Ontological Models in the Block Universe

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Introduction

Explanatory Gaps

- No-go theorems expose explanatory gaps in realist models for quantum theory.
  - *Bell's theorem*: There are nonlocal influences, but these cannot be used for signalling.
  - *Contextuality*: There are distinctions that do not make a difference.
  - *Excess baggage*: A qubit contains an infinite amount of information, but only one bit can be extracted.
  - *Reality of the wavefunction*: Many quantum phenomena are best explained if the wavefunction is epistemic, but nonetheless it must be real.
  - Operationally time symmetric experiments cannot have a time symmetric model (Matt Pusey's talk).
- Conclusion: Realist models require fine-tuning.

Possible Responses to the Gaps

- Reject realism: adopt a neo-Copenhagen approach.
- Bite the bullet:
  - Accept as a brute fact that these things exist and will remain fundamentally hidden.
  - Conjecture that these effects will be explicitly observed in the future (e.g. Valentini's approach to Bohmian mechanics).
- Reject one or more of the (perhaps implicit) assumptions in the realist frameworks used to prove these theorems.
  - Retrocausality is an obvious starting point.

Why can't I win the lottery?

- Retrocausality opens its own explanatory gap: Why can't I signal into the past?
- Two possible responses:
  - Yes, but now we have one gap rather than several.
  - Apparent retrocausality is not fundamental, but emergent from a more fundamental "block universe" theory, which has no a priori causality. We may hope that no signalling into the past emerges naturally from this.

Huw Price on Bell's theorem

- Huw Price states the logic of Bell's theorem as:

$$\text{QM} + \text{Locality} \Rightarrow \text{Retrocausality}$$

  - In Bell's framework, "free will", no superdeterminism, and no retrocausality are expressed by the same assumption, i.e. *measurement independence*.
  - Bell locality is not compelling in the presence of retrocausality. Need a more general definition.
  - As with Bell's framework, needs to be independent of the details of quantum theory, in order to support general conclusions.

Other Motivations

- Existing theories that seem retrocausal; e.g. TSVF, transactional interpretation, Wharton's models; do not *prove* retrocausality. The mathematics is merely suggestive.
  - Any physical theory has multiple mathematical formalisms. It would be wrong to draw conclusions about causality in classical physics from the Lagrangian formalism for example (c.f. FISH).
  - So our arguments should rely only on the *operational* predictions of the theory, i.e. the stuff that all formalisms must agree upon.
  - Counter Hypothesis of Instrumental Prediction Symmetry.
- Also need a framework to develop toy theories that illustrate how aspects of quantum theory can be accounted for by retrocausality, e.g. Helsinki model.
  - Hopefully, that framework will also contain a viable approach to all of quantum theory.

Overview of This Talk

Introduction

Operational Circuit Models

Block Universe Ontological Models

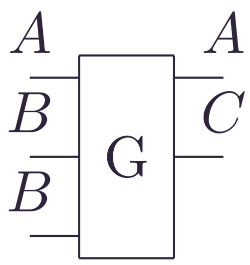
Locality

Retrocausality

Conclusions

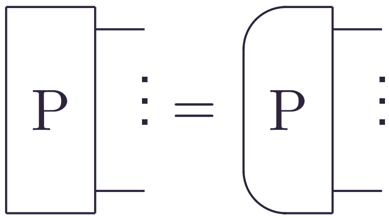
Operational Circuit Models

- The basic element of a circuit model is a *gate* G. A gate has a number of *input wires* and *output wires*, each with a *system type label*.

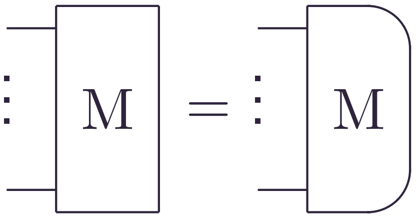


- Each gate is associated with a random variable X_{G} taking a finite number of possible values.
- If X_{G} only has one possible value then the gate is a *transformation* and we can ignore X_{G}.
- Let I_{G} and O_{G} denote the set of input and output wires of G respectively, and $W_{\mathrm{G}} = I_{\mathrm{G}} \cup O_{\mathrm{G}}$.

- A transformation with no input wires is a *preparation*. A gate with no input wires is an *ensemble preparation*.



