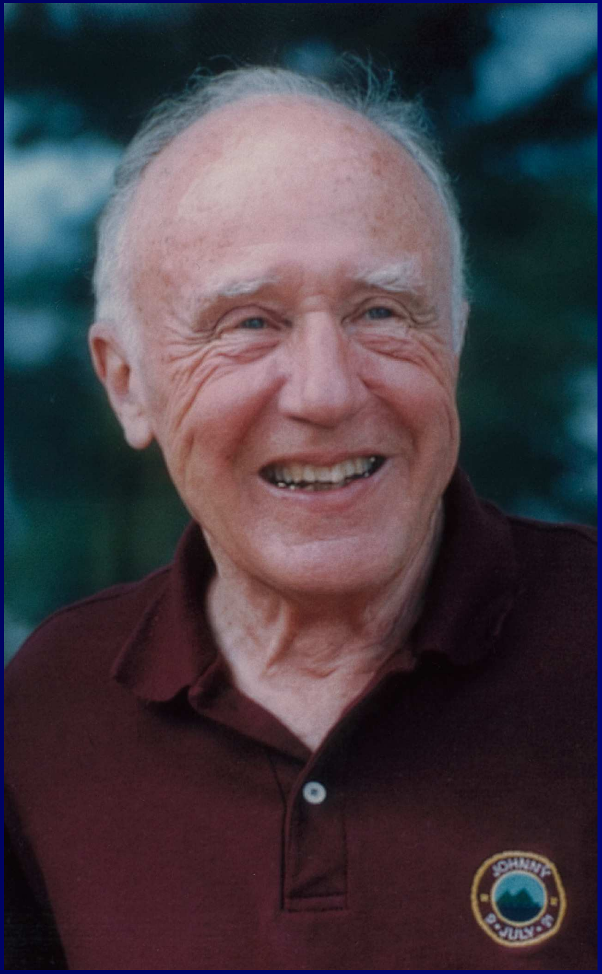
There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature. — Niels Bohr[a]

[t]he ψ-function is to be understood as the description not of a single system but of an ensemble of systems. — Albert Einstein[b]

[a] Quoted in A. Petersen, "The philosophy of Niels Bohr", *Bulletin of the Atomic Scientists* Vol. 19, No. 7 (1963)

[b] P. A. Schilpp, ed., *Albert Einstein: Philosopher Scientist* (Open Court, 1949)

Wheeler: Yet another ψ-epistemicist



Courtesy of the Wheeler family (1991)

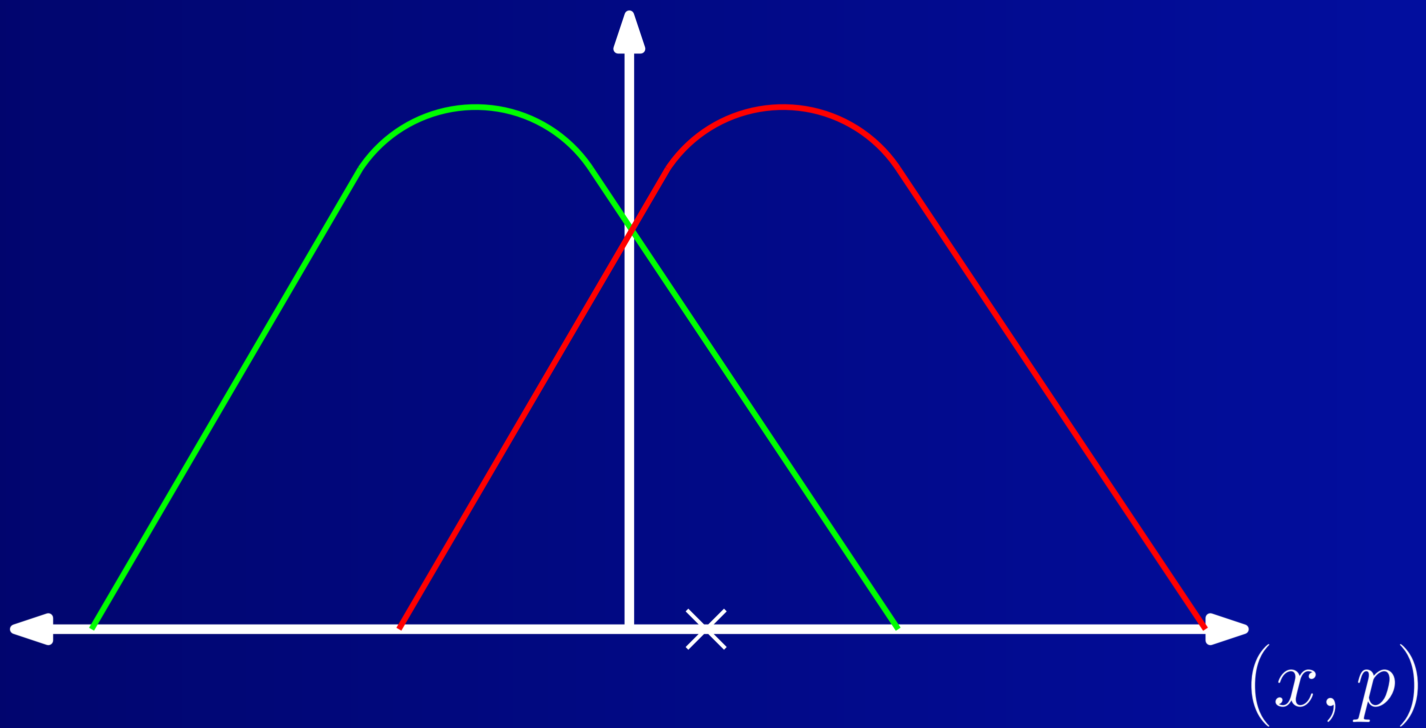
It from bit. Otherwise put, every it—every particle, every field of force, even the spacetime continuum itself—derives its function, its meaning, its very existence entirely—even if in some contexts indirectly—from the apparatus-elicited answers to yes or no questions, binary choices, bits. — John A. Wheeler[a]

[a] *Proc. 3rd Int. Symposium on Foundations of Quantum Mechanics in Light of New Technology* (Physical Society of Japan, 1990) pp. 354–368.

