Formulating Quantum Theory as a Causally Neutral Theory of Bayesian Inference

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23rd June 2011

Outline

Topic

Quantum Theology

- The Church of the Larger Hilbert Space (J. Smolin)
  - Quantum theory is "about" a pure state vector of the universe that evolves unitarily.
  - Schrödinger, Everett, Zurek, . . .
- The Church of the Smaller Hilbert Space
  - Quantum theory is a noncommutative generalization of classical probability theory.
  - Heisenberg, von Neumann, . . .

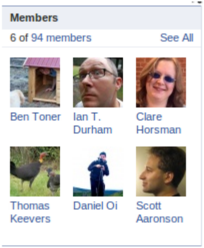
Quantum Theology on Facebook

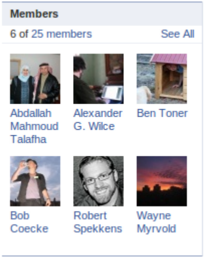
The Church of the Larger Hilbert Space Join

http://www.facebook.com/group.php?gid=5658946617

The Church of The Smaller Hilbert Space

http://www.facebook.com/group.php?gid=5965533115





A Problem for the Smaller Church

- Classical probability theory does not care about causality
  - $P(Z, W, \ldots)$
- Conventional quantum formalism does...

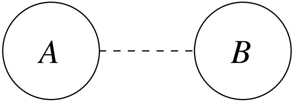


Figure: "Spacelike" correlations

ρ_{AB}

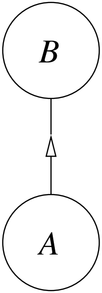


Figure: "Timelike" correlations

$\rho_B = \mathcal{E}_{B|A}(\rho_A)$

A note on notation

- Conventional Formalism: Hilbert spaces are attached to systems that persist in time.
  - States are a catalogue of probabilities for potential future measurement outcomes.
- Conditional States Formalism: Hilbert spaces are attached to systems at a specific time, or more generally to spacetime regions.
  - Always use a distinct label to distinguish input and output systems of a channel.
  - Always combine regions via the tensor product.
  - States are a catalogue of probabilities for any classical variables correlated with the region.

Classical vs. quantum Probability

Table: Basic definitions

Classical Probability	Quantum Theory
Sample space $\Omega_Z = \{1, 2, \ldots, d_Z\}$	Hilbert space $\mathcal{H}_A = \mathbb{C}^{d_A} = \text{span}\left(\|1\rangle, \|2\rangle, \ldots, \|d_A\rangle\right)$
Probability distribution $P(Z = z) \geq 0$, $\sum_{z \in \Omega_Z} P(Z = z) = 1$	Quantum state $\rho_A \in \mathfrak{L}^+\left(\mathcal{H}_A\right)$, $\text{Tr}_A\left(\rho_A\right) = 1$

Classical vs. quantum Probability

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Table: Composite systems

Classical Probability	Quantum Theory
Cartesian product $\Omega_{ZW} = \Omega_Z \times \Omega_W$	Tensor product $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$
Joint distribution $P(Z, W)$	Bipartite state ρ_{AB}
Marginal distribution $P(W) = \sum_{z \in \Omega_Z} P(Z = z, W)$	Reduced state $\rho_B = \text{Tr}_A (\rho_{AB})$
Conditional distribution $P(W\|Z) = \frac{P(Z,W)}{P(Z)}$	Conditional state $\rho_{B\|A} = ?$

Topic

Spatial correlations

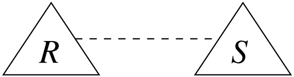

Figure: Classical correlations

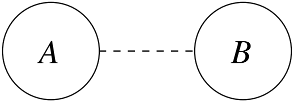

Figure: Quantum correlations

$$P(Z, W) = P(W|Z)P(Z)$$

$$\rho_{AB} = ?$$

Definition of Spatial QCS

Definition

A spatial quantum conditional state of B given A is a positive operator $\rho_{B|A}$ on $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$ that satisfies

$$\mathrm{Tr}_B\left(\rho_{B|A}\right) = I_A.$$

c.f. $P(W|Z)$ is a positive function on $\Omega_{ZW} = \Omega_Z \times \Omega_W$ that satisfies