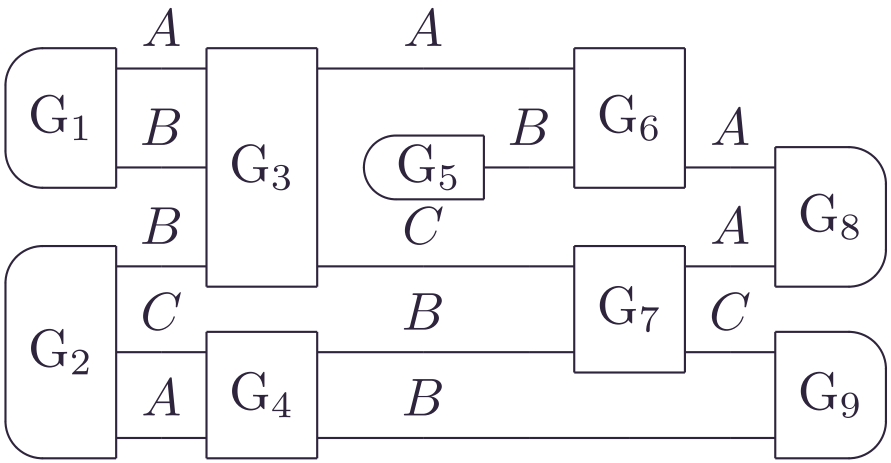
- A transformation with no output wires is a *unit* corresponding to throwing the system away. A gate with no output wires is a (destructive) *measurement*.



Example: Quantum Theory

Circuit Framework		Quantum Case	
System type	A	Hilbert space	$\mathcal{H}_A$
Input wires	I_{G}	Input space	$\mathcal{H}_{I_{\mathrm{G}}} = \bigotimes_{A \in I_{\mathrm{G}}} \mathcal{H}_A$
Output wires	O_{G}	Output space	$\mathcal{H}_{O_{\mathrm{G}}} = \bigotimes_{A \in O_{\mathrm{G}}} \mathcal{H}_A$
Preparation	P	State	$\rho_{O_{\mathrm{P}}}$
Ensemble Preparation	$\mathrm{P}, X_{\mathrm{P}}$	Ensemble of states	$\rho^x_{O_{\mathrm{P}}}$, Prob(X_{P})
Unit	M	Trace	$\mathrm{Tr}_{I_{\mathrm{M}}}(\cdot)$
Measurement	$\mathrm{M}, X_{\mathrm{M}}$	POVM	$\mathrm{Tr}_{I_{\mathrm{M}}}\left(E^x_{I_{\mathrm{M}}}(\cdot)\right)$
Transformation	G	CPT map	$\mathcal{G} : \mathcal{L}(\mathcal{H}_{I_{\mathrm{G}}}) \to \mathcal{L}(\mathcal{H}_{O_{\mathrm{G}}})$
Gate	$\mathrm{G}, X_{\mathrm{G}}$	Instrument	$\mathcal{G}^x : \mathcal{L}(\mathcal{H}_{I_{\mathrm{G}}}) \to \mathcal{L}(\mathcal{H}_{O_{\mathrm{G}}})$

- Let $\mathfrak{G} = (\mathrm{G}_1, \mathrm{G}_2, \ldots)$ be a tuple of gates.
- Denote X_{G_j} by X_j. Similarly for, I_j, O_j and W_j.
- A *wiring* of $\mathfrak{G}$ is an identification of the output wires of each gate with the input wires of others so that the system type labels match, there are no dangling wires, and no causal loops.

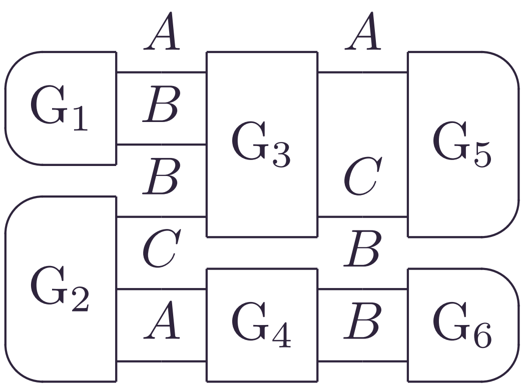


- A *circuit* $\boldsymbol{C}$ is a tuple of gates $\mathfrak{G}_{\boldsymbol{C}}$ together with a wiring. $W_{\boldsymbol{C}}$ denotes the set of wires of $\boldsymbol{C}$. Let $X_{\boldsymbol{C}} = (X_1, X_2, \ldots)$.