Interpretations of quantum theory

	ψ-epistemic	ψ-ontic
Copenhagenish	Copenhagen neo-Copenhagen (e.g. Bohr, Eddington, Wheeler, QBism)	
Straightforwardly Realist	Einstein Ballentine? Spekkens Me ?	Dirac-von Neumann Many worlds Bohmian mechanics Spontaneous collapse Modal interpretations

Epistemic states overlap

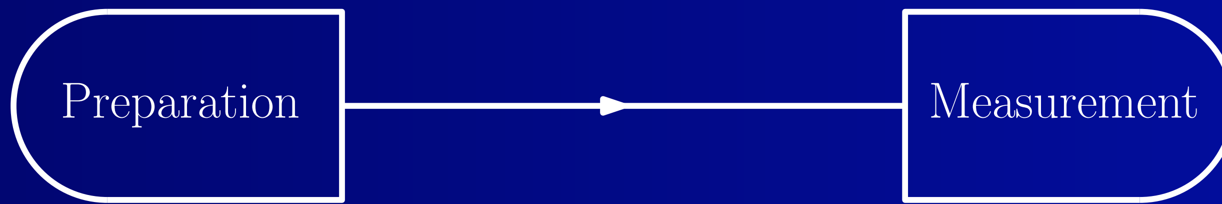


Other arguments

- Collapse of the wavefunction
- Generalized probability theory
- Excess baggage

Ontological Models

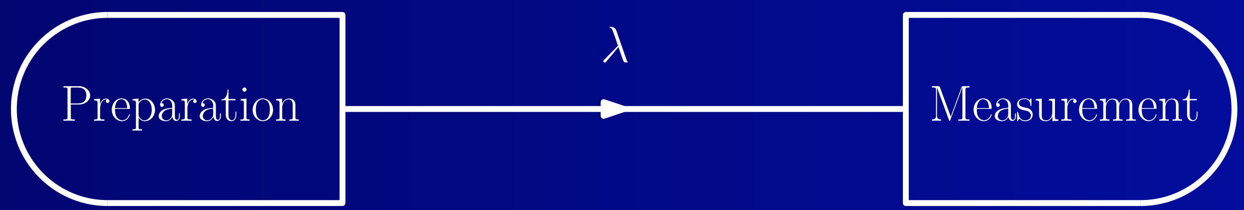
Prepare-and-measure experiments: Quantum description



$|\psi\rangle \in \mathbb{C}^d$ $\qquad$ $M = \{|a\rangle, |b\rangle, \ldots\}$

$$\mathrm{Prob}(a|\psi, M) = |\langle a|\psi\rangle|^2$$

Prepare-and-measure experiments: Ontological description



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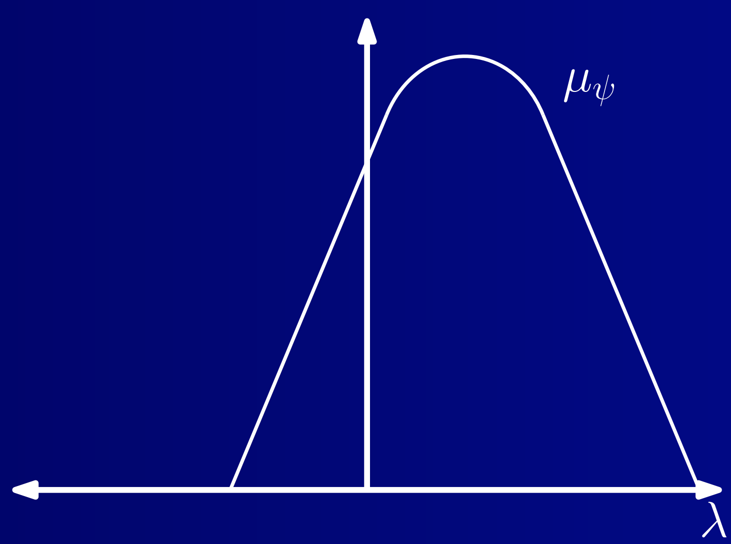