$$\sum_{w \in \Omega_W} P(W = w|Z) = 1.$$

Relation to reduced and joint States

$$(\rho_A, \rho_{B|A}) \quad \rightarrow \quad \rho_{AB} = (\sqrt{\rho_A} \otimes I_B)\, \rho_{B|A}\, (\sqrt{\rho_A} \otimes I_B)$$

$$\rho_{AB} \quad \rightarrow \quad \rho_A = \text{Tr}_B\,(\rho_{AB})$$

$$\rho_{B|A} = \left(\sqrt{\rho_A^{-1}} \otimes I_B\right) \rho_{AB} \left(\sqrt{\rho_A^{-1}} \otimes I_B\right)$$

Relation to reduced and joint States

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$$\rho_{B|A} = \left(\sqrt{\rho_A^{-1}} \otimes I_B\right) \rho_{AB} \left(\sqrt{\rho_A^{-1}} \otimes I_B\right)$$

Note: $\rho_{B|A}$ defined from ρ_{AB} is a QCS on supp$(\rho_A) \otimes \mathcal{H}_B$.

Relation to reduced and joint States

Table: Comparison of relations between joints, conditionals and marginals

Classical Probability	Quantum Theory
$P(Z,W) = P(W\|Z)P(Z)$	$\rho_{AB} = \left(\sqrt{\rho_A} \otimes I_B\right) \rho_{B\|A} \left(\sqrt{\rho_A} \otimes I_B\right)$
$P(W\|Z) = \frac{P(Z,W)}{P(Z)}$	$\rho_{B\|A} = \left(\sqrt{\rho_A^{-1}} \otimes I_B\right) \rho_{AB} \left(\sqrt{\rho_A^{-1}} \otimes I_B\right)$

Notation

- Drop implied identity operators, e.g.
  - $I_A \otimes M_{BC} N_{AB} \otimes I_C \quad \rightarrow \quad M_{BC} N_{AB}$
  - $M_A \otimes I_B = N_{AB} \quad \rightarrow \quad M_A = N_{AB}$
- Define non-associative "product"
  - $M \star N = \sqrt{N} M \sqrt{N}$

Relation to reduced and joint States

Table: Comparison of relations between joints, conditionals and marginals

Classical Probability	Quantum Theory
$P(Z, W) = P(W \vert Z)P(Z)$	$\rho_{AB} = \left(\sqrt{\rho_A} \otimes I_B\right) \rho_{B\vert A} \left(\sqrt{\rho_A} \otimes I_B\right)$
$P(W \vert Z) = \frac{P(Z,W)}{P(Z)}$	$\rho_{B\vert A} = \left(\sqrt{\rho_A^{-1}} \otimes I_B\right) \rho_{AB} \left(\sqrt{\rho_A^{-1}} \otimes I_B\right)$

Relation to reduced and joint states

Table: Comparison of relations between joints, conditionals and marginals

Classical Probability	Quantum Theory
$P(Z,W) = P(W\|Z)P(Z)$	$\rho_{AB} = \rho_{B\|A} \star \rho_A$
$P(W\|Z) = \frac{P(Z,W)}{P(Z)}$	$\rho_{B\|A} = \rho_{AB} \star \rho_A^{-1}$

Classical Probability as a Special Case

- Given a classical variable Z, define a Hilbert space $\mathcal{H}_Z$ with a preferred basis $\{|1\rangle_Z, |2\rangle_Z, \ldots, |d_Z\rangle_Z\}$ labeled by elements of Ω_Z. Then,

$$\rho_Z = \sum_{z \in \Omega_Z} P(Z = z) \, |z\rangle \langle z|_Z$$

- Similarly,

$$\rho_{ZW} = \sum_{z \in \Omega_Z, w \in \Omega_W} P(Z = z, W = w) \, |zw\rangle \langle zw|_{ZW}$$

$$\rho_{W|Z} = \sum_{z \in \Omega_Z, w \in \Omega_W} P(W = w | Z = z) \, |zw\rangle \langle zw|_{ZW}$$