- An *operational circuit model* consists of a set of gates and, for every circuit C that can be formed from tuples of those gates, a joint probability distribution

$$\text{Prob}(X_C|\mathfrak{G}_C) = \text{Prob}(X_1, X_2, \ldots |\mathfrak{G}_C).$$

- Quantum example:



$$\text{Prob}(X_1 = x_1, X_2 = x_2, X_3 = x_3, X_4 = x_4, X_5 = x_5, X_6 = x_6|\mathfrak{G}_C)$$
$$= \text{Tr}_{I_5 I_6}\left(E^{x_5}_{I_5} \otimes E^{x_6}_{I_6}\mathcal{G}^{x_4}_4\left(\mathcal{G}^{x_3}_3\left(\rho^{x_1}_{O_1} \otimes \rho^{x_2}_{O_2}\right)\right)\right)$$
$$\times \text{Prob}(X_1 = x_1)\text{Prob}(X_2 = x_2)$$

- We should impose additional causality constraints, e.g. no *operational* signalling into the past.

Block Universe Ontological Models

Block Universe Ontological Models

- Two circuits are of the same *type* if their wirings and system type labels are the same.
- An *ontological model* for an operational circuit model specifies, for each circuit type:
  - A probability space (Λ, Σ, μ) where
    - Λ is the *ontic state space*.
    - μ is some notion of the "uniform" measure.
  - A rule for computing the outcome probabilities in terms of μ that reproduces the operational predictions.
- We consider two classes of model:
  - *Constraint models*
  - *Probabilistic constraint models*

- In a constraint model, each circuit $\boldsymbol{C}$ is associated with a constraint (relation) $\Gamma \subseteq \Lambda$.
- Let $x = (x_1, x_2, \ldots)$. $X_{\boldsymbol{C}} = x$ denotes $(X_1 = x_1, X_2 = x_2, \ldots)$.
- Each tuple of values x for the random variables is associated with a constraint $\Gamma_x \subseteq \Gamma$ such that, for $x \neq x'$,

$$\Gamma_x \cap \Gamma_{x'} = \emptyset.$$

and

$$\cup_x \Gamma_x = \Gamma.$$

- The probability rule is then given by

$$P(X_{\boldsymbol{C}} = x | \mathfrak{G}_{\boldsymbol{C}}) = \int_\Lambda P(X_{\boldsymbol{C}} = x | \mathfrak{G}_{\boldsymbol{C}}, \lambda) d\mu(\lambda | \mathfrak{G}_{\boldsymbol{C}})$$
$$= \int_\Lambda \chi_{\Gamma_x}(\lambda) \frac{\chi_\Gamma(\lambda) d\mu(\lambda)}{\mu(\Gamma)} = \frac{\mu(\Gamma_x)}{\mu(\Gamma)}.$$

Probabilistic Constraint Models

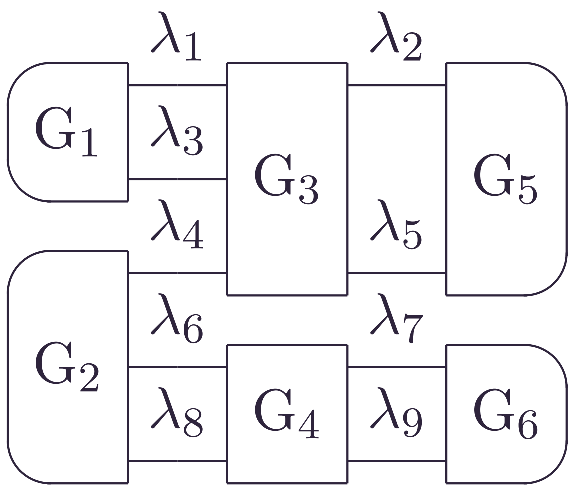
- In a probabilistic constraint model, the constraint associated with a circuit is not fixed, but drawn according to a probability measure.
- Finite example: The circuit C is associated with constraints $(\Gamma^{(j)}, \Gamma_x^{(j)})$ with probability p_j. Then,

$$P(X_{\mathfrak{C}} = x) = \sum_j p_j \frac{\mu(\Gamma_x^{(j)})}{\mu(\Gamma^{(j)})}.$$

Locality

- Let W_C be the set of wires in a circuit C. A model is *separable* if

$$\Lambda = \times_{w \in W_C} \Lambda_w .$$



- Typically, we want Λ_w to only depend on the system type label of the wire.