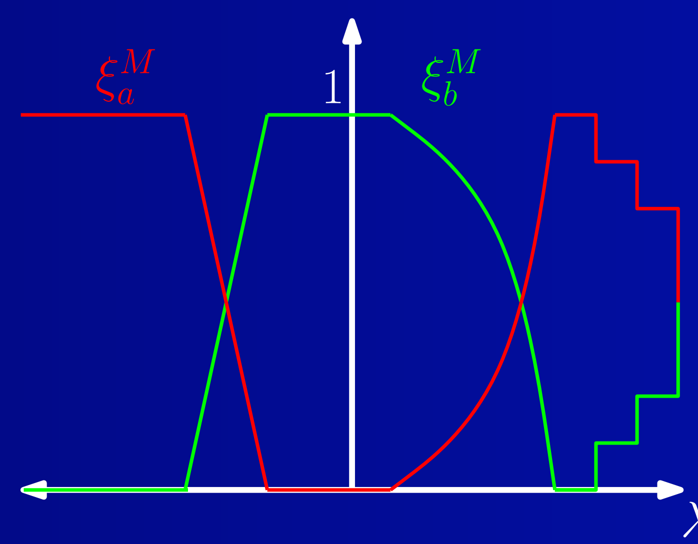

$$\mathrm{Prob}(a|\psi, M) = \int \xi_a^M(\lambda) d\mu_\psi$$

Formal definition

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- For each orthonormal basis $M = \{|a\rangle, |b\rangle, \ldots\}$, a set of response functions $\xi_a^M : \Lambda \to [0, 1]$ satisfying

$$\forall \lambda, \sum_{|a\rangle \in M} \xi_a^M(\lambda) = 1.$$

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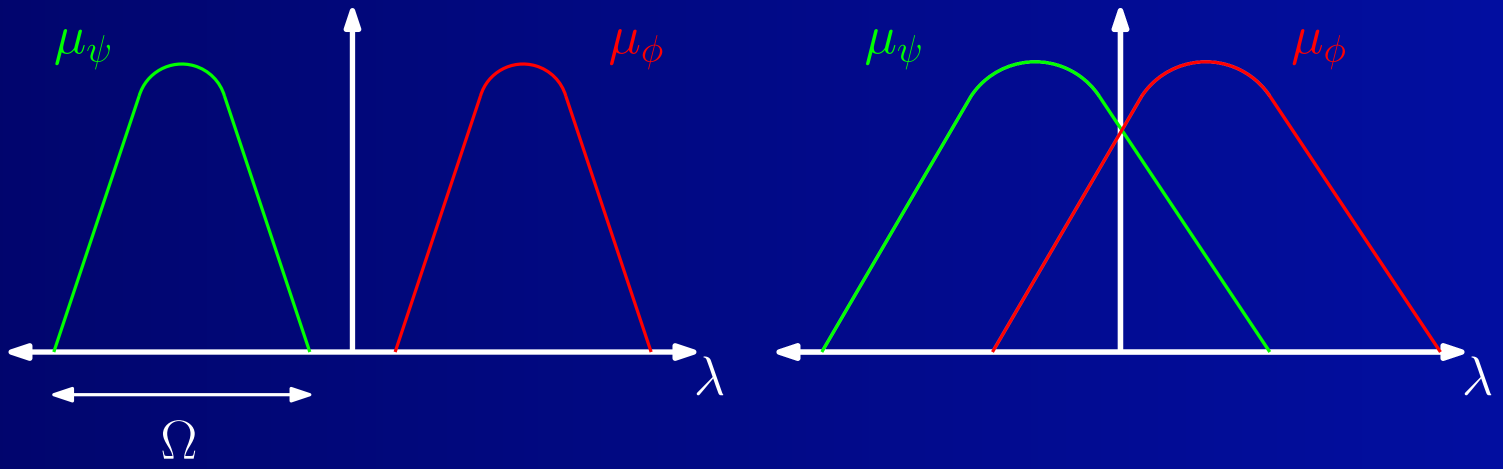
The model is required to reproduce the quantum predictions, i.e.

$$\int_\Lambda \xi_a^M(\lambda) d\mu_\psi = |\langle a|\psi\rangle|^2 .$$

ψ-ontic and ψ-epistemic models

- $|\psi\rangle$ and $|\phi\rangle$ are *ontologically distinct* in an ontological model if there exists $\Omega \in \Sigma$ s.t.

$$\mu_\psi(\Omega) = 1 \qquad \mu_\phi(\Omega) = 0.$$



- An ontological model is *ψ-ontic* if every pair of states is ontologically distinct. Otherwise it is *ψ-epistemic*.