Topic

Correlations between subsystems

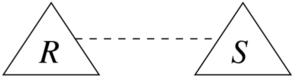

Figure: Classical correlations

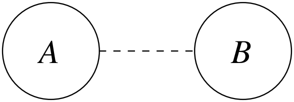

Figure: Quantum correlations

$$P(Z, W) = P(W|Z)P(Z)$$

$$\rho_{AB} = \rho_{B|A} \star \rho_A$$

Dynamics

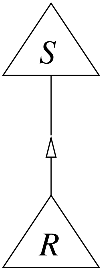

Figure: Classical stochastic map

$$P(W) = \Gamma_{W|Z}\left(P(Z)\right) = \sum_Z P(W|Z)P(Z)$$

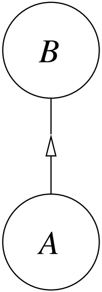

Figure: Quantum CPT map

$$\rho_B = \mathcal{E}_{B|A}(\rho_A) = \mathrm{Tr}_A\left(\rho_{B|A} \star \rho_A\right)\ ?$$

Temporal Conditionals and Joints

Theorem (Jamiołkowski isomorphism)

Let $\mathcal{E}_{B|A} : \mathfrak{L}(\mathcal{H}_A) \to \mathfrak{L}(\mathcal{H}_B)$ *be a CPT map and define*

$$\varrho_{B|A} = \mathcal{E}_{B|A'} \otimes \mathcal{I}_A \left(\left| \Phi^+ \right\rangle \left\langle \Phi^+ \right|_{A'A}^{T_A} \right),$$

where $|\Phi^+\rangle_{A'A} = \sum_j |jj\rangle_{A'A}$. *Then,*

$$\mathcal{E}_{B|A}(\rho_A) = \mathrm{Tr}_A \left(\varrho_{B|A} \star \rho_A \right).$$

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$$\mathcal{E}_{B|A}(\rho_A) = \mathrm{Tr}_A \left(\varrho_{B|A} \star \rho_A \right).$$

- $\varrho_{B|A} = \rho_{B|A}^{T_A}$ for some spatial QCS $\rho_{B|A}$. Call such operators temporal conditional states.

Temporal Conditionals and Joints

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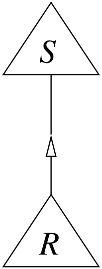
- $\varrho_{B|A} = \rho_{B|A}^{T_A}$ for some spatial QCS $\rho_{B|A}$. Call such operators temporal conditional states.
- Can also define temporal joint state: $\varrho_{AB} = \varrho_{B|A} \star \rho_A$.

Dynamics



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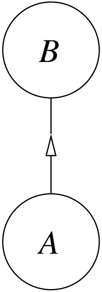

Figure: Quantum CPT map

$$\rho_B = \mathcal{E}_{B|A}\left(\rho_A\right) = \text{Tr}_A\left(\varrho_{B|A} \star \rho_A\right)$$

Topic

Quantum Classical Hybrids

- Composite of a quantum region and a classical random variable.
- Classical r.v. X has Hilbert space $\mathcal{H}_X$ with preferred basis $\{|1\rangle_X, |2\rangle_X, \ldots, |d_X\rangle_X\}$.
- Quantum region A has Hilbert space $\mathcal{H}_A$.
- Hybrid has Hilbert space $\mathcal{H}_{XA} = \mathcal{H}_X \otimes \mathcal{H}_A$

Quantum Classical Hybrids

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- Classical r.v. X has Hilbert space $\mathcal{H}_X$ with preferred basis $\{|1\rangle_X, |2\rangle_X, \ldots, |d_X\rangle_X\}$.
- Quantum region A has Hilbert space $\mathcal{H}_A$.
- Hybrid has Hilbert space $\mathcal{H}_{XA} = \mathcal{H}_X \otimes \mathcal{H}_A$
- Operators on $\mathcal{H}_{XA}$ restricted to be of the form