- Note: Separability is not essential for defining locality, but it simplifies matters to assume it.

- In a separable constraint model, let $\Lambda^j = \times_{w \in W_j} \Lambda_w$.
- A separable constraint model is *local* if, for a circuit $\boldsymbol{C}$, each gate G_j is associated with constraints $(\Gamma^j, \Gamma^j_{x_j})$ defined on Λ^j, i.e.

$$\Gamma^j \subseteq \Lambda^j, \qquad \Gamma^j_{x_j} \cap \Gamma^j_{x'_j} = \emptyset \qquad \text{and} \qquad \cup_{x_j} \Gamma^j_{x_j} = \Gamma^j.$$

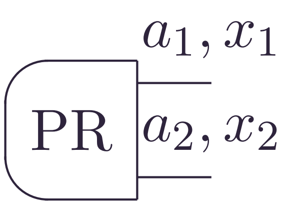
- The circuit constraints are then of the form

$$\Gamma_x = \cap_j \left(\Gamma^j_{x_j} \times_{w \notin W_j} \Lambda_w \right) \qquad \Gamma = \cap_j \left(\Gamma^j \times_{w \notin W_j} \Lambda_w \right).$$

- For a probabilistic constraint model, the constraints are drawn independently for each gate according to a local probability measure.

PR-Box Model

- To model the PR-Box, there is one system type. Each wire has ontic state space $\{0,1\} \times \{0,1\}$. Label the first bit a and the second x.
- μ is the counting measure.
- There is one preparation gate with two output wires



- Its constraint is $x_1 \oplus x_2 = a_1 a_2$, i.e. using order (a_1, a_2, x_1, x_2).

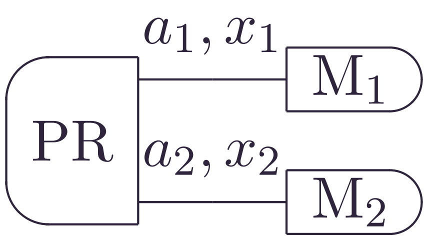
$$\Gamma^{\mathrm{PR}} = \{(0,0,0,0),(0,0,1,1),(0,1,0,1),(0,1,1,0),\\ (1,0,0,1),(1,0,1,0),(1,1,0,1),(1,1,1,0)\}$$

PR-Box Model

- There are two measurement gates with one input wire and binary valued random variable.

and constraints $\Gamma_x^a = \{(a, x)\}$.

- Consider the circuits



with $\mathrm{M}_1, \mathrm{M}_2 \in \{0, 1\}$.

- This gives the PR-Box correlations.

A Quantum Model

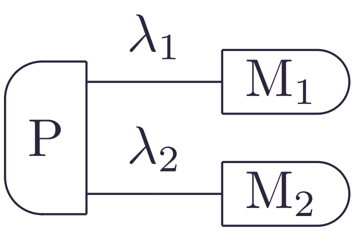
- Singlet correlations can be simulated using shared randomness and one PR-Box[1].
- Thus, we can convert the PR-Box model into a singlet model just by adding shared randomness and modifying the constraints.
  - Wires have two additional variables $\vec{\lambda}, \vec{\eta}$, which are 3D real unit vectors with a priori Haar measure.
  - Singlet and measurement gates need to impose some additional constraints relating the $\vec{\lambda}$'s and $\vec{\eta}$'s to the x's and a's. These are as in the CGMP model.
- Note: The measurement directions are not present in the ontic states.

[1] N. J. Cerf, N. Gisin, S. Massar and S. Popescu, PRL 94:220403 (2005)