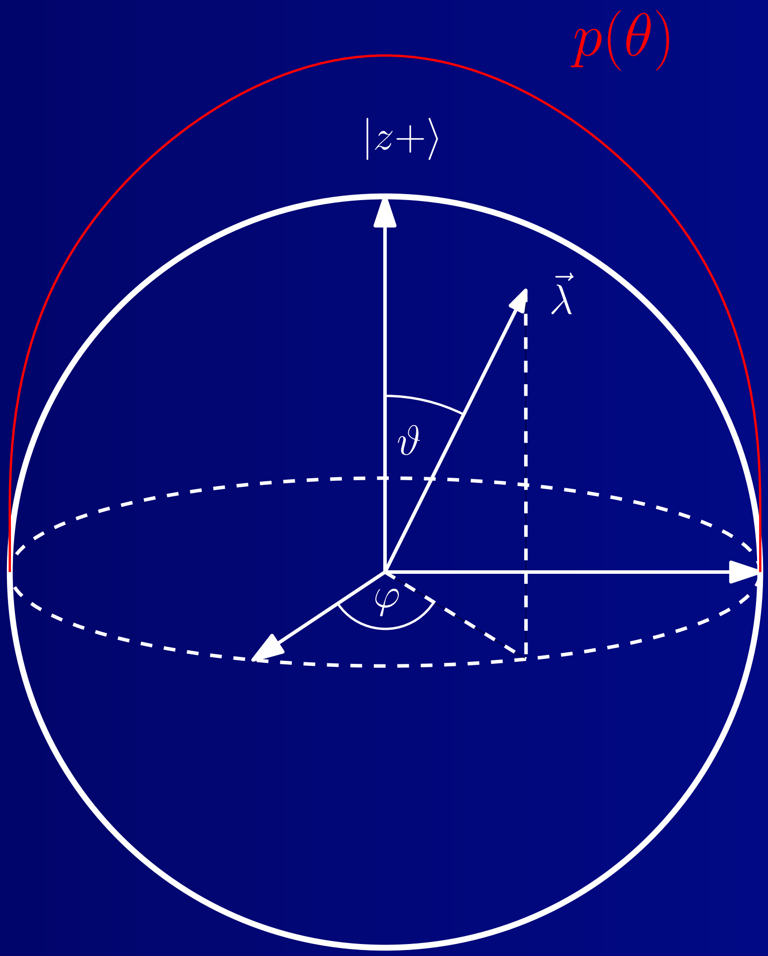
ψ-ontology theorems

ψ-ontology theorems

- The Pusey-Barrett-Rudolph theorem: M. Pusey et. al., *Nature Physics*, 8:475–478 (2012) arXiv:1111.3328
- Hardy's theorem: L. Hardy, *Int. J. Mod. Phys. B*, 27:1345012 (2013) arXiv:1205.1439
- The Colbeck-Renner theorem: R. Colbeck and R. Renner, arXiv:1312.7353 (2013).

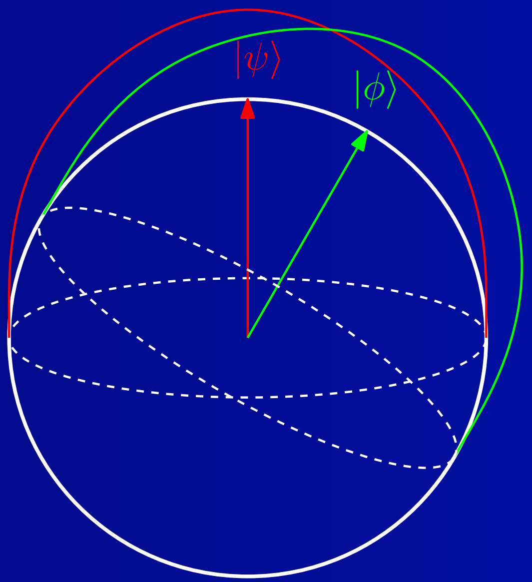
ψ-epistemic models

The Kochen-Specker model for a qubit



$$\mu_{z+}(\Omega) = \int_\Omega p(\vartheta) \sin\vartheta \, d\vartheta \, d\varphi$$

$$p(\vartheta) = \begin{cases} \frac{1}{\pi}\cos\vartheta, & 0 \leq \vartheta \leq \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < \vartheta \leq \pi \end{cases}$$



S. Kochen and E. Specker, *J. Math. Mech.*, 17:59–87 (1967)

Models for arbitrary finite dimension

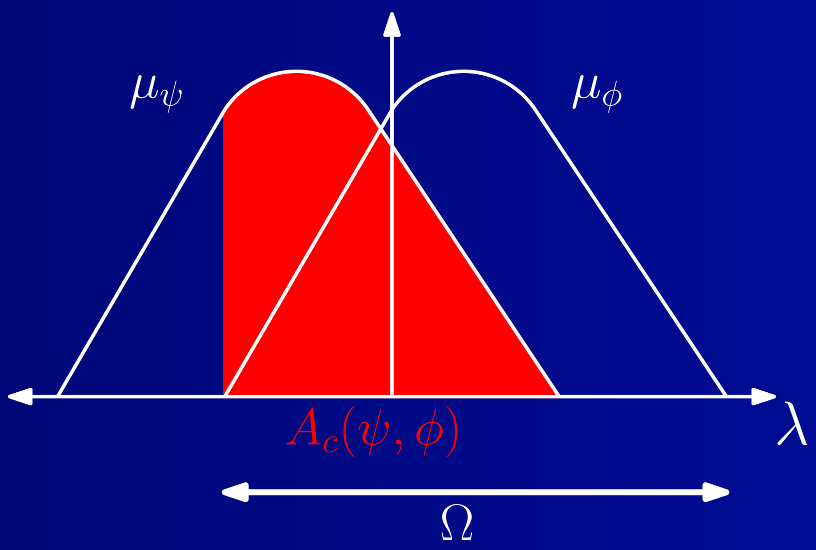
- Lewis et. al. provided a ψ-epistemic model for all finite d.
  - P. G. Lewis et. al., *Phys. Rev. Lett.* 109:150404 (2012) arXiv:1201.6554
- Aaronson et. al. provided a similar model in which every pair of nonorthogonal states is ontologically indistinct.
  - S. Aaronson et. al., *Phys. Rev. A* 88:032111 (2013) arXiv:1303.2834
- These models have the feature that, for a fixed inner product, the amount of overlap decreases with d.