$$M_{XA} = \sum_{x \in \Omega_X} |x\rangle \langle x|_X \otimes M_{X=x,A}$$

Preparations

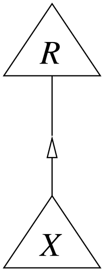


Figure: Classical preparation

$$P(Z) = \sum_X P(Z|X)P(X)$$

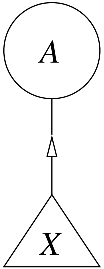


Figure: Quantum preparation

$$\rho_A = \sum_x P(X = x)\rho_A^{(x)}$$

$$\rho_A = \mathrm{Tr}_X \left(\varrho_{A|X} \star \rho_X\right)$$

Measurements

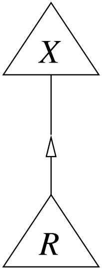


Figure: Noisy measurement

$$P(Y) = \sum_Z P(Y|Z)P(Z)$$

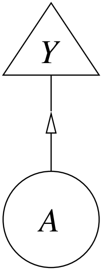


Figure: POVM measurement

$$P(Y = y) = \mathrm{Tr}_A\left(E_A^{(y)}\rho_A\right)$$

$$\rho_Y = \mathrm{Tr}_A\left(\varrho_{Y|A} \star \rho_A\right)$$

State Update rules

Definition (Quantum Instrument)

A quantum instrument is a set of CP-maps $\mathcal{E}_{B|A}^{(y)}$ such that the operators $E_A^{(y)}$ form a POVM, where

$$E_A^{(y)} = \left(\mathcal{E}_{B|A}^{(y)}\right)^{\dagger}(I_B).$$

- On obtaining y in a measurement of $E_A^{(y)}$:

$$\rho_A \to \rho_B^{(y)} = \frac{\mathcal{E}_{B|A}^{(y)}(\rho_A)}{\text{Tr}_A\left(E_A^{(y)}\rho_A\right)}$$

- The projection postulate is the special case:

$$\mathcal{E}_{B|A}^{(y)}(\rho_A) = \mathcal{I}_{B|A}\left(\Pi_A^{(y)}\rho_A\Pi_A^{(y)}\right)$$

Instruments (Update Rules)

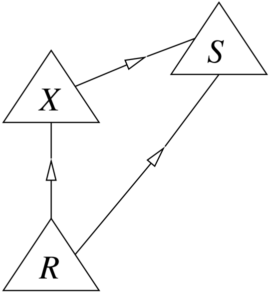


Figure: Classical instrument

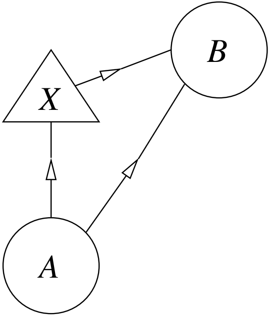


Figure: Quantum instrument

$$P(Y, W) = \sum_{Z} P(Y, W|Z) P(Z)$$

$$P(Y|Z) = \sum_{W} P(Y, W|Z)$$

$$\rho_{YB} = \mathrm{Tr}_A \left(\varrho_{YB|A} \star \rho_A \right)$$

$$\varrho_{Y|A} = \mathrm{Tr}_B \left(\varrho_{YB|A} \right)$$

Comparison of notation

Dynamics	$\rho_B = \mathcal{E}_{B\|A}(\rho_A)$	$\rho_B = \text{Tr}_A(\varrho_{B\|A} \star \rho_A)$
Preparation	$\rho_A = \sum_x P(X = x)\rho_A^{(x)}$	$\rho_A = \text{Tr}_X(\varrho_{A\|X} \star \rho_X)$
Measurement	$P(Y = y) = \text{Tr}_A(E^{(y)}\rho_A)$	$\rho_Y = \text{Tr}_A(\varrho_{Y\|A} \star \rho_A)$
Update	$P(Y = y)\rho_B^{(y)} = \mathcal{E}_{B\|A}^{(y)}(\rho_A)$	$\rho_{YB} = \text{Tr}_A(\varrho_{YB\|A} \star \rho_A)$