Retrocausality

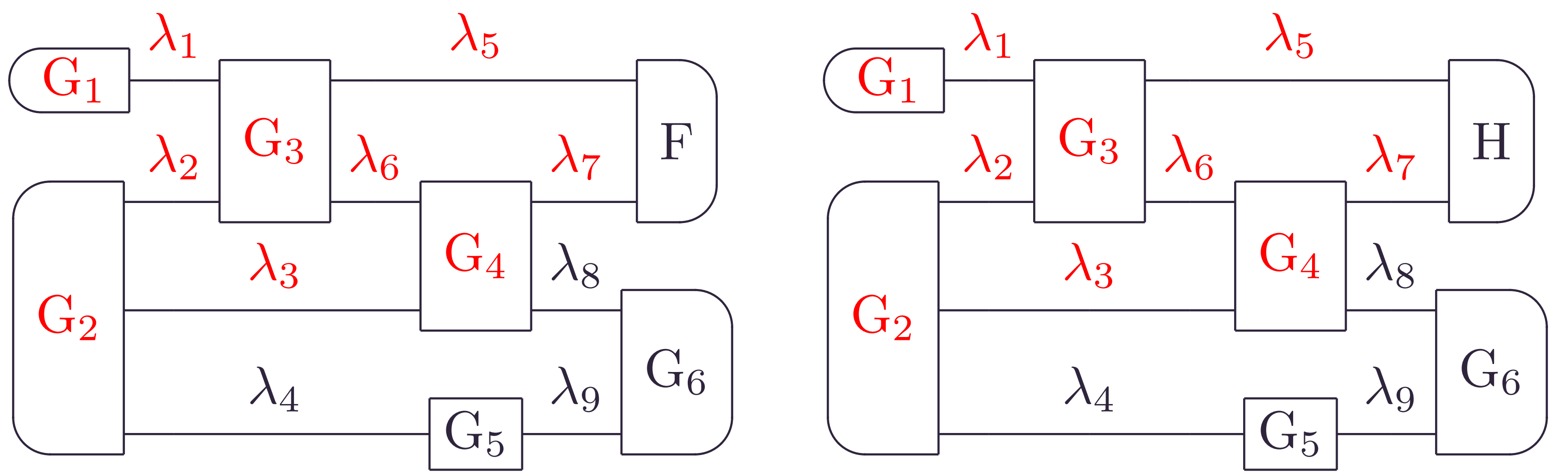
Causal Models

- A local constraint model is *causal* if, for every gate G_j, Γ^j is a function. This means that, ignoring the outcome, the transformation is a deterministic function from input to output.
- A local probabilistic constraint model is *causal* if the probabilities are nonzero only on such functional constraints. This means that the transformation acts like a stochastic transition from input to output.
- Causal models satisfy Bell's definition of local causality.



  - Constraint models become *deterministic* LHVTs.
  - Probabilistic constraint models become general (possibly stochastic) LHVTs.
- Thus we have a true generalization of Bell's local causality.

Identifying Retrocausality



$$\lambda^- = (\lambda_1, \lambda_2, \lambda_3, \lambda_5, \lambda_6, \lambda_7)$$

$$X^- = (X_1, X_2, X_3, X_4)$$

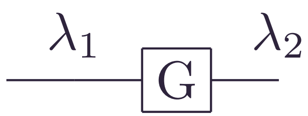
- A model is retrocausal if

$$P(\lambda^-, X^- | \mathrm{F}) \neq P(\lambda^-, X^- | \mathrm{H}).$$

- Causal models are not retrocausal, but the PR-Box and singlet models are.

Are Non-Causal Models Always Retrocausal?

- Causal models are not retrocausal, but are non-causal models always retrocausal?
  - No, because there could be an equivalent model that is causal.
  - Example:



    - Constraint model: $\Gamma^{\mathfrak{G}} = \{(0,0),(0,1),(1,0),(1,1)\}$.
    - Probabilistic causal model: $p(0|0) = p(1|0) = p(0|1) = p(1|1) = \frac{1}{2}$.
- Two models are equivalent if they have the same probabilities $P(\lambda, X_C|\mathfrak{G}_C)$.
- A model is *implicitly causal* if it has an equivalent causal model. Implicitly causal models are not retrocausal.
- Conjecture: Non implicitly causal models are retrocausal.

Huw Price's take on Bell's Theorem Revisited

- We can now take "Locality" in Huw's implication to mean generalized Bell locality. Then, if the conjecture is true, we would have

$$\text{QM} + \text{Locality} \Rightarrow \text{Non implicitly causal} \Rightarrow \text{Retrocausal}$$

Conclusions

Further Ideas and Open Questions

- Further ideas:
  - Can define generalized notions of noncontextuality, ψ-epistemic models, etc. Toy models that satisfy them can be found.
- Open questions:
  - Do communication complexity results block resolving excess baggage in these models?
  - Extension of Spekkens' toy theory.
  - Are there natural principles that can be used to derive quantum theory in this framework?
  - Formulating similar models in spacetime rather than circuits.