Overlap measures

Asymmetric overlap

- *Classical asymmetric overlap*:

$$A_c(\psi, \phi) := \inf_{\{\Omega \in \Sigma | \mu_\phi(\Omega) = 1\}} \mu_\psi(\Omega)$$



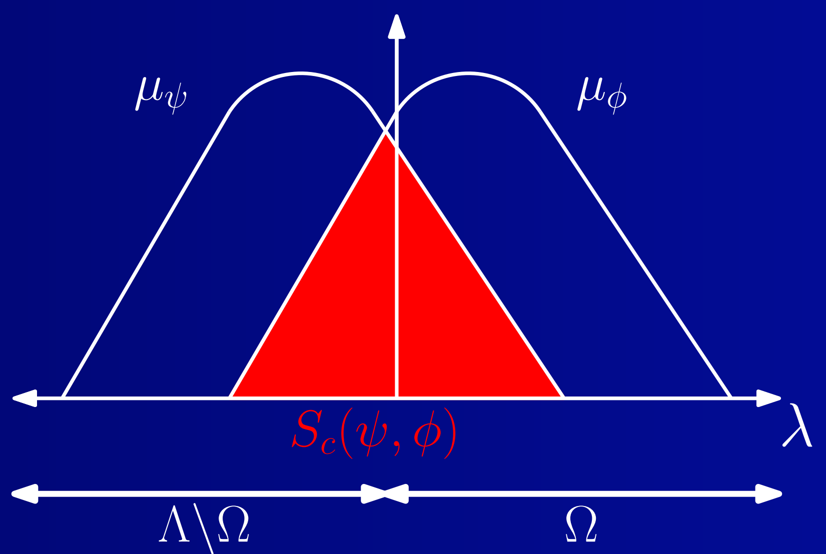
- An ontological model is *maximally ψ-epistemic* if

$$A_c(\psi, \phi) = |\langle \phi | \psi \rangle|^2$$

Classical Symmetric overlap

- *Classical symmetric overlap*:

$$S_c(\psi,\phi) := \inf_{\Omega\in\Sigma}\left[\mu_\psi(\Omega)+\mu_\phi(\Lambda\backslash\Omega)\right]$$



- Optimal success probability of distinguishing $|\psi\rangle$ and $|\phi\rangle$ if you know λ:

$$p_c(\psi,\phi) = \frac{1}{2}\left(2 - S_c(\psi,\phi)\right)$$

Quantum Symmetric overlap

- *Classical symmetric overlap*:

$$S_c(\psi, \phi) := \inf_{\Omega \in \Sigma} \left[\mu_\psi(\Omega) + \mu_\phi(\Lambda \backslash \Omega) \right]$$

- *Quantum symmetric overlap*:

$$S_q(\psi, \phi) := \inf_{0 \leq E \leq I} \left[\langle \psi | E | \psi \rangle + \langle \phi | (I - E) | \phi \rangle \right]$$

- Optimal success probability of distinguishing $|\psi\rangle$ and $|\phi\rangle$ based on a quantum measurement:

$$p_q(\psi, \phi) = \frac{1}{2} \left(2 - S_q(\psi, \phi) \right)$$

Relationships between overlap measures

- Classical overlap measures:

$$S_c(\psi, \phi) \leq A_c(\psi, \phi)$$

- Quantum overlap measures:
  - $S_q(\psi, \phi) = 1 - \sqrt{1 - |\langle\phi|\psi\rangle|^2}$
  - $S_q(\psi, \phi) \geq \frac{1}{2} |\langle\phi|\psi\rangle|^2$
- Hence:

$$\frac{S_c(\psi, \phi)}{S_q(\psi, \phi)} \leq 2 \frac{A_c(\psi, \phi)}{|\langle\phi|\psi\rangle|^2}.$$