Topic

Classical Bayes' rule

- Two expressions for joint probabilities:

$$P(Z, W) = P(W|Z)P(Z) = P(Z|W)P(W)$$

- Bayes' rule:

$$P(Z|W) = \frac{P(W|Z)P(Z)}{P(W)}$$

- Alternative form of Bayes' rule:

$$P(Z|W) = \frac{P(W|Z)P(Z)}{\sum_Z P(W|Z)P(Z)}$$

Spatial Quantum Bayes' rule

- Two expressions for bipartite states:

$$\rho_{AB} = \rho_{B|A} \star \rho_A$$
$$= \rho_{A|B} \star \rho_B$$

- Bayes' rule:

$$\rho_{A|B} = \rho_{B|A} \star \left(\rho_A \otimes \rho_B^{-1} \right)$$

- Alternative form of Bayes' rule

$$\rho_{A|B} = \rho_{B|A} \star \left(\rho_A \otimes \mathrm{Tr}_A \left(\rho_{B|A} \star \rho_A \right)^{-1} \right)$$

Temporal Quantum Bayes' rule

- Given an input state ρ_A and a temporal QCS $\varrho_{B|A}$, define

$$\varrho_{A|B} = \varrho_{B|A} \star \left(\rho_A \otimes \mathrm{Tr}_B\left(\varrho_{B|A} \star \rho_A\right)\right)$$

- Temporal joint state now has two decompositions:

$$\varrho_{AB} = \varrho_{B|A} \star \rho_A = \varrho_{A|B} \star \rho_B$$

State/POVM duality

- A temporal hybrid joint state can be written two ways:

$$\varrho_{XA} = \varrho_{A|X} \star \rho_X = \varrho_{X|A} \star \rho_A$$

- The two representations are connected via Bayes' rule:

$$\varrho_{X|A} = \varrho_{A|X} \star \left(\rho_X \otimes \mathrm{Tr}_X \left(\varrho_{A|X} \star \rho_X\right)^{-1}\right)$$
$$\varrho_{A|X} = \varrho_{X|A} \star \left(\mathrm{Tr}_A \left(\varrho_{X|A} \star \rho_A\right)^{-1} \otimes \rho_A\right)$$

$$\varrho_{X=x|A} = \frac{P(X=x)\varrho_{A|X=x}}{\sum_{x' \in \Omega_X} P(X=x')\varrho_{A|X=x'}} \qquad \varrho_{A|X=x} = \frac{\sqrt{\rho_A}\varrho_{X=x|A}\sqrt{\rho_A}}{\mathrm{Tr}_A\left(\varrho_{X=x|A}\rho_A\right)}$$

Topic

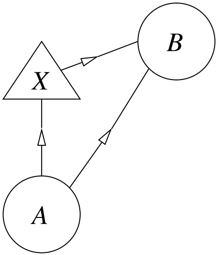
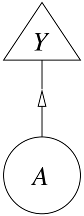
Quantum (Hybrid) Bayesian Conditioning

- Classically, upon learning $X = x$:

$$P(Z) \to P(Z|X = x)$$

- Quantumly: $\rho_A \to \varrho_{A|X=x}$?

Projection postulate vs. Bayes' rule

Projection Postulate	Bayesian Conditioning
$\rho_A \to \frac{\mathcal{I}_{B\|A}\left(\sqrt{E_A^{(y)}}\rho_A\sqrt{E_A^{(y)}}\right)}{\text{Tr}_A\left(E_A^{(y)}\rho_A\right)}$	$\rho_A \to \frac{\sqrt{\rho_A}E_A^{(y)}\sqrt{\rho_A}}{\text{Tr}_A\left(E_A^{(y)}\rho_A\right)}$
	

Conditioning on a Preparation Variable

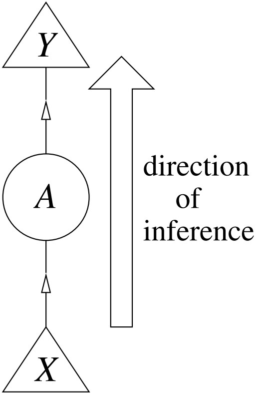


Figure: Prep. & meas. experiment

- Joint probability:

$$\varrho_{XY} = \text{Tr}_A \left(\varrho_{Y|A} \star \left(\varrho_{A|X} \star \rho_X \right) \right)$$