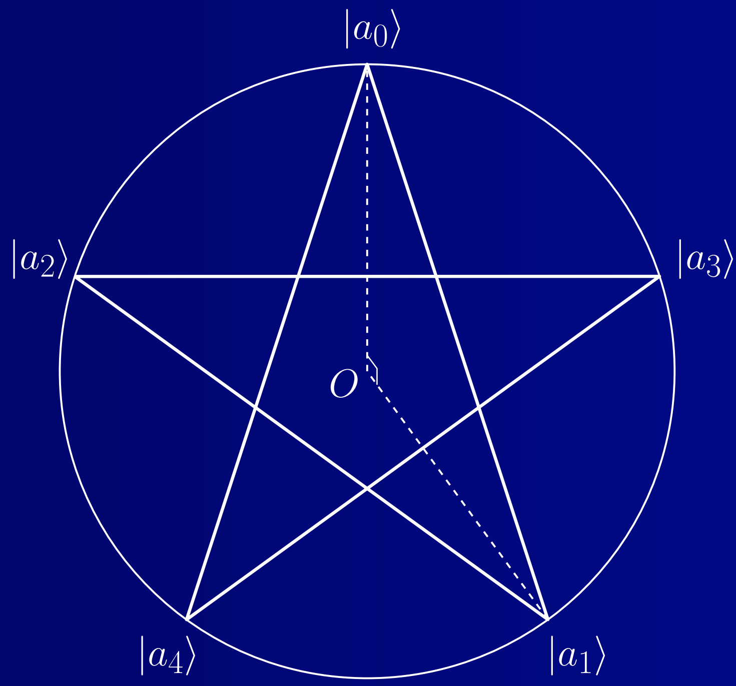
Overlap bounds

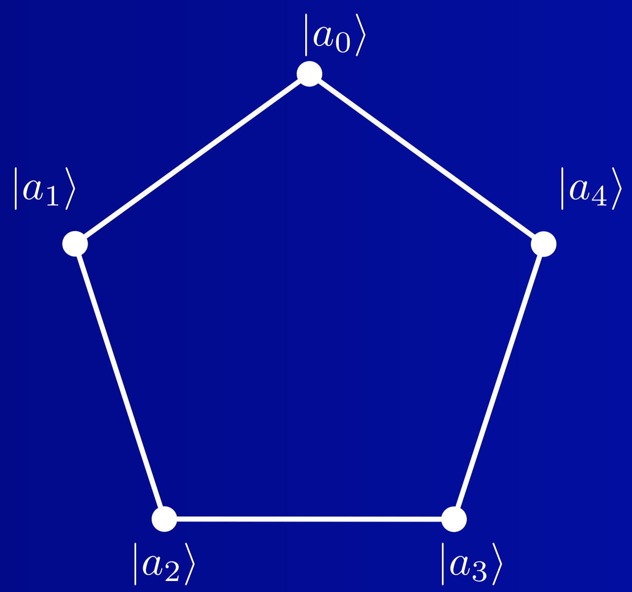
Previous results

- Define:
$$k(\psi, \phi) = \frac{A_c(\psi, \phi)}{|\langle\phi|\psi\rangle|^2}.$$
- Maroney showed $k(\psi, \phi) < 1$ for some states. ML and Maroney showed this follows from KS theorem.
- Barrett et. al. exhibited a family of states in $\mathbb{C}^d$ such that, for $d \geq 4$:
$$k(\psi, \phi) \leq \frac{4}{d-1}.$$
- Today: $k(\psi, \phi) \leq de^{-cd}$ for d divisible by 4.

Orthogonality graphs

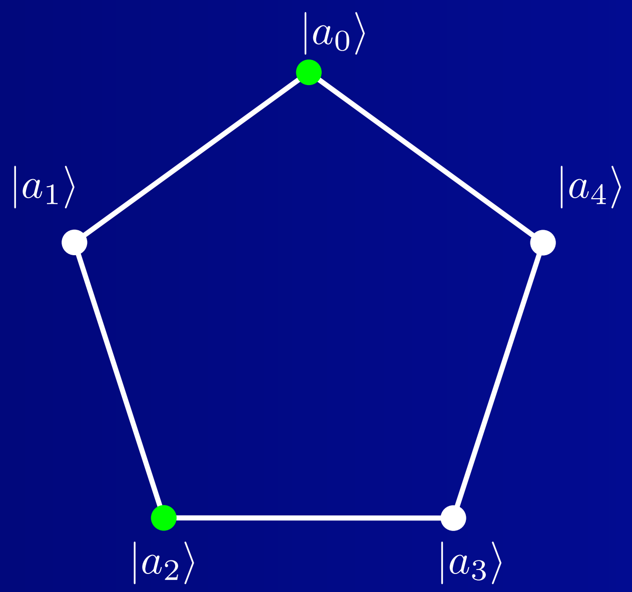
- Example: Klyachko states
  - $|a_j\rangle = \sin\vartheta\cos\varphi_j\,|0\rangle + \sin\vartheta\sin\varphi_j\,|1\rangle + \cos\vartheta\,|2\rangle$
  - $\varphi_j = \frac{4\pi j}{5}$ and $\cos\vartheta = \frac{1}{\sqrt[4]{5}}$





Independence number

- The *independence number* $\alpha(G)$ of a graph G is the cardinality of the largest subset of vertices such that no two vertices are connected by an edge.
- Example: $\alpha(G) = 2$



Main result

Theorem: Let V be a finite set of states in $\mathbb{C}^d$ an let $G = (V, E)$ be its orthogonality graph. For $|\psi\rangle \in \mathbb{C}^d$ define

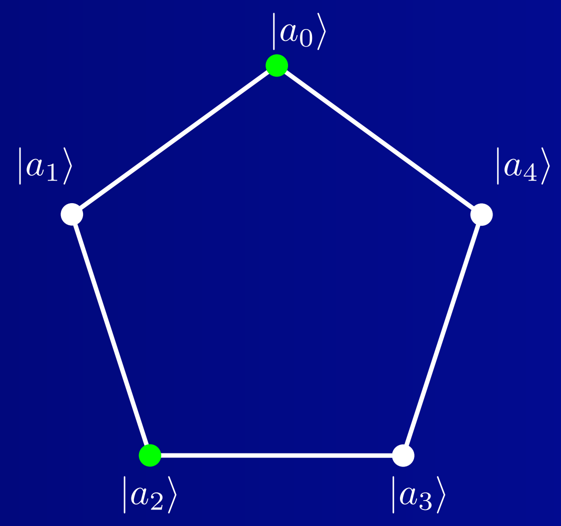
$$\bar{k}(\psi) = \frac{1}{|V|} \sum_{|a\rangle \in V} k(\psi, a).$$

Then, in any ontological model

$$\bar{k}(\psi) \leq \frac{\alpha(G)}{|V| \min_{|a\rangle \in V} |\langle a|\psi\rangle|^2}.$$

Bound from Klyatchko states

- $|a_j\rangle = \sin\vartheta\cos\varphi_j\,|0\rangle + \sin\vartheta\sin\varphi_j\,|1\rangle + \cos\vartheta\,|2\rangle$
- $\varphi_j = \frac{4\pi j}{5}$ and $\cos\vartheta = \frac{1}{\sqrt[4]{5}}$
- $|\psi\rangle = |2\rangle$



$$\bar{k}(\psi) \leq \frac{\alpha(G)}{5\min_j |\langle a_j|\psi\rangle|^2} = \frac{2}{5\times\frac{1}{\sqrt{5}}} \sim 0.8944$$

Exponential bound: Hadamard states

- For $\boldsymbol{x} = (x_0, x_1, \ldots, x_{d-1}) \in \{0,1\}^d$, let

$$|a_{\boldsymbol{x}}\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} (-1)^{x_j} |j\rangle .$$

- Let $|\psi\rangle = |0\rangle$.
- By Frankl-Rödl theorem[1], for d divisible by 4, there exists an $\epsilon > 0$ such that $\alpha(G) \leq (2-\epsilon)^d$.

$$\bar{k}(\psi) \leq \frac{\alpha(G)}{2^d \min_{\boldsymbol{x}\in\{0,1\}^d} |\langle a_{\boldsymbol{x}}|\psi\rangle|^2} = \frac{(2-\epsilon)^d}{2^d \times \frac{1}{d}} = de^{-cd}$$

$$c = \ln 2 - \ln(2-\epsilon)$$

[1] P. Frankl and V. Rödl, *Trans. Amer. Math. Soc.* 300:259 (1987)

The connection to contextuality

- An ontological model for a set of bases $\mathcal{M}$ is *Kochen-Specker noncontextual* if it is:
  - *Outcome deterministic*: $\xi_a^M(\lambda) \in \{0, 1\}$.
  - *Measurement noncontextual*: $\xi_a^M = \xi_a^N$.

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- In any ontological model $A_c(\psi, \phi) \leq \max \text{Prob}_{\text{N.C.}}(\phi|\psi, M)$
- Therefore, any KS contextuality inequality gives an overlap bound.

Conclusions

Summary and Open questions

- Summary
  - There exist pairs of states such that $k(\psi, \phi) \leq de^{-cd}$. The ψ-epistemic explanations of indistinguishability, no-cloning, etc. get implausible for these states very radpidly for large d.
  - Any contextuality inequality can be used to derive an overlap bound.
- Open questions
  - Error analysis.
  - Best bounds in small dimensions.
  - Bounds with a fixed inner product.
  - Connection to communication complexity.

What now for ψ-epistemicists?

- Become neo-Copenhagen.
- Adopt a more exotic ontology:
  - Nonstandard logics and probability theories.
  - Ironic many-worlds.
  - Retrocausality.
  - Relationalism.

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  - Retrocausality.
  - Relationalism.
- Principle of minimal weirdness: QM is weird but an interpretation of QM should not be more weird than it has to be.
  - Suggests exploring exotic ontologies.

References

- Review article:
  - ML, “Is the wavefunction real? A review of ψ-ontology theorems”, to appear.
- Connection to contextuality:
  - ML and O. Maroney, *Phys. Rev. Lett.* 110:120401 (2013) arXiv:1208.5132
- Exponential overlap bound:
  - ML arXiv:1401.7996 (2014)

Extra Slides

Main result

Theorem: Let V be a finite set of states in $\mathbb{C}^d$ an let $G = (V, E)$ be its orthogonality graph. For $|\psi\rangle \in \mathbb{C}^d$ define

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Proof of main result:1

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  - $\mu_a(\Gamma_a^{\mathcal{M}}) = 1$ also.
- Hence, $A_c(\psi, a) = \inf_{\{\Omega \in \Sigma | \mu_a(\Omega) = 1\}} \mu_\psi(\Omega) \leq \mu_\psi(\Gamma_a^{\mathcal{M}})$

Proof of main result:2

- $$A_c(\psi, a) \leq \mu_\psi(\Gamma_a^{\mathcal{M}})$$
$$\sum_{|a\rangle \in V} A_c(\psi, a) \leq \sum_{a \in V} \mu_\psi(\Gamma_a^{\mathcal{M}})$$

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  $$\chi_a(\lambda) = \begin{cases} 1, & \lambda \in \Gamma_a^{\mathcal{M}} \\ 0, & \lambda \notin \Gamma_a^{\mathcal{M}} \end{cases}$$

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- Then,
  $$\sum_{a \in V} \mu_\psi(\Gamma_a^{\mathcal{M}}) = \int_\Lambda \left[\sum_{a \in V} \chi_a(\lambda) \right] d\mu_\psi \leq \sup_{\lambda \in \Lambda} \left[\sum_{a \in V} \chi_a(\lambda) \right].$$

Proof of main result:3

- If $\langle a|b\rangle = 0$ then $\Gamma_a^M \cap \Gamma_b^M = \emptyset$ because $\xi_a^M(\lambda) + \xi_b^M(\lambda) \leq 1$.

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- Hence, if $\lambda \in \Gamma_a^{\mathcal{M}}$ then $\lambda \notin \Gamma_b^{\mathcal{M}}$ for any $|b\rangle \in V$ such that $(|a\rangle, |b\rangle) \in E$.