- Marginal for Y:

$$\rho_Y = \text{Tr}_A \left(\varrho_{Y|A} \star \rho_A \right)$$

- Conditional probabilities:

$$\varrho_{Y|X} = \text{Tr}_A \left(\varrho_{Y|A} \star \varrho_{A|X} \right)$$

- Bayesian update:

$$\rho_A \to \varrho_{A|X=x}$$

Conditioning on a Direct Measurement

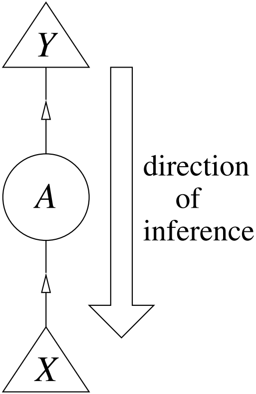


Figure: Prep. & meas. experiment

- Apply Bayes' rule to $\varrho_{A|X}$ and $\varrho_{Y|A}$:

$$\rho_{XY} = \text{Tr}_A\left(\varrho_{X|A} \star \left(\varrho_{A|Y} \star \rho_Y\right)\right)$$

- Marginal for X:

$$\rho_X = \text{Tr}_A\left(\varrho_{X|A} \star \rho_A\right)$$

- Conditional probabilities:

$$\varrho_{X|Y} = \text{Tr}_A\left(\varrho_{X|A} \star \varrho_{A|Y}\right)$$

- Bayesian update:

$$\rho_A \rightarrow \varrho_{A|Y=y}$$

- c.f. Barnett, Pegg & Jeffers, J. Mod. Opt. 47:1779 (2000).

Conditioning on a Remote Measurement

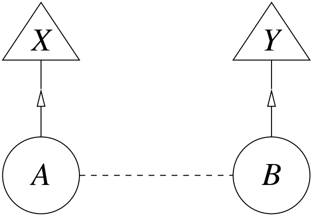


Figure: Bipartite experiment

- Joint probability: $\rho_{XY} = \text{Tr}_{AB}\left(\left(\varrho_{X|A} \otimes \varrho_{Y|B}\right) \star \rho_{AB}\right)$
- B can be factored out: $\rho_{XY} = \text{Tr}_{A}\left(\varrho_{Y|A} \star \left(\varrho_{A|X} \star \rho_{X}\right)\right)$
- where $\varrho_{Y|A} = \text{Tr}_{B}\left(\varrho_{Y|B}\rho_{B|A}\right)$

Topic

Expert Advice

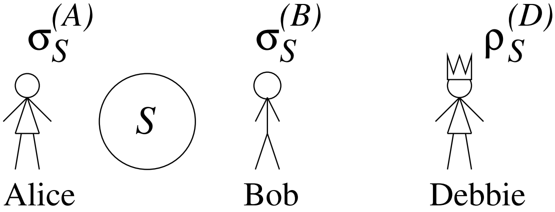


Figure: Initial State Assignments

Expert Advice

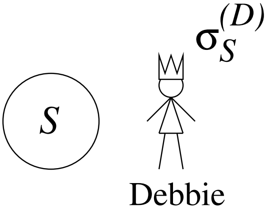


Figure: Final State Assignment

$$\sigma_S^{(D)} = f(\sigma_S^{(A)}, \sigma_S^{(B)}, \rho_S^{(D)})$$

Classical Expert Advice

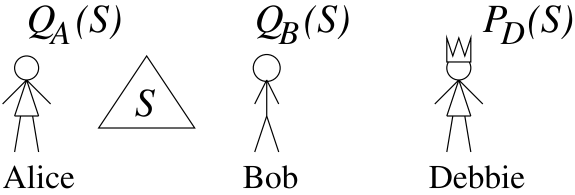


Figure: Initial State Assignments

Classical Expert Advice

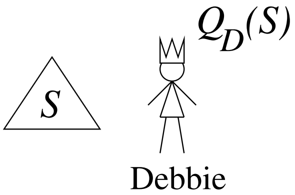


Figure: Final State Assignment

$$Q_D(S) = f(Q_A(S), Q_B(S), P_D(S))$$

Diplomatic Rule

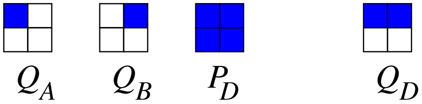


Figure: Combining Incompatible Assignments

- Linear pool:

$$Q_D(S) = w_A Q_A(S) + w_B Q_B(S) + w_D P_D(S)$$

Scientific Rule

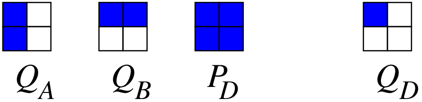


Figure: Combining Independent Evidence

- Multiplicative (log-linear) pool:

$$Q_D(S) \propto \frac{Q_A(S)Q_B(S)}{P_D(S)}$$

Bayesian Approach

- Bayesian inference says that:

$$Q_D(S) = P_D(S|R_A = Q_A(S), R_B = Q_B(S))$$
$$= \frac{P_D(R_A = Q_A(S), R_B = Q_B(S)|S)P_D(S)}{\sum_S P_D(R_A = Q_A(S), R_B = Q_B(S)|S)P_D(S)}$$

- Similarly:

$$\sigma_S^{(D)} = \rho^{(D)}_{S|R_A=\sigma_S^{(A)},R_B=\sigma_S^{(B)}}$$
$$= \rho^{(D)}_{R_A=\sigma_S^{(A)},R_B=\sigma_S^{(B)}|S} \star \left(\rho_S^{(D)} \otimes \mathrm{Tr}_S\left(\rho^{(D)}_{R_A=\sigma_S^{(A)},R_B=\sigma_S^{(B)}|S} \star \rho_S^{(D)}\right)\right)$$

Quantum conditional independence

- Quantum Chain Rule:

$$\rho_{ABC} = \rho_{C|AB} \star \left(\rho_{B|A} \star \rho_A\right)$$

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If $\rho_{C|AB} = \rho_{C|B}$ then C is conditionally independent of A given B.

Quantum conditional independence

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Theorem

The following conditions are equivalent:

- $\rho_{C|AB} = \rho_{C|B}$
- $\rho_{A|BC} = \rho_{A|B}$
- $I(A : C|B) = 0.$

Further, conditional independence implies that

- $\rho_{AC|B} = \rho_{A|B}\rho_{C|B}.$

Shared priors

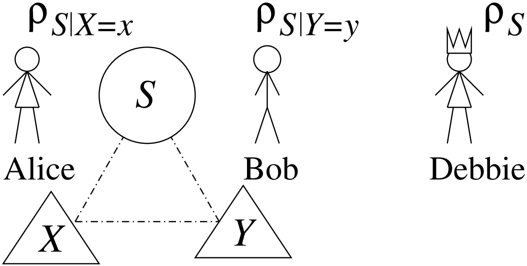

Figure: The Case of Shared Priors

Theorem

If X and Y are conditionally independent given S then

$$\rho^{(D)}_{S|R_A=\rho_{S|X=x},R_B=\rho_{S|Y=y}} \propto \rho_{S|X=x}\rho_S^{-1}\rho_{S|Y=y}$$

Shared Priors

UCL

Theorem (Stronger Version)

If the minimal sufficient statistics for X and Y with respect to S are conditionally independent given S then

$$\rho^{(D)}_{S|R_A=\rho_{S|X=x},R_B=\rho_{S|Y=y}} \propto \rho_{S|X=x}\rho_S^{-1}\rho_{S|Y=y}$$

Topic

Further results

Forthcoming paper(s) with R. W. Spekkens also include:

- Quantum sufficient statistics
- Quantum state compatibility
- Quantum pooling

Earlier papers with related ideas:

- M. Asorey et. al., Open.Syst.Info.Dyn. 12:319–329 (2006).
- M. S. Leifer, Phys. Rev. A 74:042310 (2006).
- M. S. Leifer, AIP Conference Proceedings 889:172–186 (2007).
- M. S. Leifer & D. Poulin, Ann. Phys. 323:1899 (2008).

Open question

What is the meaning of fully quantum Bayesian conditioning?

$$\rho_B \to \rho_{B|A} = \rho_{A|B} \star \left(\mathrm{Tr}_B \left(\rho_{A|B} \star \rho_B \right)^{-1} \otimes \rho_B \right)$$

Thanks for your attention!

People who gave me money

- Foundational Questions Institute (FQXi) Grant RFP1-06-006

People who gave me office space when I didn't have any money

- Perimeter Institute
- University College London