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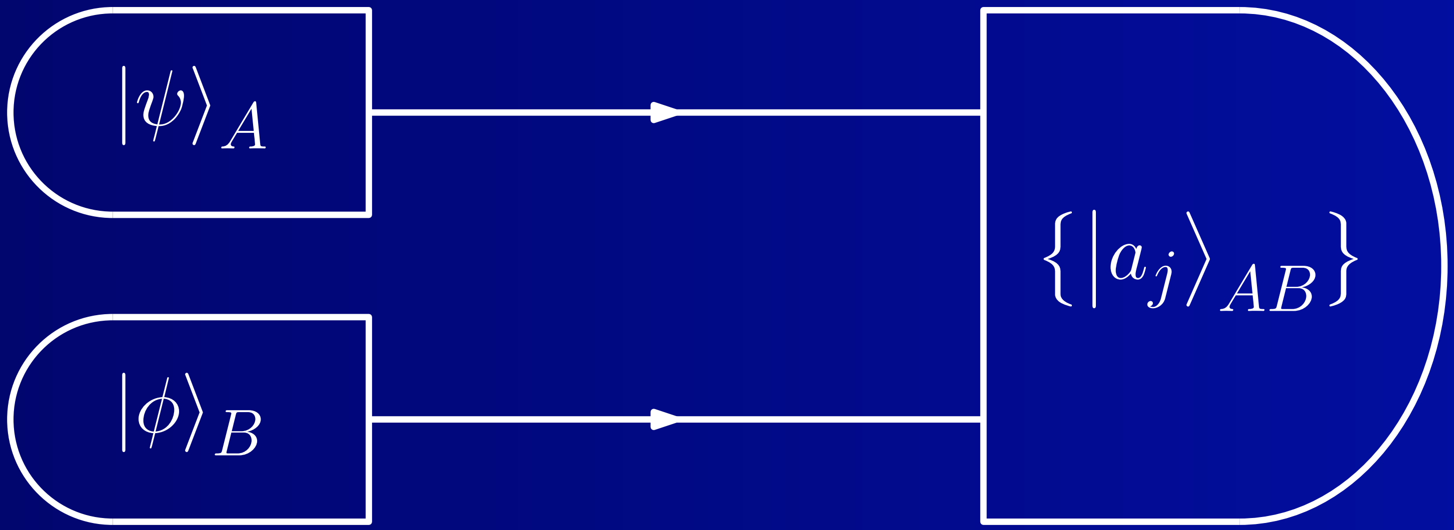
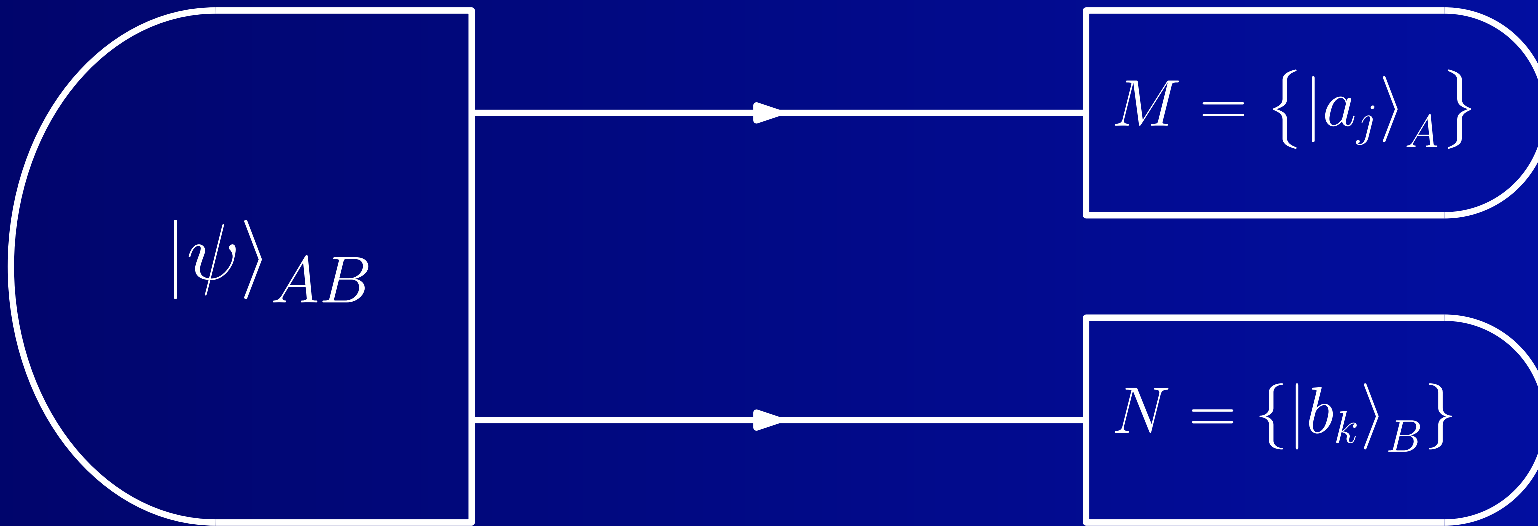
- Hence, if $\lambda \in \Gamma_a^{\mathcal{M}}$ then $\lambda \notin \Gamma_b^{\mathcal{M}}$ for any $|b\rangle \in V$ such that $(|a\rangle, |b\rangle) \in E$.
- Hence, $\sup_{\lambda \in \Lambda} \left[\sum_{a \in V} \chi_a(\lambda)\right] \leq \alpha(G)$.

The PBR Theorem



- The *Preparation Independence Postulate*:
  - $(\Lambda_{AB}, \Sigma_{AB}) = (\Lambda_A \times \Lambda_B, \Sigma_A \otimes \Sigma_B)$
  - $\mu_{AB} = \mu_A \times \mu_B$

The Colbeck-Renner Theorem



- *Parameter Independence*:
  - $P(a_j|M,N,\lambda) = P(a_j|M,\lambda)$
  - $P(b_k|M,N,\lambda) = P(b_k|N,\lambda)$